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A TEXTBOOK

ON

MARINE ENGINEERING

INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

ARITHMETIC
MENSURATION
MECHANICS
STEAM AND STEAM-BOILERS
WITH PRACTICAL QUESTIONS AND EXAMPLES

SCRANTON
INTERNATIONAL TEXTBOOK COMPANY
B–2

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NOTICE.

This set of volumes differs from those printed for the second edition by reason of the fact that Volumes IV and V, the Tables and Formulas and Key Volumes, have been placed under one cover, thus making four volumes in the set instead of five, as formerly. Otherwise there are no changes, except the correction of a few typographical errors.

PREFACE TO SECOND EDITION.

Owing to the introduction of new devices and new auxiliary machinery and changes in the Government regulations regarding the construction of marine boilers, we have completely revised the Marine Engineering Course and added considerable new matter to it in order to bring it up to date and to extend its scope. With the changes and additions that have been made, the volumes form the most complete and up-to-date treatise on the subject of marine engineering that has yet appeared.

The subject of marine engineering proper is treated in six Papers. All the matter appearing therein has been selected with special reference to the requirements of persons wishing to obtain government license as marine engineers, or desiring a higher-grade license for ocean, lake, bay, sound, and river steamers. These Papers give the theoretical knowledge that is essential in order to pass the examination for any grade of license up to and including that for chief engineer of the largest ocean steamers. The Paper on The Machinery of Western River Steamboats is of special value to persons engaged on the steamboats of the Southern and Western Rivers; it also gives to those holding license for ocean, lake, bay, and sound steamers the knowledge that is essential in order to take charge of the machinery of river steamboats.

A knowledge of the theory and practice of refrigeration is becoming imperative, owing to the rapidly extending use of refrigerating machines on ships; for this reason, a chapter on refrigeration has been inserted. The information contained in this chapter will enable the engineer to understand and intelligently direct the operation of refrigerating apparatus.

The purification of feed-water being a subject of vital importance, considerable space has been devoted to modern methods of purification, the neutralization of corrosive water, and the testing of feed-water.

With the exception of those portions that were revised, Volume I has been printed from the same plates that were used in the first edition.

Volume II, on the contrary, with the exception of the two Papers on Steam Engines, has been printed from new plates. As marine engineering practice is rapidly changing, we have thought best to provide for future revisions by making each paper and part independent of the others so far as article numbers, page numbers, figure numbers, etc., are concerned. To accomplish this we have given each paper and part in Volume II a number. This number has been placed at the top of each page on the headline, opposite the page number, and to distinguish it from the page number it is preceded by the printer's section mark (§). The object to be attained by using this mark is to make the index intelligible. All references in the index have two numbers, one referring to the section and the other to the page, consequently a reference such as § 11, page 14, may be readily found as follows: Look through the volume along the inside edges of the headlines until § 11 is found, then look through § 11 until page 14 is found. The Examination Questions in this volume have been arranged in the same way. They have been given the same section numbers as the Instruction Papers to which they belong and are grouped together at the end of the volume.

The volumes for this course are four in number.

Volume I contains the Instruction and Examination Questions on Arithmetic, Mensuration, Mechanics, and Steam and Steam Boilers.

Volume II (§§ 9–12 and §§ 28–30) contains the Instruction and Examination Questions on Steam Engines, The Machinery of Western River Steamboats, Recent Developments in Marine Engineering, and Dynamos and Motors.

Volume III contains the plates and the Instructions for drawing them. It forms a very complete course in Mechanical Drawing, and is the same in all respects as in the first edition.

Volume IV contains the principal tables, rules, and formulas that occur in the Instruction Papers of Volumes I and II. When the student has finished his Course this volume will be of considerable value to him. The tables and also the rules and formulas with the definitions of the letters used in the formulas are conveniently arranged for reference, so that the student can save the time and labor of searching for them in the Instruction Papers. This volume also contains the answers to the questions and the solutions to the examples in the Examination Questions.

INTERNATIONAL CORRESPONDENCE SCHOOLS.

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ARITHMETIC.

DEFINITIONS.

1. **Arithmetic** is the art of reckoning, or the study of numbers.

2. A **unit** is *one*, or a single thing, as *one*, *one* bolt, *one* pulley, *one* dozen.

3. A **number** is a unit or a collection of units, as *one*, *three* engines, *five* boilers.

4. The **unit of a number** is one of the collection of units which constitutes the number. Thus, the unit of *twelve* is *one*, of *twenty* dollars is *one* dollar, of *one hundred* bolts is *one* bolt.

5. A **concrete number** is a number applied to some particular kind of object or quantity, as three *grate bars*, five *dollars*, ten *pounds*.

6. An **abstract number** is a number that is not applied to any object or quantity, as *three*, *five*, *ten*.

7. **Like numbers** are numbers which express units of the *same kind*, as 6 *days* and 10 *days*, 2 *feet* and 5 *feet*.

8. **Unlike numbers** are numbers which express units of *different kinds*, as ten *months* and eight *miles*, seven *wrenches* and five *bolts*.

NOTATION AND NUMERATION.

9. Numbers are expressed in three ways: (1) by words; (2) by figures; (3) by letters.

10. **Notation** is the art of expressing numbers by figures or letters.

11. **Numeration** is the art of reading the numbers which have been expressed by figures or letters.

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12. The **Arabic notation** is the method of expressing numbers by figures. This method employs ten different **figures** to represent numbers, viz.:

Figures	0	1	2	3	4	5	6	7	8	9
Names	*naught,* *cipher* *or zero;*	*one*	*two*	*three*	*four*	*five*	*six*	*seven*	*eight*	*nine*

The first character (0) is called **naught, cipher,** or **zero,** and, when standing alone, has no value.

The other nine figures are called **digits,** and each one has a value of its own.

Any whole number is called an **integer.**

13. As there are only ten *figures* used in expressing numbers, each *figure* must express a different *value* at different times.

14. The value of a figure depends upon its *position* in relation to others.

15. Figures have **simple** values and **local** or **relative** values.

16. The **simple** value of a figure is the value it expresses when standing alone.

17. The **local** or **relative** value is the *increased* value it expresses by having other figures placed on its right.

For instance, if we see the figure 6 standing alone, thus. 6

we consider it as *six units*, or simply **six.**

Place another 6 to the *left* of it; thus. 66

The original figure is still *six units*, but the second one is *ten times* 6, or 6 **tens.**

If a third 6 be now placed still one place further to the *left*, it is increased in value *ten times* more, thus making it 6 **hundreds** 666

A fourth 6 would be 6 **thousands** 6666

A fifth 6 would be 6 **tens of thousands,** or **sixty thousand** 66666

A sixth 6 would be 6 **hundreds of thousands** . 666666

A seventh 6 would be 6 **millions** 6666666

The entire line of seven figures is read *six millions, six hundred sixty-six thousands, six hundred sixty-six.*

18. The **increased value** of each of these figures is its *local* or *relative* value. Each figure is *ten times* greater in value than the one immediately on its *right*.

19. The **cipher** (0) has no value itself, but it is useful in determining the place of other figures. To represent the number *four hundred five*, two digits only are necessary, one to represent *four hundred*, and the other to represent *five units;* but if these two digits are placed together, as 45, the 4 (being in the second place) will mean 4 *tens*. To mean 4 *hundreds*, the 4 should have two figures on its right, and a *cipher* is therefore inserted in the place usually given to *tens*, to show that the number is composed of *hundreds* and *units* only, and that there are no *tens*. *Four hundred five* is therefore expressed as 405. If the number were *four thousand and five*, two ciphers would be inserted; thus, 4005. If it were *four hundred fifty*, it would have the *cipher* at the right-hand side to show that there were no *units*, and only *hundreds and tens;* thus, 450. *Four thousand and fifty* would be expressed 4050, the first cipher indicating that there are no hundreds and the second that there are no units.

Note.—When speaking of the figures of a number by referring to them as first figure, second figure, etc., always begin to count at the *left*. Thus, in the number 41,625, 4 is the first figure, 6 the third figure, 5 the fifth or last figure, etc.

20. In *reading* figures, it is usual to point off the number into groups of three figures each, beginning with the right-hand or **units** column, a comma (,) being used to point off these groups.

Billions.			*Millions.*			*Thousands.*			*Units.*		
Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds of Units.	Tens of Units.	Units.
4	3	2,	1	9	8.	7	6	5,	4	3	2

In *pointing off* these figures, begin at the right-hand figure and count—*units*, *tens*, *hundreds;* the next group of three figures is *thousands*, therefore, we insert a comma (,) before beginning with them. Beginning at the figure 5, we say *thousands*, *tens of thousands*, *hundreds of thousands*, and insert another comma; we next read *millions*, *tens of millions*, *hundreds of millions*, and insert another comma; we then read *billions*, *tens of billions*, *hundreds of billions*.

The entire line of figures would be read: *Four hundred thirty-two billions*, *one hundred ninety-eight millions*, *seven hundred sixty-five thousands*, *four hundred thirty-two.* When we thus *read* a line of figures it is called **numeration,** and if the **numeration** be changed back to *figures*, it is called **notation.**

For instance, the writing of the figures,

72,584,623,

would be the **notation,** and the **numeration** would be *seventy-two millions*, *five hundred eighty-four thousands*, *six hundred twenty-three.*

21. NOTE.—It is customary to leave the *s* off the words millions, thousands, etc., in cases like the above, both in speaking and writing; hence, the above would usually be expressed, seventy-two million, five hundred eighty-four thousand, six hundred twenty-three.

22. The four fundamental processes of Arithmetic are **addition, subtraction, multiplication,** and **division.** They are called fundamental processes, because all operations in Arithmetic are based upon them.

ADDITION.

23. **Addition** is the *process* of *finding* the *sum* of *two* or *more* numbers. The sign of addition is $+$. It is read *plus*, and means *more.* Thus, $5+6$ is read 5 *plus* 6, and means that 5 and 6 are to be added.

24. The sign of equality is $=$. It is read *equals* or *is equal to.* Thus, $5+6=11$ may be read 5 plus 6 *equals* 11.

25. *Like numbers* can be added, but *unlike numbers* cannot. Thus, 6 dollars *can* be added to 7 dollars, and the *sum* will be 13 dollars, but 6 dollars *cannot* be added to 7 *feet.*

26. The following table gives the sum of any two numbers from 1 to 12:

TABLE 1.

1 and 1 are 2	2 and 1 are 3	3 and 1 are 4	4 and 1 are 5
1 and 2 are 3	2 and 2 are 4	3 and 2 are 5	4 and 2 are 6
1 and 3 are 4	2 and 3 are 5	3 and 3 are 6	4 and 3 are 7
1 and 4 are 5	2 and 4 are 6	3 and 4 are 7	4 and 4 are 8
1 and 5 are 6	2 and 5 are 7	3 and 5 are 8	4 and 5 are 9
1 and 6 are 7	2 and 6 are 8	3 and 6 are 9	4 and 6 are 10
1 and 7 are 8	2 and 7 are 9	3 and 7 are 10	4 and 7 are 11
1 and 8 are 9	2 and 8 are 10	3 and 8 are 11	4 and 8 are 12
1 and 9 are 10	2 and 9 are 11	3 and 9 are 12	4 and 9 are 13
1 and 10 are 11	2 and 10 are 12	3 and 10 are 13	4 and 10 are 14
1 and 11 are 12	2 and 11 are 13	3 and 11 are 14	4 and 11 are 15
1 and 12 are 13	2 and 12 are 14	3 and 12 are 15	4 and 12 are 16
5 and 1 are 6	6 and 1 are 7	7 and 1 are 8	8 and 1 are 9
5 and 2 are 7	6 and 2 are 8	7 and 2 are 9	8 and 2 are 10
5 and 3 are 8	6 and 3 are 9	7 and 3 are 10	8 and 3 are 11
5 and 4 are 9	6 and 4 are 10	7 and 4 are 11	8 and 4 are 12
5 and 5 are 10	6 and 5 are 11	7 and 5 are 12	8 and 5 are 13
5 and 6 are 11	6 and 6 are 12	7 and 6 are 13	8 and 6 are 14
5 and 7 are 12	6 and 7 are 13	7 and 7 are 14	8 and 7 are 15
5 and 8 are 13	6 and 8 are 14	7 and 8 are 15	8 and 8 are 16
5 and 9 are 14	6 and 9 are 15	7 and 9 are 16	8 and 9 are 17
5 and 10 are 15	6 and 10 are 16	7 and 10 are 17	8 and 10 are 18
5 and 11 are 16	6 and 11 are 17	7 and 11 are 18	8 and 11 are 19
5 and 12 are 17	6 and 12 are 18	7 and 12 are 19	8 and 12 are 20
9 and 1 are 10	10 and 1 are 11	11 and 1 are 12	12 and 1 are 13
9 and 2 are 11	10 and 2 are 12	11 and 2 are 13	12 and 2 are 14
9 and 3 are 12	10 and 3 are 13	11 and 3 are 14	12 and 3 are 15
9 and 4 are 13	10 and 4 are 14	11 and 4 are 15	12 and 4 are 16
9 and 5 are 14	10 and 5 are 15	11 and 5 are 16	12 and 5 are 17
9 and 6 are 15	10 and 6 are 16	11 and 6 are 17	12 and 6 are 18
9 and 7 are 16	10 and 7 are 17	11 and 7 are 18	12 and 7 are 19
9 and 8 are 17	10 and 8 are 18	11 and 8 are 19	12 and 8 are 20
9 and 9 are 18	10 and 9 are 19	11 and 9 are 20	12 and 9 are 21
9 and 10 are 19	10 and 10 are 20	11 and 10 are 21	12 and 10 are 22
9 and 11 are 20	10 and 11 are 21	11 and 11 are 22	12 and 11 are 23
9 and 12 are 21	10 and 12 are 22	11 and 12 are 23	12 and 12 are 24

This table should be carefully committed to memory. Since 0 has no value, the sum of any number and 0 is the number itself; thus, 17 and 0 are 17.

27. For *addition*, place the numbers to be added directly under each other, taking care to place *units* under *units*, *tens* under *tens*, *hundreds* under *hundreds*, and so on.

When the numbers are thus written, the *right-hand figure* of *one number* is placed *directly under the right-hand figure* of the *number above it*, thus bringing the unit figures of all the numbers to be added in the same vertical line. Proceed as in the following examples:

28. **Example.—What is the sum of 181, 222, 21, 2, and 418?**

Solution.—

```
       181
       222
        21
         2
       418
       ———
  sum  789  Ans.
```

Explanation.—After placing the numbers in proper order, begin at the bottom of the right-hand or *units* column, and add, mentally repeating the different sums. Thus, three and two are five and one are six and two are eight and one are nine, the sum of the numbers in *units* column. Place the 9 directly beneath as the first or *units* figure in the sum.

The sum of the numbers in the next or *tens* column equals 8 *tens*, which is the second or *tens* figure in the sum.

The sum of the numbers in the next or *hundreds* column equals 7 *hundreds*, which is the third or *hundreds* figure in the sum.

The sum or answer is 789.

29. **Example.—What is the sum of 425, 86, 9,215, 4, and 907?**

Solution.—

```
        425
         86
       9215
          4
        907
      —————
         27
         60
       1500
       9000
      —————
  sum 10587  Ans.
```

Explanation.—The sum of the numbers in the first or units column is seven and four are eleven and five are sixteen and six are twenty-two and five are twenty-seven, or 27 units; i. e., two tens and seven units. Write 27 as shown.

The sum of the numbers in the second or tens column is six tens, or 60. Write 60 underneath 27 as shown. The sum of the numbers in the third or hundreds column is 15 hundreds, or 1,500. Write 1,500 under the two preceding results as shown. There is only one number in the fourth or thousands column, nine, which represents 9,000. Write 9,000 under the three preceding results. Adding these four results, the sum is 10,587, which is the sum of 425, 36, 9,215, 4, and 907.

NOTE.—It frequently happens, when adding a long column of figures, that the sum of two numbers, one of which does not occur in the addition table, is required. Thus, in the first column above, the sum of 16 and 6 was required. We know from the table that 6 + 6 = 12; hence, the first figure of the sum is 2. Now, the sum of any number less than 20 and of any number less than 10 must be less than thirty, since 20 + 10 = 30; therefore, the sum is 22. Consequently, in cases of this kind, add the first figure of the larger number to the smaller number and, if the result is greater than 9, increase the second figure of the larger number by 1. Thus, 44 + 7 = ? 4 + 7 = 11; hence, 44 + 7 = 51.

30. The addition may also be performed as follows:

```
       425
        36
      9215
         4
       907
      ----
sum  10587 Ans.
```

EXPLANATION.—The sum of the numbers in *units* column = 27 *units*, or 2 *tens* and 7 *units*. Write the 7 *units* as the first or right-hand figure in the sum. Reserve the two *tens* and add them to the figures in *tens* column. The sum of the figures in the *tens* column, plus the 2 *tens* reserved and carried from the *units* column = 8, which is written down as the second figure in the sum. There is nothing to carry to the next column, because 8 is less than 10. The sum of the numbers in the next column is 15 *hundreds*, or 1 *thousand* and 5 *hundreds*. Write down the 5 as the third or *hundreds* figure in the sum and carry the 1 to the next column. 1 + 9 = 10, which is written down at the left of the other figures.

The second method saves space and figures, but the first is to be preferred when adding a long column.

31. **Example.**—Add the numbers in the column below.

Solution.—

$$
\begin{array}{r}
890 \\
82 \\
90 \\
393 \\
281 \\
80 \\
770 \\
83 \\
492 \\
80 \\
383 \\
84 \\
191 \\
\hline
\textit{sum}\ 3899
\end{array}
$$
Ans.

Explanation.—The sum of the digits in the first column equals 19 *units*, or 1 *ten* and 9 *units*. Write down the 9 and carry 1 to the next column. The sum of the digits in the second column + 1 = 109 *tens*, or 10 *hundreds* and 9 *tens*. Write down the 9 and carry the 10 to the next column. The sum of the digits in this column plus the 10 reserved = 38. The entire sum is 3,899.

32. **Rule 1.**—(*a*) *Begin at the right, add each column separately, and write the sum, if it be only one figure, under the column added.*

(*b*) *If the sum of any column consists of two or more figures, put the right-hand figure of the sum under that column, and add the remaining figure or figures to the next column.*

33. **Proof.**—*To prove addition, add each column from top to bottom. If you obtain the same result as by adding from bottom to top, the work is probably correct.*

EXAMPLES FOR PRACTICE.

34. Find the sum of

	Ans.
(*a*) 104 + 203 + 613 + 214.	(*a*) 1,134.
(*b*) 1,875 + 3,143 + 5,826 + 10,832.	(*b*) 21,676.
(*c*) 4,865 + 2,145 + 8,173 + 40,084.	(*c*) 55,267.
(*d*) 14,204 + 8,173 + 1,065 + 10,042.	(*d*) 33,484.

(*e*) 10,832 + 4,145 + 8,133 + 5,872.	Ans.	(*e*) 28,982.
(*f*) 214 + 1,231 + 141 + 5,000.		(*f*) 6,586.
(*g*) 123 + 104 + 425 + 126 + 327.		(*g*) 1,105.
(*h*) 6,354 + 2,145 + 2,042 + 1,111 + 3,333.		(*h*) 14,985.

1. A week's record of coal burned in an engine room is as follows: Monday, 1,800 pounds; Tuesday, 1,655 pounds; Wednesday, 1,725 pounds; Thursday, 1,690 pounds; Friday, 1,648 pounds; Saturday, 1,020 pounds. How much coal was burned during the week?
Ans. 9,538 pounds.

2. A steam pump pumps out of a cistern in one hour 4,200 gallons; in the next hour, 5,420 gallons, and in 45 minutes more, an additional 3,600 gallons, when the cistern becomes empty. How many gallons were in the cistern at first? Ans. 13,220 gallons.

3. What is the total cost of a steam plant, the several items of expense being as follows: Steam engine, $900; boiler, $775; fittings and connections, $225; erecting the plant, $125; engine house, $650?
Ans. $2,675.

SUBTRACTION.

35. In Arithmetic, **subtraction** is the process of finding how much greater one number is than another.

The greater of the two numbers is called the **minuend.**

The smaller of the two numbers is called the **subtrahend.**

The number left after subtracting the *subtrahend* from the *minuend* is called the **difference** or **remainder.**

36. The sign of subtraction is —. It is read **minus,** and means *less*. Thus, 12 — 7 is read 12 *minus* 7, and means that 7 is to be taken from 12.

37. EXAMPLE.—From 7,568 take 3,425.

SOLUTION.—

$$\begin{array}{rr} \textit{minuend} & 7568 \\ \textit{subtrahend} & 3425 \\ \hline \textit{remainder} & 4143 \end{array} \quad \text{Ans.}$$

EXPLANATION.—Begin at the right-hand or *units* column and subtract in succession each figure in the subtrahend from the one directly above it in the minuend, and write the remainders below the line. The result is the entire remainder.

38. When there are more figures in the *minuend* than in the *subtrahend*, and when some figures in the *minuend* are *less* than the figures directly under them in the *subtrahend*, proceed as in the following example:

EXAMPLE.—From 8,453 take 844.

SOLUTION.—
minuend 8453
subtrahend 844
remainder 7609 Ans.

EXPLANATION.—Begin at the right-hand or *units* column to subtract. We can not take 4 from 3, and must, therefore, borrow 1 from 5 in *tens* column and annex it to the 3 in *units* column. The 1 *ten* = 10 *units*, which added to the 3 in the *units* column = 13 *units*. 4 from 13 = 9, the first or *units* figure in the remainder.

Since we borrowed 1 from the 5, only 4 remains; 4 from 4 = 0, the second or *tens* figure. We can not take 8 from 4, so borrow 1 *thousand* or 10 *hundreds* from 8; 10 *hundreds* + 4 *hundreds* = 14 *hundreds*.

8 from 14 = 6, the third or *hundreds* figure in the remainder.

Since we borrowed 1 from 8 only 7 remains, from which there is nothing to subtract; therefore, 7 is the next figure in the remainder or answer.

The operation of borrowing is placing 1 before the figure following the one from which it is borrowed. In the above example the one borrowed from 5 is placed before 3, making it 13, from which we subtract 4. The 1 borrowed from 8 is placed before 4, making 14, from which 8 is taken.

39. EXAMPLE.—Find the difference between 10,000 and 8,763.

SOLUTION.—
minuend 10000
subtrahend 8763
remainder 1237 Ans.

EXPLANATION.—In the above example we borrow 1 from the second column and place it before 0, making 10; 3 from 10 = 7. In the same way we borrow 1 and place it before the next cipher, making 10; but as we have borrowed 1 from this column and taken it to the *units* column, only 9 remains, from which to subtract 6, 6 from 9 = 3. For the same reason we subtract 7 from 9 and 8 from 9 for the next two figures, and obtain a total remainder of 1,237.

40. Rule 2.—*Place the subtrahend or smaller number under the minuend or larger number, in the same manner as for addition, and proceed as in Arts.* ***37, 38,*** *and* ***39.***

41. Proof.—*To prove an example in subtraction, add the subtrahend and remainder. The sum should equal the minuend. If it does not, a mistake has been made, and the work should be done over.*

Proof of the above example:

subtrahend	8763
remainder	1237
minuend	10000

EXAMPLES FOR PRACTICE.

42. From

(*a*) 94,278 take 62,574.		(*a*) 31,704.
(*b*) 53,714 take 25,824.		(*b*) 27,890.
(*c*) 71,832 take 58,109.		(*c*) 13,723.
(*d*) 20,804 take 10,408.	Ans.	(*d*) 10,396.
(*e*) 310,465 take 102,141.		(*e*) 208,324.
(*f*) (81,043 + 1,041) take 14,831.		(*f*) 67,253.
(*g*) (20,482 + 18,216) take 21,214.		(*g*) 17,484.
(*h*) (2,040 + 1,213 + 542) take 3,791.		(*h*) 4.

1. A cistern is fed by two pipes which supply 1,200 and 2,250 gallons per hour, respectively, and is being emptied by a pump which delivers 5,800 gallons per hour. Starting with 8,000 gallons in the cistern, how much water does it contain at the end of an hour? Ans. 5,650 gallons.

2. A train in running from New York to Buffalo travels 38 miles the first hour, 42 the second, 39 the third, 56 the fourth, 52 the fifth, and 48 the sixth hour. How many miles remain to be traveled at the end of the sixth hour, the distance between the two places being 410 miles? Ans. 135 miles.

8. On Monday morning a bank had on hand $2,862. During the day $1,831 were deposited and $2,172 drawn out; on Tuesday, $3,126 were deposited, and $1,954 drawn out. How many dollars were on hand Wednesday morning? Ans. $3,693.

MULTIPLICATION.

43. To **multiply** a number is to *add* it to itself a certain number of times.

44. **Multiplication** is the process of multiplying one number by another.

The *number* thus added to itself, or the number to be multiplied, is called the **multiplicand.**

The *number* which shows how many times the *multiplicand* is to be taken, or the *number* by which we *multiply*, is called the **multiplier.**

The result obtained by multiplying is called the **product.**

45. The sign of multiplication is ×. It is read *times* or *multiplied by*. Thus, 9 × 6 is read 9 *times* 6, or 9 *multiplied by* 6.

46. It matters not in what order the numbers to be multiplied together are placed. Thus, 6 × 9 is the same as 9 × 6.

47. In the following table, the product of any two numbers (neither of which exceeds twelve) may be found:

TABLE 2.

1 times 1 is 1	2 times 1 are 2	3 times 1 are 3
1 times 2 are 2	2 times 2 are 4	3 times 2 are 6
1 times 3 are 3	2 times 3 are 6	3 times 3 are 9
1 times 4 are 4	2 times 4 are 8	3 times 4 are 12
1 times 5 are 5	2 times 5 are 10	3 times 5 are 15
1 times 6 are 6	2 times 6 are 12	3 times 6 are 18
1 times 7 are 7	2 times 7 are 14	3 times 7 are 21
1 times 8 are 8	2 times 8 are 16	3 times 8 are 24
1 times 9 are 9	2 times 9 are 18	3 times 9 are 27
1 times 10 are 10	2 times 10 are 20	3 times 10 are 30
1 times 11 are 11	2 times 11 are 22	3 times 11 are 33
1 times 12 are 12	2 times 12 are 24	3 times 12 are 36
4 times 1 are 4	5 times 1 are 5	6 times 1 are 6
4 times 2 are 8	5 times 2 are 10	6 times 2 are 12
4 times 3 are 12	5 times 3 are 15	6 times 3 are 18
4 times 4 are 16	5 times 4 are 20	6 times 4 are 24
4 times 5 are 20	5 times 5 are 25	6 times 5 are 30
4 times 6 are 24	5 times 6 are 30	6 times 6 are 36
4 times 7 are 28	5 times 7 are 35	6 times 7 are 42
4 times 8 are 32	5 times 8 are 40	6 times 8 are 48
4 times 9 are 36	5 times 9 are 45	6 times 9 are 54
4 times 10 are 40	5 times 10 are 50	6 times 10 are 60
4 times 11 are 44	5 times 11 are 55	6 times 11 are 66
4 times 12 are 48	5 times 12 are 60	6 times 12 are 72
7 times 1 are 7	8 times 1 are 8	9 times 1 are 9
7 times 2 are 14	8 times 2 are 16	9 times 2 are 18
7 times 3 are 21	8 times 3 are 24	9 times 3 are 27
7 times 4 are 28	8 times 4 are 32	9 times 4 are 36
7 times 5 are 35	8 times 5 are 40	9 times 5 are 45
7 times 6 are 42	8 times 6 are 48	9 times 6 are 54
7 times 7 are 49	8 times 7 are 56	9 times 7 are 63
7 times 8 are 56	8 times 8 are 64	9 times 8 are 72
7 times 9 are 63	8 times 9 are 72	9 times 9 are 81
7 times 10 are 70	8 times 10 are 80	9 times 10 are 90
7 times 11 are 77	8 times 11 are 88	9 times 11 are 99
7 times 12 are 84	8 times 12 are 96	9 times 12 are 108
10 times 1 are 10	11 times 1 are 11	12 times 1 are 12
10 times 2 are 20	11 times 2 are 22	12 times 2 are 24
10 times 3 are 30	11 times 3 are 33	12 times 3 are 36
10 times 4 are 40	11 times 4 are 44	12 times 4 are 48
10 times 5 are 50	11 times 5 are 55	12 times 5 are 60
10 times 6 are 60	11 times 6 are 66	12 times 6 are 72
10 times 7 are 70	11 times 7 are 77	12 times 7 are 84
10 times 8 are 80	11 times 8 are 88	12 times 8 are 96
10 times 9 are 90	11 times 9 are 99	12 times 9 are 108
10 times 10 are 100	11 times 10 are 110	12 times 10 are 120
10 times 11 are 110	11 times 11 are 121	12 times 11 are 132
10 times 12 are 120	11 times 12 are 132	12 times 12 are 144

This table should be carefully committed to memory.

Since 0 has no value, the product of 0 and any number is 0.

48. To multiply a number by one figure only:

Example.—Multiply 425 by 5.

Solution.— *multiplicand* 425
multiplier 5
product 2125 Ans.

Explanation.—For convenience, the *multiplier* is generally written *under* the *right-hand figure* of the *multiplicand*. On looking in the multiplication table, we see that 5 × 5 are 25. *Multiplying* the *first figure* at the *right* of the *multiplicand*, or 5, by the *multiplier* 5, it is seen that 5 times 5 units are 25 units, or 2 tens and 5 units. Write the 5 *units* in *units place* in the *product*, and *reserve* the 2 tens to *add* to the *product* of *tens*. Looking in the multiplication table again, we see that 5 × 2 are 10. *Multiplying* the *second figure* of the *multiplicand* by the *multiplier* 5, we see *that* 5 *times* 2 tens are 10 tens, *plus* the 2 tens *reserved*, are 12 tens, or 1 hundred plus 2 tens. Write the *2 tens* in tens place, and *reserve* the 1 hundred to *add* to the product of *hundreds*. Again, we see by the multiplication table that 5 × 4 are 20. *Multiplying* the *third* or *last figure* of the *multiplicand* by the *multiplier* 5, we see that 5 times 4 hundreds are 20 hundreds, *plus* the 1 hundred *reserved*, are 21 hundreds, or 2 thousands *plus* 1 hundred, which we write in *thousands* and *hundreds places*, respectively.

Hence, the *product* is 2,125.

This *result* is the same as adding 425 five times. Thus,

425
425
425
425
425
sum 2125 Ans.

EXAMPLES FOR PRACTICE.

49. Find the product of

(*a*)	61,483 × 6.	Ans. (*a*)	368,898.
(*b*)	12,375 × 5.	(*b*)	61,875.
(*c*)	10,426 × 7.	(*c*)	72,982.
(*d*)	10,835 × 3.	(*d*)	32,505.

(*e*) 98,376 × 4.	Ans.	(*e*) 393,504.
(*f*) 10,873 × 8.		(*f*) 86,984.
(*g*) 71,543 × 9.		(*g*) 643,887.
(*h*) 218,734 × 2.		(*h*) 437,468.

1. A stationary engine makes 5,520 revolutions per hour. Running 9 hours a day, 5 days in the week, and 5 hours on Saturday, how many revolutions would it make in 4 weeks? Ans. 1,104,000 revolutions.

2. An engineer earns $650 a year, and his average expenses are $548. How much could he save in 8 years at that rate? Ans. $816.

3. The connection between an engine and boiler is made up of 5 lengths of pipe, three of which are 12 feet long, one 2 feet 6 inches long, and one 8 feet 6 inches long. If the pipe weighs 9 pounds per foot, what is the total weight of the pipe used? Ans. 423 pounds.

50. To multiply a number by two or more figures:

EXAMPLE.—Multiply 475 by 234.

SOLUTION.—

$$\begin{array}{rr} \textit{multiplicand} & 475 \\ \textit{multiplier} & 234 \\ \hline & 1900 \\ & 1425 \\ & 950 \\ \hline \textit{product} & 111150 \end{array}$$ Ans.

EXPLANATION.—For convenience, the *multiplier* is generally written *under* the *multiplicand*, placing units under units, tens under tens, etc.

We *can not* multiply by 234 at one operation; we must, therefore, *multiply* by the *parts* and then *add* the **partial products.**

The parts by which we are to multiply are 4 units, 3 tens, and 2 hundreds. 4 times 475 = 1,900, the *first partial product;* 3 times 475 = 1,425, the *second partial product*, the *right-hand figure* of which is *written directly under* the *figure multiplied by*, or 3; 2 times 475 = 950, the *third partial product*, the *right-hand figure* of which is *written directly under* the *figure multiplied by*, or 2.

The sum of these *three partial products* is 111,150, which is the *entire product.*

51. Rule 3.—(**a**) *Write the multiplier under the multiplicand, so that units are under units, tens under tens, etc.*

(**b**) *Begin at the right and multiply each figure of the multiplicand by each successive figure of the multiplier, placing the right-hand figure of each partial product directly under the figure used as a multiplier.*

(**c**) *The sum of the partial products will equal the required product.*

52. Proof.—*Review the work carefully, or multiply the multiplier by the multiplicand; if the results agree, the work is correct.*

53. When there is a *cipher* in the *multiplier*, multiply the entire multiplicand by it; since the result will be zero, place a cipher under the cipher in the multiplier. Thus,

$$\begin{array}{r} (a)\\ 0\\ \times 0\\ \hline 0 \end{array}\text{ Ans.}\qquad \begin{array}{r} (b)\\ 2\\ \times 0\\ \hline 0 \end{array}\text{ Ans.}\qquad \begin{array}{r} (c)\\ 15\\ \times\ 0\\ \hline 0 \end{array}\text{ Ans.}\qquad \begin{array}{r} (d)\\ 708\\ \times\ \ 0\\ \hline 0 \end{array}\text{ Ans.}$$

$$\begin{array}{r} (e)\\ 3114\\ 203\\ \hline 9342\\ 62280\\ \hline 632142 \end{array}\text{ Ans.}\qquad \begin{array}{r} (f)\\ 4008\\ 305\\ \hline 20040\\ 120240\\ \hline 1222440 \end{array}\text{ Ans.}\qquad \begin{array}{r} (g)\\ 31264\\ 1002\\ \hline 62528\\ 3126400\\ \hline 31326528 \end{array}\text{ Ans.}$$

In examples (*e*), (*f*), and (*g*), we multiply by 0 as directed above; then multiply by the next figure of the multiplier and place the first figure of the product alongside the 0, as shown.

EXAMPLES FOR PRACTICE.

54. Find the product of

(*a*) 3,842 × 26.	Ans.	(*a*)	99,892.
(*b*) 3,716 × 45.		(*b*)	167,220.
(*c*) 1,817 × 124.		(*c*)	225,308.
(*d*) 675 × 38.		(*d*)	25,650.

		Ans.
(*e*)	1,875 × 33.	61,875.
(*f*)	4,836 × 47.	227,292.
(*g*)	5,682 × 543.	3,085,326.
(*h*)	3,257 × 246.	801,222.
(*i*)	2,875 × 302.	868,250.
(*j*)	17,819 × 1,004.	17,890,276.
(*k*)	38,674 × 205.	7,928,170.
(*l*)	18,304 × 100.	1,830,400.
(*m*)	7,834 × 10.	78,340.
(*n*)	87,543 × 1,000.	87,543,000.
(*o*)	48,763 × 100.	4,876,300.

1. If the area of a steam-engine piston is 113 square inches, what is the total pressure upon it when the steam pressure is 85 pounds per square inch? Ans. 9,605 pounds.

2. A steam engine, which indicated 164 horsepower, was found to consume 4 pounds of coal per horsepower per hour. Being replaced by a new engine, which was of the same horsepower as the other, another test was made, which showed a consumption of 3 pounds per horsepower per hour. What was the saving of coal for a year of 309 days, if the engine averaged to run 14 hours a day? Ans. 709,464 pounds.

3. Two steamers are 7,846 miles apart, and are sailing towards each other, one at the rate of 18 miles an hour, and the other at the rate of 15 miles an hour. How far apart will they be at the end of 205 hours? Ans. 1,081 miles.

DIVISION.

55. Division is the process of finding how many times one number is contained in another of the same kind.

The number to be *divided* is called the **dividend.**

The number by which we *divide* is called the **divisor.**

The number which *shows* how many times the *divisor* is *contained* in the *dividend* is called the **quotient.**

56. The sign of division is ÷. It is read *divided by*. $54 \div 9$ is read 54 *divided by* 9. Another way to write 54 *divided by* 9 is $\frac{54}{9}$. Thus, $54 \div 9 = 6$, or $\frac{54}{9} = 6$.

In both of these cases 54 is the *dividend* and 9 is the *divisor*.

Division is the *reverse* of multiplication.

57. To divide when the divisor consists of but one figure, proceed as in the following example:

Example.—What is the quotient of 875 ÷ 7?

divisor dividend quotient

Solution.— 7)875(125 Ans.

```
 7)875(125 Ans.
   7
   17
   14
   ——
    35
    35
    ——
remainder 0
```

Explanation.—7 is contained in 8 *hundreds* 1 *hundred* times. Place the one as the first, or left-hand, figure of the quotient. Multiply the divisor 7 by the 1 *hundred* of the quotient, and place the product 7 *hundreds* under the 8 *hundreds* in the dividend, and subtract. Beside the remainder 1, bring down the 7 *tens*, making 17 *tens;* 17 divided by 7 = 2 times. Write the two as the second figure of the quotient. Multiply the divisor 7 by the 2, and subtract the product from 17. Beside the remainder 3, bring down the 5 *units* of the dividend, making 35 *units.* 7 is contained in 35, 5 times, which is placed in the quotient. Multiplying the divisor by the last figure of the quotient, 5 times 7 = 35, which subtracted from 35, under which it is placed, leaves 0. Therefore, the quotient is 125. This method is called **long division.**

58. In **short division,** only the divisor, dividend, and quotient are written, the operations being performed mentally.

dividend

divisor $7\,)\,8^{1}7^{3}5$

quotient 1 2 5 Ans.

The mental operation is as follows: 7 is contained in 8, once and one remainder; 1 placed before 7 makes [illegible] 7 is contained in 17, 2 times and 3 over; the 3 place[illegible] 5 makes 35; 7 is contained in 35, 5 times. These p[illegible] tients placed in order as they are found, make [illegible] quotient 125.

The small figures are placed in the example given to better illustrate the explanation; they are never written when actually performing division in this way.

59. If the *divisor* consists of *2 or more* figures, proceed as in the following example:

Example.—Divide 2,702,826 by 63.

```
               divisor    dividend    quotient
SOLUTION.—        63 ) 2 7 0 2 8 2 6 ( 4 2 9 0 2  Ans.
                       2 5 2
                       -----
                         1 8 2
                         1 2 6
                         -----
                           5 6 8
                           5 6 7
                           -----
                               1 2 6
                               1 2 6
                               -----
                                   0
```

Explanation.—As 63 is not contained in the first two figures, 27, we must use the first three figures, 270. Now, by trial, we must find how many times 63 is contained in 270; 6 is contained in the first two figures of 270, 4 times. Place the 4 as the first or left-hand figure in the quotient. Multiply the divisor 63 by 4, and subtract the product 252 from 270. The remainder is 18, beside which we write the next figure of the dividend, 2, making 182. Now, 6 is contained in the first two figures of 182, 3 times, but on multiplying 63 by 3, we see that the product 189 is too great, so we try 2 as the second figure of the quotient. Multiplying the divisor 63 by 2, and subtracting the product 126 from 182, the remainder is 56, beside which we bring down the next figure of the dividend, making 568; 6 is contained in 56 about 9 times. Multiply the divisor 63 by 9 and subtract the product 567 from 568. The remainder is 1, and bringing down the next figure of the dividend, 2, gives 12. As 12 is smaller than 63, we write 0 in the quotient and bring down the next figure, 6, making 126. 63 is contained in 126, 2 times, without a remainder. Therefore, 42,902 is the quotient.

60. Rule 4.—(**a**) *Write the divisor at the left of the dividend, with a line between them.*

(**b**) *Find how many times the divisor is contained in the lowest number of the left-hand figures of the dividend that will contain it, and write the result at the right of the dividend, with a line between, for the first figure of the quotient.*

(**c**) *Multiply the divisor by this quotient; write the product under the partial dividend used, and subtract, annexing to the remainder the next figure of the dividend. Divide as before, and thus continue until all the figures of the dividend have been used.*

(**d**) *If any partial dividend will not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend and proceed as before.*

(**e**) *If there be a remainder at last, write it after the quotient, with the divisor underneath.*

61. Proof.—*Multiply the quotient by the divisor, and add the remainder, if there be any, to the product. The result will be the dividend.*

divisor dividend quotient
Thus, 63)4285(67$\frac{14}{63}$ Ans.
378
455
441
remainder 14
Proof, *quotient* 67
divisor 63
201
402
4221
remainder 14
dividend 4285

EXAMPLES FOR PRACTICE.

62. Divide the following:

(*a*) 126,498 by 58.	Ans. (*a*) 2,181.
(*b*) 3,207,594 by 767.	(*b*) 4,182.
(*c*) 11,408,202 by 234.	(*c*) 48,753.
(*d*) 2,100,315 by 581.	(*d*) 3,615.
(*e*) 969,936 by 4,008.	(*e*) 242.
(*f*) 7,481,888 by 1,021.	(*f*) 7,328.
(*g*) 1,525,915 by 5,003.	(*g*) 305.
(*h*) 1,646,301 by 381.	(*h*) 4,321.

1. In a mile there are 5,280 feet. How many rails would it take to lay a double row one mile long, each rail being 30 feet long? Ans. 352 rails.

2. How many rivets will be required for the longitudinal seams of a cylindrical boiler 20 feet long, the joint being double riveted, and the rivets being spaced 4 inches apart? Ans. 120 rivets.

NOTE.—First find the length of the boiler in inches.

3. It requires 7,020,000 bricks to build a large foundry. How many teams will it require to draw the bricks in 60 days, if each team draws 6 loads per day and 1,500 bricks at a load? Ans. 13 teams.

NOTE.—Find how many loads 7,020,000 bricks make; then, how many days it will take one team to draw the brick.

CANCELATION.

63. Cancelation is the process of shortening operations in division by casting out equal factors from both dividend and divisor.

64. The **factors** of a number are *those numbers* which, when *multiplied* together, will *equal that number*. Thus, 5 and 3 are factors of 15, since 5 × 3 = 15. Likewise, 8 and 7 are the factors of 56, since 8 × 7 = 56.

65. A **prime number** is one which can not be divided by any number except itself and 1. Thus, 2, 3, 11, 29, etc., are prime numbers.

66. A **prime factor** is any factor that is a prime number.

Any number that is not a prime is called a **composite** number, and may be produced by multiplying together its prime factors. Thus, 60 is a composite number, and is equal to the product of its prime factors, 2 × 2 × 3 × 5.

Numbers are said to be **prime to each other** when no two of them can be divided by any number except 1; the numbers themselves *may* be either prime or composite. Thus, the numbers 3, 5, and 11 are prime to each other, so also are 22, 25, and 21 — all composite numbers.

67. Canceling *equal factors* from *both dividend and divisor* does *not* change the *quotient.*

The *canceling* of a *factor* in *both dividend and divisor* is the *same* as *dividing them both* by the *same number*, which, by the principle of division, does not *change the quotient.*

Write the *numbers* which make the *dividend* above the *line,* and those which make the *divisor* below it.

68. EXAMPLE.—Divide 4 × 45 × 60 by 9 × 24.

SOLUTION.—Placing the dividend over the divisor, and canceling

$$\frac{\cancel{4} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\cancel{9} \times \underset{\underset{1}{\cancel{6}}}{\cancel{24}}} = \frac{50}{1} = 50. \quad \text{Ans.}$$

EXPLANATION.—The 4 in the dividend and 24 in the divisor are both divisible by 4, since 4 divided by 4 equals 1, and 24 divided by 4 equals 6. Cross off the four and write the 1 over it; also, cross off the 24 and write the 6 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times 45 \times 60}{9 \times \underset{6}{\cancel{24}}}$$

60 in the dividend and 6 in the divisor are divisible by 6, since 60 divided by 6 equals 10, and 6 divided by 6 equals 1. Cross off the 60 and write 10 over it; also, cross off the 6 and write 1 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times 45 \times \overset{10}{\cancel{60}}}{9 \times \underset{\underset{1}{\cancel{6}}}{\cancel{24}}}$$

Again, 45 in the dividend and 9 in the divisor are divisible by 9, since 45 divided by 9 equals 5, and 9 divided by 9 equals 1. Cross off the 45 and write the 5 over it; also, cross off the 9 and write the 1 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{\underset{1}{\cancel{6}}}{\cancel{24}}}$$

Since there are no two remaining numbers (one in the dividend and one in the divisor) divisible by any number except 1, without a remainder, it is impossible to cancel further.

Multiply all the uncanceled numbers in the dividend together, and divide their product by the product of all the uncanceled numbers in the divisor. The result will be the quotient. The product of all the uncanceled numbers in the dividend equals 5 × 1 × 10 = 50; the product of all the uncanceled numbers in the divisor equals 1 × 1 = 1.

$$\text{Hence, } \frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{\underset{1}{\cancel{6}}}{\cancel{24}}} = \frac{1 \times 5 \times 10}{1 \times 1} = 50. \quad \text{Ans.}$$

It is usual to omit the 1's when canceling them, instead of writing them as above.

69. Rule 5.—(*a*) *Cancel the common factors from both the dividend and divisor.*

(*b*) *Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

EXAMPLES FOR PRACTICE.

70. Divide

		Ans.
(*a*)	14 × 18 × 16 × 40 by 7 × 8 × 6 × 5 × 8.	(*a*) 32.
(*b*)	8 × 65 × 50 × 100 × 60 by 30 × 60 × 13 × 10.	(*b*) 250.
(*c*)	8 × 4 × 3 × 9 × 11 by 11 × 9 × 4 × 3 × 8.	(*c*) 1.
(*d*)	164 × 321 × 6 × 7 × 4 by 82 × 321 × 7.	(*d*) 48.
(*e*)	50 × 100 × 200 × 72 by 1,000 × 144 × 100.	(*e*) 5.
(*f*.)	48 × 63 × 55 × 49 by 7 × 21 × 11 × 48.	(*f*) 105.
(*g*)	110 × 150 × 84 × 32 by 11 × 15 × 100 × 64.	(*g*) 42.
(*h*)	115 × 120 × 400 × 1,000 by 23 × 1,000 × 60 × 800.	(*h*) 5.

FRACTIONS.

71. A **fraction** is a *part* of a *whole number: One-half, one-third, two-fifths* are fractions.

72. *Two* numbers are required to express a fraction, one called the **numerator,** and the other the **denominator.**

73. The *numerator* is placed above the *denominator*, with a *line* between them; as, $\frac{2}{3}$. 3 is the *denominator*, and shows into how many *equal parts* the *unit* or *one* is divided. The *numerator* 2 shows how many of these *equal parts* are taken or considered. The *denominator* also indicates the *names* of the parts.

$\frac{1}{2}$ is read one-half.
$\frac{3}{4}$ is read three-fourths.
$\frac{3}{8}$ is read three-eighths.
$\frac{5}{16}$ is read five-sixteenths.
$\frac{29}{47}$ is read twenty-nine-forty-sevenths.

In the expression "$\frac{3}{4}$ of an apple," the *denominator* 4 shows that the apple is to be (or has been) cut into 4 *equal parts*, and the *numerator* 3 shows that *three of these parts*, or *fourths*, are taken or considered.

If each of the *parts*, or *fourths*, of the apple were cut in *two equal pieces*, there would then be twice as many pieces as before, or $4 \times 2 = 8$ pieces in all; one of these pieces would be called one-eighth, and would be expressed in figures as $\frac{1}{8}$. Three of these pieces would be called three-eighths, and written $\frac{3}{8}$. The words three-fourths, three-eighths, five-sixteenths, etc., are abbreviations of three one-fourths, three one-eighths, five one-sixteenths, etc. It is evident that the larger the *denominator*, the greater is the number of parts into which anything is divided; consequently, the parts themselves are smaller, and the value of the fraction is less for the same number of parts taken. In other words, $\frac{7}{9}$, for example, is smaller than $\frac{7}{8}$, because if an object be divided into 9 parts, the parts are smaller than if the same object had been divided into 8 parts; and, since $\frac{1}{9}$ is smaller than $\frac{1}{8}$, it is clear that 7 one-ninths is a smaller amount than 7 one-eighths. Hence, also, $\frac{3}{8}$ is less than $\frac{3}{4}$.

74. The **value** of a fraction is the result obtained by dividing the *numerator* by the *denominator;* as, $\frac{4}{2} = 2$, $\frac{9}{3} = 3$.

75. The line between the *numerator* and *denominator* means *divided by*, or ÷.

$\frac{3}{4}$ is equivalent to $3 \div 4$.
$\frac{5}{8}$ is equivalent to $5 \div 8$.

76. The *numerator* and *denominator* of a fraction are called the **terms** of a fraction.

77. The *value* of a fraction whose *numerator* and *denominator* are equal, is 1.

$\frac{4}{4}$, or four-fourths, =1.
$\frac{8}{8}$, or eight-eighths, = 1.
$\frac{64}{64}$, or sixty-four sixty-fourths, = 1.

78. A **proper fraction** is a fraction whose *numerator* is *less* than its *denominator*. Its *value* is *less* than 1; as, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{16}$.

79. An **improper fraction** is a fraction whose *numerator equals* or is *greater than* the *denominator*. Its *value* is *one* or *more* than *one;* as, $\frac{4}{4}$, $\frac{9}{8}$, $\frac{48}{33}$.

80. A **mixed number** is a *whole* number and a *fraction* united. $4\frac{2}{3}$ is a mixed number, and is equivalent to $4 + \frac{2}{3}$. It is read *four and two-thirds.*

REDUCTION OF FRACTIONS.

81. **Reduction of fractions** is the process of changing their form without changing their *value.*

82. A *fraction* is reduced to *higher terms* by *multiplying both terms* of the *fraction* by the *same number.* Thus, $\frac{3}{4}$ is reduced to $\frac{6}{8}$ by multiplying both terms by 2.

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}.$$

The *value* is not changed, since $\frac{3}{4} = \frac{6}{8}$. For, suppose that an object, say an apple, is divided into 8 equal parts. If these parts be arranged into 4 piles, each containing 2 parts, it is evident each pile will be composed of the same amount of the entire apple as would have been the case had the apple been originally cut into 4 equal parts. Now, if one of these piles (containing 2 parts) be removed, there will be 3 piles left each containing 2 equal parts, or 6 equal parts in all, i. e., six-eighths. But, since one pile, or one quarter, was removed, there are three-quarters left. Hence, $\frac{3}{4} = \frac{6}{8}$.

The same course of reasoning may be applied to any similar case. Therefore, multiplying both terms of a fraction by the same number does not alter its value.

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}.$$

83. A *fraction* is reduced to *lower terms* by *dividing both terms* by the *same number*. Thus, $\frac{8}{10}$ is reduced to $\frac{4}{5}$ by dividing both terms by 2.

$$\frac{8 \div 2}{10 \div 2} = \frac{4}{5}.$$

That $\frac{8}{10} = \frac{4}{5}$ is readily seen from the explanation given in Art. **82**; for, multiplying both terms of the fraction $\frac{4}{5}$ by 2, $\frac{4 \times 2}{5 \times 2} = \frac{8}{10}$, and, if $\frac{4}{5} = \frac{8}{10}$, $\frac{8}{10}$ must equal $\frac{4}{5}$. Hence, dividing both terms of a fraction by the same number does not alter its value.

84. A *fraction* is reduced to its *lowest terms* when its *numerator and denominator* can not be *divided* by the *same number*, as $\frac{3}{4}$, $\frac{5}{8}$, $\frac{11}{17}$.

85. To reduce a whole number or mixed number to an improper fraction:

EXAMPLE.—How many *fourths* in 5?

SOLUTION.—Since there are 4 *fourths* in 1 ($\frac{4}{4} = 1$). in 5 there will be 5×4 fourths, or 20 fourths, $5 \times \frac{4}{4} = \frac{20}{4}$. Ans.

EXAMPLE.—Reduce $8\frac{3}{4}$ to an improper fraction.

SOLUTION.—$8 \times \frac{4}{4} = \frac{32}{4}$. $\frac{32}{4} + \frac{3}{4} = \frac{35}{4}$. Ans.

86. Rule 6.—*Multiply the whole number by the denominator of the fraction, add the numerator to the product, and place the denominator under the result.*

EXAMPLES FOR PRACTICE.

87. Reduce to improper fractions:

(*a*) $4\frac{1}{8}$.	Ans.	(*a*) $\frac{33}{8}$.
(*b*) $5\frac{1}{9}$.		(*b*) $\frac{46}{9}$.
(*c*) $10\frac{2}{10}$.		(*c*) $\frac{102}{10}$.
(*d*) $37\frac{3}{4}$.		(*d*) $\frac{151}{4}$.
(*e*) $50\frac{4}{5}$.		(*e*) $\frac{254}{5}$.

88. To reduce an improper fraction to a whole or mixed number:

Example.—Reduce $\frac{21}{4}$ to a mixed number.

Solution.—4 is contained in 21, 5 times and 1 remaining; as this is also divided by 4, its value is $\frac{1}{4}$. Therefore, $5 + \frac{1}{4}$, or $5\frac{1}{4}$, is the number. Ans.

89. Rule 7.—*Divide the numerator by the denominator; the quotient will be the whole number; the remainder, if there be any, will be the numerator of the fractional part of which the denominator is the same as the denominator of the improper fraction.*

EXAMPLES FOR PRACTICE.

90. Reduce to whole or mixed numbers:

(*a*) $\frac{145}{6}$.	Ans.	(*a*) $24\frac{1}{6}$.
(*b*) $\frac{185}{3}$.		(*b*) $61\frac{2}{3}$.
(*c*) $\frac{701}{6}$.		(*c*) $116\frac{5}{6}$.
(*d*) $\frac{148}{3}$.		(*d*) $49\frac{1}{3}$.
(*e*) $\frac{76}{19}$.		(*e*) 4.
(*f*) $\frac{125}{25}$.		(*f*) 5.

---

91. A **common denominator** of *two* or *more fractions* is a number which will contain all of the *denominators* of the *fractions* without a remainder. The **least common denominator** is the least number that will contain all of the denominators of the fractions without a remainder.

92. To find the least common denominator:

Example.—Find the least common denominator of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{16}$.

Solution.—We first place the denominators in a row, separated by commas.

$$\begin{array}{rl}
2\,) & \underline{4,\ 3,\ 9,\ 16} \\
2\,) & \underline{2,\ 3,\ 9,\ \ 8} \\
3\,) & \underline{1,\ 3,\ 9,\ \ 4} \\
3\,) & \underline{1,\ 1,\ 3,\ \ 4} \\
4\,) & \underline{1,\ 1,\ 1,\ \ 4} \\
 & 1,\ 1,\ 1,\ \ 1
\end{array}$$

$2 \times 2 \times 3 \times 3 \times 4 = 144$, the least common denominator. Ans.

Explanation.—Divide each of them by some prime number which will divide at least two of them without a remainder (if possible), bringing down those denominators to the row below which will not contain the divisor without a remainder. Dividing each of the numbers by 2, the second row becomes 2, 3, 9, 8, since 2 will not divide 3 and 9 without a remainder. Dividing again by 2, the result is 1, 3, 9, 4. Dividing the third row by 3, the result is 1, 1, 3, 4. So continue until the last row contains only 1's. The product of all the divisors, or 2 × 2 × 3 × 3 × 4 = 144, is the least common denominator.

93. Example.—Find the least common denominator of $\frac{4}{9}$, $\frac{5}{12}$, $\frac{7}{18}$.

Solution.—

$$\begin{array}{r|l} 3 & 9,\ 12,\ 18 \\ \hline 3 & 3,\ \ 4,\ \ 6 \\ \hline 2 & 1,\ \ 4,\ \ 2 \\ \hline 2 & 1,\ \ 2,\ \ 1 \\ \hline & 1,\ \ 1,\ \ 1 \end{array}$$

3 × 3 × 2 × 2 = 36. Ans.

94. To reduce two or more fractions to fractions having a common denominator:

Example.—Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ to fractions having a common denominator.

Solution.—The common denominator is a number which will contain 3, 4, and 2. The least common denominator is 12, because it is the smallest number which can be divided by 3, 4, and 2 without a remainder.

$$\frac{2}{3} = \frac{8}{12},\ \frac{3}{4} = \frac{9}{12},\ \frac{1}{2} = \frac{6}{12}.$$

Reducing $\frac{2}{3}$, 3 is contained in 12, 4 times. By multiplying both numerator and denominator of $\frac{2}{3}$ by 4, we find

$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$. In the same way we find $\frac{3}{4} = \frac{9}{12}$ and $\frac{1}{2} = \frac{6}{12}$.

95. **Rule 8.**—*Divide the common denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

EXAMPLES FOR PRACTICE.

96. Reduce to fractions having a common denominator:

(*a*) $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$.	Ans.	(*a*)	$\frac{6}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.
(*b*) $\frac{3}{16}$, $\frac{3}{4}$, $\frac{7}{32}$.		(*b*)	$\frac{6}{32}$, $\frac{24}{32}$, $\frac{7}{32}$.
(*c*) $\frac{7}{8}$, $\frac{7}{88}$, $\frac{10}{11}$.		(*c*)	$\frac{77}{88}$, $\frac{7}{88}$, $\frac{80}{88}$.
(*d*) $\frac{3}{5}$, $\frac{5}{8}$, $\frac{11}{40}$.		(*d*)	$\frac{24}{40}$, $\frac{25}{40}$, $\frac{11}{40}$.
(*e*) $\frac{4}{10}$, $\frac{6}{40}$, $\frac{9}{20}$.		(*e*)	$\frac{16}{40}$, $\frac{6}{40}$, $\frac{18}{40}$.
(*f*) $\frac{7}{15}$, $\frac{17}{30}$, $\frac{21}{30}$.		(*f*)	$\frac{14}{30}$, $\frac{17}{30}$, $\frac{21}{30}$.

ADDITION OF FRACTIONS.

97. *Fractions can not be added unless they have a common denominator.* We can not add $\frac{3}{4}$ to $\frac{7}{8}$ as they now stand, since the denominators represent parts of different sizes. Fourths can not be added to eighths.

Suppose we divide an apple into 4 equal parts, and then divide 2 of these parts into two equal parts. It is evident that we shall have 2 one-fourths and 4 one-eighths. Now if we add these parts the result is $2 + 4 = 6$ something. But what is this something? It is not fourths, for six fourths are $1\frac{1}{2}$, and we had only 1 apple to begin with; neither is it eighths, for six eighths are $\frac{6}{8}$, which is less than 1 apple. By reducing the quarters to eighths, we have $\frac{2}{4} = \frac{4}{8}$, and adding the other 4 eighths, $4 + 4 = 8$ eighths. The result is correct, since $\frac{8}{8} = 1$. Or we can, in this case, reduce the eighths to quarters. Thus, $\frac{4}{8} = \frac{2}{4}$; whence, adding, $2 + 2 =$ 4 quarters, a correct result, since $\frac{4}{4} = 1$.

Before adding, fractions should be reduced to a common denominator, preferably the *least* common denominator.

98. EXAMPLE.—Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$.

SOLUTION.—The *least common denominator* or the *least number* which will contain all the *denominators* is 8.

$$\frac{1}{2} = \frac{4}{8},\ \frac{3}{4} = \frac{6}{8},\text{ and } \frac{5}{8} = \frac{5}{8}.$$

$$\frac{4}{8} + \frac{6}{8} + \frac{5}{8} = \frac{4+6+5}{8} = \frac{15}{8} = 1\frac{7}{8}.\quad \text{Ans.}$$

EXPLANATION.—As the *denominator* tells or indicates the names of the *parts*, the *numerators* only are added to obtain the total number of *parts* indicated by the *denominator*.

99. EXAMPLE.—What is the sum of $12\frac{3}{4}$, $14\frac{5}{8}$, and $7\frac{5}{16}$?

SOLUTION.—The least common denominator in this case is 16.

$$\begin{aligned} 12\tfrac{3}{4} &= 12\tfrac{12}{16} \\ 14\tfrac{5}{8} &= 14\tfrac{10}{16} \\ 7\tfrac{5}{16} &= \ \ 7\tfrac{5}{16} \\ \hline \textit{sum}\ 33 + \tfrac{27}{16} &= 33 + 1\tfrac{11}{16} = 34\tfrac{11}{16}.\quad \text{Ans.} \end{aligned}$$

The sum of the fractions $= \frac{27}{16}$ or $1\frac{11}{16}$, which added to the sum of the whole numbers $= 34\frac{11}{16}$.

Example.—What is the sum of 17, $13\frac{3}{16}$, $\frac{9}{32}$, and $3\frac{1}{4}$?

Solution.—The least common denominator is 32. $13\frac{3}{16} = 13\frac{6}{32}$, $3\frac{1}{4} = 3\frac{8}{32}$.

$$\begin{array}{rr} & 17 \\ & 13\frac{6}{32} \\ & \frac{9}{32} \\ & 3\frac{8}{32} \\ \hline \textit{sum} & 33\frac{23}{32}. \end{array} \quad \text{Ans.}$$

100. Rule 9.—(*a*) *Reduce the given fractions to fractions having the least common denominator, and write the sum of the numerators over the common denominator.*

(*b*) *When there are mixed numbers and whole numbers, add the fractions first, and if their sum is an improper fraction, reduce it to a mixed number and add the whole number with the other whole numbers.*

EXAMPLES FOR PRACTICE.

101. Find the sum of

	Ans.
(*a*) $\frac{1}{6}$, $\frac{7}{24}$, $\frac{5}{8}$.	(*a*) $1\frac{1}{12}$.
(*b*) [illegible]	(*b*) [illegible]
(*c*) $\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$.	(*c*) $1\frac{3}{16}$.
(*d*) [illegible]	(*d*) [illegible]
(*e*) $\frac{10}{11}$, [illegible]	(*e*) [illegible]
(*f*) [illegible]	(*f*) [illegible]
(*g*) $\frac{4}{11}$, $\frac{7}{33}$, [illegible]	(*g*) $1\frac{7}{33}$.
(*h*) [illegible]	(*h*) 1.

1. The weights of a number of castings were $412\frac{3}{4}$ lb., $270\frac{1}{2}$ lb., 1,020 lb., $75\frac{1}{4}$ lb., and $68\frac{1}{2}$ lb. What was their total weight?
Ans. 1,847 lb.

2. Four bolts are required, $2\frac{3}{4}$, $1\frac{7}{8}$, $2\frac{5}{16}$, and $1\frac{13}{32}$ inches long. How long a piece of iron will be required to cut them from, allowing $\frac{3}{4}$ of an inch altogether for cutting off and finishing the ends? Ans. $9\frac{3}{32}$ in.

SUBTRACTION OF FRACTIONS.

102. Fractions can not be *subtracted* without first reducing them to a *common denominator*. This can be shown in the same manner as in the case of addition of fractions.

Example.—Subtract $\frac{3}{8}$ from $\frac{13}{16}$.

Solution.—The common denominator is 16.

$$\frac{3}{8} = \frac{6}{16}. \quad \frac{13}{16} - \frac{6}{16} = \frac{13 - 6}{16} = \frac{7}{16}. \quad \text{Ans.}$$

103. Example.—From 7 take $\frac{3}{8}$.

Solution.—$1 = \frac{8}{8}$; therefore, since $7 = 6 + 1$ we see that $7 = 6 + \frac{8}{8}$, so that $6\frac{8}{8} - \frac{3}{8} = 6\frac{5}{8}$. Ans.

104. Example.—What is the difference between $17\frac{9}{16}$ and $9\frac{15}{32}$?

Solution.—The common denominator of the fractions is 32. $17\frac{9}{16} = 17\frac{18}{32}$.

$$\begin{array}{rr} \textit{minuend} & 17\frac{18}{32} \\ \textit{subtrahend} & 9\frac{15}{32} \\ \hline \textit{difference} & 8\frac{3}{32} \end{array} \text{ Ans.}$$

105. Example.—From $9\frac{1}{4}$ take $4\frac{7}{16}$.

Solution.—The common denominator of the fractions is 16. $9\frac{1}{4} = 9\frac{4}{16}$.

$$\begin{array}{rrcr} \textit{minuend} & 9\frac{4}{16} & & 8\frac{20}{16} \\ \textit{subtrahend} & 4\frac{7}{16} & \text{or} & 4\frac{7}{16} \\ \hline \textit{difference} & 4\frac{13}{16} & & 4\frac{13}{16} \end{array} \text{ Ans.}$$

Explanation.—As the *fraction* in the *subtrahend* is greater than the fraction in the *minuend*, it *can not* be subtracted; therefore, *borrow* 1, or $\frac{16}{16}$, from the 9 in the *minuend* and *add* it to the $\frac{4}{16}$; $\frac{4}{16} + \frac{16}{16} = \frac{20}{16}$. $\frac{7}{16}$ from $\frac{20}{16} = \frac{13}{16}$. Since 1 was *borrowed* from 9, 8 *remains;* 4 from $8 = 4$; $4 + \frac{13}{16} = 4\frac{13}{16}$.

106. Example.—From 9 take $8\frac{3}{16}$.

Solution.—

$$\begin{array}{rrcr} \textit{minuend} & 9 & & 8\frac{16}{16} \\ \textit{subtrahend} & 8\frac{3}{16} & \text{or} & 8\frac{3}{16} \\ \hline \textit{difference} & \frac{13}{16} & & \frac{13}{16} \end{array} \text{ Ans.}$$

Explanation.—As there is no *fraction* in the *minuend* from which to take the *fraction* in the *subtrahend*, *borrow* 1, or $\frac{16}{16}$, from 9. $\frac{3}{16}$ from $\frac{16}{16} = \frac{13}{16}$. Since 1 was *borrowed* from 9, only 8 is left. 8 from $8 = 0$.

107. Rule 10.—(*a*) *Reduce the fractions to fractions having a common denominator. Subtract one numerator from the other and place the remainder over the common denominator.*

(*b*) *When there are mixed numbers, subtract the fractions and whole numbers separately, and place the remainders side by side.*

(*c*) *When the fraction in the subtrahend is greater than the fraction in the minuend, borrow 1 from the whole number in the minuend and add it to the fraction in the minuend, from which subtract the fraction in the subtrahend.*

(*d*) *When the minuend is a whole number, borrow 1; reduce it to a fraction whose denominator is the same as the denominator of the fraction in the subtrahend, and place it over that fraction for subtraction.*

EXAMPLES FOR PRACTICE.

108. Subtract

(*a*) $\frac{10}{32}$ from $\frac{11}{16}$.	(*a*) $\frac{3}{8}$.
(*b*) $\frac{7}{12}$ from $\frac{11}{16}$.	(*b*) $\frac{5}{48}$.
(*c*) $\frac{4}{30}$ from $\frac{5}{10}$.	(*c*) $\frac{11}{30}$.
(*d*) $\frac{13}{35}$ from $\frac{41}{70}$.	Ans. (*d*) $\frac{3}{14}$.
(*e*) $\frac{13}{16}$ from $\frac{47}{48}$.	(*e*) $\frac{1}{6}$.
(*f*) $13\frac{1}{4}$ from $30\frac{1}{2}$.	(*f*) $17\frac{1}{4}$.
(*g*) $12\frac{5}{8}$ from 27.	(*g*) $14\frac{3}{8}$.
(*h*) $5\frac{3}{4}$ from 30.	(*h*) $24\frac{1}{4}$.

1. An engineer found that he had on hand $48\frac{3}{4}$ gallons of cylinder oil. During the following week he used $\frac{7}{8}$ of a gallon a day for three days, $\frac{7}{8}$ of a gallon on the fourth day, $\frac{13}{16}$ of a gallon on the fifth day, and $\frac{5}{8}$ of a gallon on the sixth day. How much oil remained at the end of the week? Ans. $43\frac{13}{16}$ gallons.

2. The main line shaft of a manufacturing plant is run by an engine and water wheel. A test of the plant showed that the engine was capable of developing $251\frac{3}{8}$ H. P. (horsepower), and the water wheel, under full gate, $67\frac{3}{4}$ H. P. It was also found that the machinery consumed $210\frac{11}{16}$ H. P., and the friction of the shafting and belting was $32\frac{1}{3}$ H. P. How much power remained unused? Ans. $76\frac{5}{48}$ H. P.

MULTIPLICATION OF FRACTIONS.

109. In *multiplication* of fractions it is not necessary to *reduce* the *fractions* to fractions having a *common denominator.*

110. Multiplying the *numerator* or *dividing* the *denominator multiplies* the fraction.

Example.—Multiply $\frac{3}{4}\times 4$.

Solution.—

$$\tfrac{3}{4}\times 4=\frac{3\times 4}{4}=\tfrac{12}{4}=3,\quad \text{Ans.}$$

$$\text{or,}\quad \tfrac{3}{4}\times 4=\frac{3}{4\div 4}=\tfrac{3}{1}=3.\quad \text{Ans.}$$

The word "of" in multiplication of fractions means the same as ×, or times. Thus,

$$\tfrac{3}{4}\text{ of }4=\tfrac{3}{4}\times 4=3.$$

$$\tfrac{1}{8}\text{ of }\tfrac{5}{16}=\tfrac{1}{8}\times\tfrac{5}{16}=\tfrac{5}{128}.$$

Example.—Multiply $\frac{3}{8}$ by 2.

Solution.—

$$\tfrac{3}{8}\times 2=\frac{3\times 2}{8}=\tfrac{6}{8}=\tfrac{3}{4},\quad \text{Ans.}$$

$$\text{or,}\quad \tfrac{3}{8}\times 2=\frac{3}{8\div 2}=\tfrac{3}{4}.\quad \text{Ans.}$$

111. **Example.**—What is the product of $\frac{4}{16}$ and $\frac{7}{8}$?

Solution.—

$$\tfrac{4}{16}\times\tfrac{7}{8}=\frac{4\times 7}{16\times 8}=\tfrac{28}{128}=\tfrac{7}{32},$$

$$\text{or, by cancelation,}\quad \frac{\cancel{4}\times 7}{\underset{4}{\cancel{16}}\times 8}=\frac{7}{4\times 8}=\tfrac{7}{32}.\quad \text{Ans.}$$

112. **Example.**—What is $\frac{4}{8}$ of $\frac{3}{4}$ of $\frac{16}{32}$?

Solution.—

$$\frac{\cancel{4}\times 3\times\cancel{16}}{8\times\cancel{4}\times\underset{2}{\cancel{32}}}=\frac{3}{8\times 2}=\tfrac{3}{16}.\quad \text{Ans.}$$

113. **Example.**—What is the product of $9\frac{3}{4}$ and $5\frac{5}{8}$?

Solution.—

$$9\tfrac{3}{4}=\tfrac{39}{4};\ 5\tfrac{5}{8}=\tfrac{45}{8}.$$

$$\tfrac{39}{4}\times\tfrac{45}{8}=\frac{39}{4}\times\frac{45}{8}=\tfrac{1755}{32}=54\tfrac{27}{32}.\quad \text{Ans.}$$

114. **Example.**—Multiply $15\frac{7}{8}$ by 3.

Solution.—

$$\begin{array}{r} 15\tfrac{7}{8}\\ 3\\ \hline 47\tfrac{5}{8} \end{array}\quad\text{or}\quad \begin{array}{r} 15\tfrac{7}{8}\\ 3\\ \hline 45 \end{array}+\tfrac{21}{8}=45+2\tfrac{5}{8}=47\tfrac{5}{8}.\quad \text{Ans.}$$

115. **Rule 11.**—(*a*) *Divide the product of the numerators by the product of the denominators. All factors common to the numerators and denominators should first be cast out by cancelation.*

(*b*) *To multiply one mixed number by another, reduce them both to improper fractions.*

(*c*) *To multiply a mixed number by a whole number, first multiply the fractional part by the multiplier, and if the product is an improper fraction, reduce it to a mixed number and add the whole number part to the product of the multiplier and whole number.*

EXAMPLES FOR PRACTICE.

116. Find the product of

		Ans.
(*a*)	$7 \times \frac{3}{16}$.	(*a*) $1\frac{5}{16}$.
(*b*)	$14 \times \frac{5}{16}$.	(*b*) $4\frac{3}{8}$.
(*c*)	$\frac{21}{32} \times \frac{3}{14}$.	(*c*) $\frac{9}{64}$.
(*d*)	$\frac{16}{27} \times 4$.	(*d*) $2\frac{10}{27}$.
(*e*)	$\frac{17}{16} \times 7$.	(*e*) $7\frac{7}{16}$.
(*f*)	$17\frac{18}{21} \times 7$.	(*f*) 125.
(*g*)	$\frac{105}{224} \times 32$.	(*g*) 15.
(*h*)	$\frac{15}{28} \times 14$.	(*h*) $7\frac{1}{2}$.

1. A single belt can transmit $107\frac{3}{8}$ H. P., but as it is desired to use more power, a double belt of the same width is substituted for it. Supposing the double belt to be capable of transmitting $\frac{10}{7}$ as much power as the single belt, how many H. P. can be used after the change? Ans. $153\frac{11}{28}$ H. P.

2. What is the weight of $2\frac{5}{8}$ miles of copper wire weighing $5\frac{3}{4}$ pounds per 100 feet? There are 5,280 feet in a mile. Ans. $796\frac{19}{20}$ pounds.

3. The grate of a steam boiler contains $20\frac{1}{2}$ square feet. If the boiler burns $8\frac{3}{10}$ pounds of coal an hour per square foot of grate area, and can evaporate $7\frac{1}{2}$ pounds of water an hour per pound of coal burned, how many pounds of water are evaporated by the boiler in 1 hour? Ans. $1,276\frac{1}{8}$ pounds.

DIVISION OF FRACTIONS.

117. In *division* of fractions it is not necessary to *reduce* the *fractions* to fractions having a *common denominator.*

118. Dividing the *numerator* or *multiplying* the *denominator divides* the fraction.

Example.—Divide $\frac{6}{8}$ by 3.

Solution.—When *dividing* the *numerator*, we have

$$\frac{6}{8} \div 3 = \frac{6 \div 3}{8} = \frac{2}{8} = \frac{1}{4}.\quad \text{Ans.}$$

When *multiplying* the *denominator*, we have

$$\frac{6}{8} \div 3 = \frac{6}{8 \times 3} = \frac{6}{24} = \frac{1}{4}.\quad \text{Ans.}$$

Example.—Divide $\frac{3}{16}$ by 2.

Solution.— $$\frac{3}{16} \div 2 = \frac{3}{16 \times 2} = \frac{3}{32}.\quad \text{Ans.}$$

Example.—Divide $\frac{14}{32}$ by 7.

Solution.— $$\frac{14}{32} \div 7 = \frac{14 \div 7}{32} = \frac{2}{32} = \frac{1}{16}.\quad \text{Ans.}$$

119. To **invert** a fraction is to *turn* it *upside down;* that is, make the *numerator* and *denominator change places.*

Invert $\frac{3}{4}$ and it becomes $\frac{4}{3}$.

120. **Example.**—Divide $\frac{9}{16}$ by $\frac{3}{16}$.

Solution.—1. The fraction $\frac{3}{16}$ is contained in $\frac{9}{16}$, 3 times, for the *denominators* are the same, and one *numerator* is *contained* in the other 3 times. 2. If we now *invert* the *divisor*, $\frac{3}{16}$, and *multiply*, the solution is

$$\frac{9}{16} \times \frac{16}{3} = \frac{\overset{3}{\cancel{9}} \times \cancel{16}}{\cancel{16} \times \cancel{3}} = 3.\quad \text{Ans.}$$

This gives the *same quotient* as in the first case.

121. **Example.**—Divide $\frac{3}{8}$ by $\frac{1}{4}$.

Solution.—We can not divide $\frac{3}{8}$ by $\frac{1}{4}$, as in the first case above, for the *denominators* are *not* the same; therefore, we must solve as in the second case.

$$\frac{3}{8} \div \frac{1}{4} = \frac{3}{8} \times \frac{4}{1} = \frac{3 \times \cancel{4}}{\underset{2}{\cancel{8}} \times 1} = \frac{3}{2} \text{ or } 1\tfrac{1}{2}.\quad \text{Ans.}$$

122. **Example.**—Divide 5 by $\frac{10}{16}$.

Solution.—$\frac{10}{16}$ inverted becomes $\frac{16}{10}$,

$$5 \times \frac{16}{10} = \frac{\cancel{5} \times \overset{8}{\cancel{16}}}{\underset{\cancel{2}}{\cancel{10}}} = 8.\quad \text{Ans.}$$

123. EXAMPLE.—How many times is $3\frac{3}{4}$ contained in $7\frac{7}{16}$?

SOLUTION.— $3\frac{3}{4} = \frac{15}{4}$; $7\frac{7}{16} = \frac{119}{16}$.

$\frac{15}{4}$ inverted becomes $\frac{4}{15}$.

$$\frac{119}{16} \times \frac{4}{15} = \frac{119 \times \cancel{4}}{\underset{4}{\cancel{16}} \times 15} = \frac{119}{60} = 1\frac{59}{60}. \text{ Ans.}$$

124. Rule 12.—*Invert the divisor and proceed as in multiplication.*

125. We have learned that a line placed between two numbers indicates that the number above the line is to be divided by the number below it. Thus, $\frac{18}{3}$ shows that 18 is to be divided by 3. This is also true if a fraction or a fractional expression be placed above or below a line.

$\dfrac{9}{\frac{3}{8}}$ means that 9 is to be divided by $\frac{3}{8}$; $\dfrac{3 \times 7}{\frac{8+4}{16}}$ means that 3×7 is to be divided by the value of $\dfrac{8+4}{16}$.

$\dfrac{\frac{1}{4}}{\frac{3}{8}}$ is the same as $\frac{1}{4} \div \frac{3}{8}$.

It will be noticed that there is a heavy line between the 9 and the $\frac{3}{8}$. This is necessary, since otherwise there would be nothing to show whether 9 is to be divided by $\frac{3}{8}$, or $\frac{9}{3}$ by 8. Whenever a heavy line is used, as in the above case, it indicates that *all above the line* is to be divided by *all below it*.

EXAMPLES FOR PRACTICE.

126. Divide

(*a*) 15 by $6\frac{1}{4}$.	Ans.	(*a*) $2\frac{2}{5}$.
(*b*) 30 by $\frac{3}{4}$.		(*b*) 40.
(*c*) 172 by $\frac{4}{5}$.		(*c*) 215.
(*d*) $\frac{14}{18}$ by $1\frac{7}{16}$.		(*d*) $\frac{112}{207}$.
(*e*) $\frac{102}{5}$ by $14\frac{7}{8}$.		(*e*) $1\frac{13}{35}$.
(*f*) $\frac{142}{35}$ by $17\frac{1}{5}$.		(*f*) $\frac{71}{301}$.
(*g*) $\frac{14}{18}$ by $\frac{145}{72}$.		(*g*) $\frac{56}{145}$.
(*h*) $\frac{128}{18}$ by $72\frac{1}{3}$.		(*h*) $\frac{64}{651}$.

1. A $\frac{3}{8}$-inch boiler plate containing 24 square feet of surface weighs $362\frac{4}{10}$ pounds. What is its weight per square foot? Ans. $15\frac{1}{10}$ pounds.

2. A certain boiler has 927½ square feet of heating surface, which is equal to 35 times the area of the grate. What is the area of the grate in square feet? Ans. 26½ square feet.

3. If the distance around the rim of a locomotive drive wheel is $13\frac{1}{12}$ feet, how many revolutions will the wheel make in traveling 682 feet? Ans. $52\frac{20}{157}$ revolutions.

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127. Whenever an expression like one of the three following ones is obtained, it may always be simplified by transposing the denominator from *above* to *below* the line, or from *below* to *above*, as the case may be, taking care, however, to indicate that the denominator when so transferred is a multiplier.

1. $\dfrac{\frac{3}{4}}{9} = \dfrac{3}{9 \times 4} = \frac{3}{36} = \frac{1}{12}$; for, regarding the fraction above the heavy line as the numerator of a fraction whose denominator is 9, $\dfrac{\frac{3}{4} \times 4}{9 \times 4} = \dfrac{3}{9 \times 4}$, as before.

2. $\dfrac{9}{\frac{3}{4}} = \dfrac{9 \times 4}{3} = 12$. The proof is the same as in the first case.

3. $\dfrac{\frac{5}{9}}{\frac{3}{4}} = \dfrac{5 \times 4}{3 \times 9} = \frac{20}{27}$; for, regarding $\frac{5}{9}$ as the numerator of a fraction whose denominator is $\frac{3}{4}$, $\dfrac{\frac{5}{9} \times 9}{\frac{3}{4} \times 9} = \dfrac{5}{\frac{3 \times 9}{4}}$; and

$$\frac{\dfrac{5}{\frac{3 \times 9}{4}} \times 4}{\dfrac{3 \times 9}{4} \times 4} = \frac{5 \times 4}{3 \times 9} = \frac{20}{27}, \text{ as above.}$$

This principle may be used to great advantage in cases like $\dfrac{\frac{1}{4} \times 310 \times \frac{27}{12} \times 72}{40 \times 4\frac{1}{2} \times 5\frac{1}{6}}$. Reducing the mixed numbers to fractions, the expression becomes $\dfrac{\frac{1}{4} \times 310 \times \frac{27}{12} \times 72}{40 \times \frac{9}{2} \times \frac{31}{6}}$. Now transferring the denominators of the fractions and canceling,

$$\frac{1 \times 310 \times 27 \times 72 \times 2 \times 6}{40 \times 9 \times 31 \times 4 \times 12} = \frac{1 \times \overset{10}{\cancel{310}} \times \overset{3}{\cancel{27}} \times \overset{\overset{3}{\cancel{6}}}{\cancel{72}} \times \cancel{2} \times \overset{3}{\cancel{6}}}{\underset{\underset{\cancel{2}}{\cancel{4}}}{\cancel{40}} \times \cancel{9} \times \cancel{31} \times \underset{2}{\cancel{4}} \times \cancel{12}} =$$

$\dfrac{27}{2} = 13\frac{1}{2}$.

Greater exactness in results can usually be obtained by using this principle than can be obtained by reducing the fractions to decimals. The principle, however, should not be employed *if a sign of addition or subtraction occurs either above or below the dividing line.*

DECIMALS.

128. **Decimals** are *tenth* fractions ; that is, the parts of a unit are expressed on the scale of ten, as *tenths*, *hundredths*, *thousandths*, etc.

129. The *denominator*, which is always ten or a multiple of ten, as 10, 100, 1,000, etc., is *not* expressed as it would be in common fractions, by writing it under the *numerator*, with a line between them; as, $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$. The denominator is always understood, the numerator consisting of the figures on the right of the *unit* figure. In order to distinguish the unit figure, a period (.), called the **decimal point,** is placed between the unit figure and the next figure on the right. The decimal point may be regarded in two ways : first, as indicating that the number on the right is the numerator of a fraction whose denominator is 10, 100, 1,000, etc. ; and, second, as a part of the Arabic system of notation, each figure on the right being 10 times as large as the next succeeding figure, and 10 times as small as the next preceding figure, serving merely to point out the unit figure.

130. The *reading* of a *decimal number* depends upon the *number* of *decimal places* in it, or the *number* of *figures* to the *right* of the unit figure.

The first figure to the right of the unit figure expresses *tenths.*

The second figure to the right of the unit figure expresses *hundredths.*

The third figure to the right of the unit figure expresses *thousandths.*

The fourth figure to the right of the unit figure expresses *ten-thousandths.*

The fifth figure to the right of the unit figure expresses *hundred-thousandths.*

The sixth figure to the right of the unit figure expresses *millionths.*

Thus:

.3 $= \frac{3}{10}$ = 3 tenths.
.03 $= \frac{3}{100}$ = 3 hundredths.
.003 $= \frac{3}{1000}$ = 3 thousandths.
.0003 $= \frac{3}{10000}$ = 3 ten-thousandths.
.00003 $= \frac{3}{100000}$ = 3 hundred-thousandths.
.000003 $= \frac{3}{1000000}$ = 3 millionths.

The first figure to the right of the unit figure is called the *first* **decimal place**; the second figure, the *second decimal place,* etc. We see in the above that the *number* of *decimal places* in a decimal equals the *number of ciphers* to the *right* of the figure 1 in the *denominator* of its *equivalent fraction.* This fact kept in mind will be of much assistance in reading and writing decimals.

Whatever may be written to the *left* of a *decimal point* is a whole number. The decimal point affects only the figures to its *right.*

When a *whole number* and *decimal* are written together, the expression is a *mixed number.* Thus, 8.12 and 17.25 are mixed numbers.

The relation of decimals and whole numbers to each other is clearly shown by the following table:

hundreds of millions.	tens of millions.	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.	decimal point.	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	ten-millionths.	hundred-millionths.
9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9

The figures to the *left* of the *decimal point* represent *whole numbers;* those to the *right* are *decimals.*

In *both* the decimals and whole numbers, the *units* place is made the *starting point* of notation and numeration. The *decimals decrease* on the scale of *ten* to the *right* and the *whole* numbers *increase* on the scale of *ten* to the *left.* The *first* figure to the *left* of units is *tens*, and the *first* figure to the *right* of units is *tenths*. The *second* figure to the *left* of units is *hundreds*, and the *second* figure to the *right* is *hundredths*. The *third* figure to the *left* is *thousands*, and the *third* to the *right* is *thousandths*, and so on; the *whole* numbers on the *left* and the *decimals* on the *right*. The figures equally distant from units place correspond in name. The *decimals* have the ending *ths*, which distinguishes them from *whole* numbers. The following is the numeration of the number in the above table: Nine hundred eighty-seven million, six hundred fifty-four thousand, three hundred twenty-one, and twenty-three million, four hundred fifty-six thousand, seven hundred eighty-nine hundred millionths.

The *decimals* increase to the *left* on a scale of *ten*, the same as *whole* numbers, for if you begin at 4-*thousandths*, in the table, you see that the next figure is *hundredths*, which is ten times as great, and the next *tenths*, or ten times the *hundredths*, and so on through both decimals and whole numbers.

131. *Annexing* or *taking away* a *cipher* at the *right* of a *decimal* does *not* affect its value.

.5 is $\frac{5}{10}$; .50 is $\frac{50}{100}$, but $\frac{5}{10} = \frac{50}{100}$; therefore, $.5 = .50$.

132. *Inserting* a *cipher* between a *decimal* and the *decimal point divides* the decimal by 10.

$.5 = \frac{5}{10}$; $\frac{5}{10} \div 10 = \frac{5}{100} = .05$.

133. *Taking away* a *cipher* from the *left* of a *decimal multiplies* the *decimal* by 10.

$.05 = \frac{5}{100}$; $\frac{5}{100} \times 10 = \frac{5}{10} = .5$.

ADDITION OF DECIMALS.

134. The only respect in which *addition* of *decimals* differs from *addition* of *whole numbers* is in the placing of the numbers to be added.

Whole numbers begin at units and increase on the scale of 10 to the left. Decimals decrease on the scale of 10 to the right. Whole numbers are to the left of the decimal point and decimals are to the right of it. In whole numbers the *right hand* side of a column of figures to be added must be in line, and in decimals the *left hand* side must be in line, which brings the decimal points directly under each other.

whole numbers	*decimals*	*mixed numbers*
342	.342	342.032
4234	.4234	4234.5
26	.26	26.6782
3	.03	3.06
sum 4605 Ans.	*sum* 1.0554 Ans.	*sum* 4606.2702 Ans.

135. EXAMPLE.—What is the sum of 242, .36, 118.725, 1.005, 6, and 100.1?

SOLUTION.—

242.
.36
118.725
1.005
6.
100.1
sum 468.190 Ans.

136. Rule 13.—*Place the numbers to be added so that the decimal points will be directly under each other. Add as in whole numbers, and place the decimal point in the sum directly under the decimal points above.*

EXAMPLES FOR PRACTICE.

137. Find the sum of

(*a*) .2143, .105, 2.3042, and 1.1417.	Ans.	(*a*) 3.7652.
(*b*) 783.5, 21.473, .2101, and .7816.		(*b*) 805.9647.
(*c*) 21.781, 138.72, 41.8738, .72, and 1.418.		(*c*) 204.5078.
(*d*) .3724, 104.15, 21.417, and 100.042.		(*d*) 225.9814.
(*e*) 200.172, 14.105, 12.1465, .705, and 7.2.		(*e*) 234.3285.
(*f*) 1,427.16, .244, .32, .032, and 10.0041.		(*f*) 1,437.7601.
(*g*) 2,473.1, 41.65, .7243, 104.067, and 21.073.		(*g*) 2,640.6143.
(*h*) 4,107.2, .00375, 21.716, 410.072, and .0345.		(*h*) 4,539.02625.

1. The estimated weights of the parts of a return tubular boiler were as follows: Shell, 3,626 lb.; tubes, 3,564.5 lb.; manhole cover, ring, and yoke, 270.34 lb.; stays, etc., 1,089.4 lb.; steam nozzles,

236.07 lb.; handhole covers and yokes, 120.25 lb.; feed pipe, 84.75 lb.; boiler supports, 350.6 lb. What was the total estimated weight of the boiler? Ans. 9,291.91 lb.

2. A bill for engine room supplies had the following items: 1 waste can, $8.30; 20 feet of 4-inch belting, $11.20; 1 pipe wrench, $1.65; 12 pounds of waste, $0.84; 5 gallons of cylinder oil, $8.75; 20 gallons of machine oil, $24. How much did the bill amount to? Ans. $54.74.

SUBTRACTION OF DECIMALS.

138. For the same reason as in addition of decimals, the *left hand* figures of *decimal numbers* are placed in line and the *decimal points* under each other.

EXAMPLE.—Subtract .132 from .3068.

SOLUTION.—
minuend .3068
subtrahend .132
difference .1748 Ans.

139. EXAMPLE.—What is the difference between 7.895 and .725?

SOLUTION.—
minuend 7.895
subtrahend .725
difference 7.170 or 7.17 Ans.

140. EXAMPLE.—Subtract .625 from 11.

SOLUTION.—
minuend 11.000
subtrahend .625
difference 10.375

141. Rule 14.—*Place the subtrahend under the minuend, so that the decimal points will be directly under each other. Subtract as in whole numbers, and place the decimal point in the remainder, directly under the decimal points above.*

When the figures in the decimal part of the subtrahend extend beyond those in the minuend, place ciphers in the minuend above them and subtract as before.

EXAMPLES FOR PRACTICE.

142. From

	Ans.
(*a*) 407.385 take 235.0004.	(*a*) 172.3846.
(*b*) 22.718 take 1.7042.	(*b*) 21.0138.
(*c*) 1,368.17 take 13.6817.	(*c*) 1,354.4883.
(*d*) 70.00017 take 7.000017.	(*d*) 63.000153.

(*e*) 630.630 take .6304.	Ans.	(*e*) 629.9996.
(*f*) 421.73 take 217.162.		(*f*) 204.568.
(*g*) 1.000014 take .00001.		(*g*) 1.000004.
(*h*) .783652 take .542314.		(*h*) .241338.

1. If the temperature of steam at 5 pounds pressure is 227.964 degrees, and at 100 pounds pressure 337.874 degrees, how many degrees warmer is the steam at the higher pressure? Ans. 109.91 degrees.

2. The outside diameter of 2¼-inch wrought-iron pipe is 2.87 inches and the inside diameter 2.46 inches. How thick is the pipe? Ans. .41 ÷ 2 = .205 inch.

3. In a cistern that will hold 326.5 barrels of water there are 178.625 barrels. How much does it lack of being full? Ans. 147.875 barrels.

4. A wrought-iron rod is 2.53 inches in diameter. What must be the thickness of metal turned off so that the rod will be 2.495 inches in diameter? Ans. .035 ÷ 2 = .0175.

MULTIPLICATION OF DECIMALS.

143. In multiplication of decimals, we do not place the *decimal points* directly *under* each other as in addition and subtraction. We pay *no* attention for the time being to the decimal points. Place the multiplier under the multiplicand, so that the *right-hand* figure of the one is under the *right-hand* figure of the other, and proceed exactly as in multiplication of whole numbers. After multiplying, *count the number of decimal places in both multiplicand and multiplier, and point off the same number in the product.*

Example.—Multiply .825 by 13.

$$\begin{array}{lr} \textbf{Solution.}\text{—}\quad \textit{multiplicand} & .825 \\ \textit{multiplier} & 13 \\ \hline & 2475 \\ & 825 \\ \hline \textit{product} & 10.725 \text{ Ans.} \end{array}$$

In this example there are three decimal places in the multiplicand and none in the multiplier; therefore, 3 decimal places are pointed off in the product.

144. Example.—What is the product of 426 and the decimal .005?

Solution.— *multiplicand* 426
multiplier .005
product 2.130 or 2.13 Ans.

In this example there are 3 decimal places in the multiplier and none in the multiplicand; therefore, 3 decimal places are pointed off in the product.

145. It is *not* necessary to multiply by the ciphers on the *left* of a *decimal;* they merely determine the number of decimal places. Ciphers to the *right* of a decimal should be removed, as they only make more figures to deal with, and do not change the value.

146. Example.—Multiply 1.205 by 1.15.

Solution.— *multiplicand* 1.205
multiplier 1.15
6025
1205
1205
product 1.38575 Ans.

In this example there are 3 decimal places in the multiplicand, and 2 in the multiplier, therefore 3 + 2, or 5, decimal places must be pointed off in the product.

147. Example.—Multiply .232 by .001.

Solution.— *multiplicand* .232
multiplier .001
product .000232 Ans.

In this example we multiply the multiplicand by the digit in the multiplier, which makes 232 in the product, but since there are 3 decimal places in each, the multiplier and the multiplicand, we must prefix 3 ciphers to the 232, to make 3 + 3, or 6, decimal places in the product.

148. Rule 15.—*Place the multiplier under the multiplicand, disregarding the position of the decimal points. Multiply as in whole numbers, and in the product point off as* *many decimal places as there are decimal places in both multiplier and multiplicand, prefixing ciphers if necessary.*

EXAMPLES FOR PRACTICE.

149. Find the product of

	Ans.
(*a*) .000492 × 4.1418.	(*a*) .0020377656.
(*b*) 4,003.2 × 1.2.	(*b*) 4,803.84.
(*c*) 78.6531 × 1.03.	(*c*) 81.012693.
(*d*) .3685 × .042.	(*d*) .015477.
(*e*) 178,352 × .01.	(*e*) 1,783.52.
(*f*) .00045 × .0045.	(*f*) .000002025.
(*g*) .714 × .00002.	(*g*) .00001428.
(*h*) .00004 × .008.	(*h*) .00000032.

1. The stroke of an engine was found by measurement to be 2.987 feet. How many feet will the cross-head pass over in 600 revolutions? Ans. 3,584.4 feet.

2. If a steam pump delivers 2.39 gallons of water per stroke and runs at 51 strokes a minute, how many gallons of water would it pump in 58½ minutes? Ans. 7,130.565 gallons.

3. Wishing to obtain the weight of a connecting-rod from a drawing, it was calculated that the rod contained 294.8 cubic inches of wrought iron, 63.5 cubic inches of brass, and 10.4 cubic inches of babbitt. Assuming the weight of wrought iron to be .278 pound per cubic inch, of brass .303 pound, and of babbitt .264 pound, what was the weight of the rod? Ans. 103.94 pounds.

DIVISION OF DECIMALS.

150. In division of decimals we pay *no* attention to the *decimal point* until *after* the *division* is performed. The *number of decimal places* in the *dividend must equal* (*be made to equal by annexing ciphers*) the *number of decimal places in the divisor.* Divide *exactly* as in whole numbers. Subtract the *number* of *decimal places* in the *divisor* from the *number* of *decimal places* in the *dividend*, and *point off* as many *decimal* *places in the quotient as there are units in the remainder thus found.*

Example.—Divide .625 by 25.

divisor dividend quotient
Solution.— 25).625(.025 Ans.
50
125
125
remainder 0

In this example there are no decimal places in the divisor, and 3 decimal places in the dividend; therefore, there are 3 minus 0, or 3, decimal places in the quotient. One cipher has to be prefixed to the 25, to make the 3 decimal places.

151. **Example.**—Divide 6.085 by .05.

divisor dividend quotient
Solution.— .05)6.085(120.7 Ans.
5
10
10
35
35
remainder 0

In this example we divide by 5, as if the cipher were not before it. There is one more decimal place in the dividend than in the divisor; therefore, one decimal place is pointed off in the quotient.

152. **Example.**—Divide .125 by .005.

divisor dividend quotient
Solution.— .005).125(25 Ans.
10
25
25
remainder 0

In this example there are the same number of decimal places in the dividend as in the divisor; therefore, the quotient has no decimal places, and is a whole number.

153. Example.—Divide 326 by .25.

divisor dividend quotient
Solution.— .25) 326.00 (1304 Ans.
25
76
75
100
100
remainder 0

In this problem two ciphers were annexed to the dividend, to make the number of decimal places equal to the number in the divisor. The quotient is a whole number.

154. Example.—Divide .0025 by 1.25.

Solution.— 1.25) .00250 (.002 Ans.
250
remainder 0

Explanation.—In this example we are to divide .0025 by 1.25. Consider the dividend as a whole number, or 25 (disregarding the two ciphers at its left, for the present); also, consider the divisor as a whole number, or 125. It is clearly evident that the dividend 25 will not contain the divisor 125; we must, therefore, annex one cipher to the 25, thus making the dividend 250. 125 is contained twice in 250, so we place the figure 2 in the quotient. In pointing off the decimal places in the quotient, it must be remembered that there were only four decimal places in the dividend; but one cipher was annexed, thereby making 4 + 1, or 5, decimal places. Since there are 5 decimal places in the dividend and 2 decimal places in the divisor, we must point off 5 − 2, or 3, decimal places in the quotient. In order to point off 3 decimal places, two ciphers must be prefixed to the figure 2, thereby making .002 the quotient. It is not necessary to consider the ciphers at the left of a decimal when dividing, except when determining the position of the decimal point in the quotient.

155. Rule 16.—(*a*) *Place the divisor to the left of the dividend and proceed as in division of whole numbers, and in the* *quotient, point off as many decimal places as the number of decimal places in the dividend exceed those in the divisor, prefixing ciphers to the quotient, if necessary.*

(**b**) *If in dividing one number by another there be a remainder, the remainder can be placed over the divisor, as a fractional part of the quotient, but it is generally better to annex ciphers to the remainder, and continue dividing until there are 3 or 4 decimal places in the quotient, and then if there still be a remainder, terminate the quotient by the plus sign (+), which shows that it can be carried further.*

156. Example.—What is the quotient of 199 divided by 15?

Solution.— $15)199(13+\frac{4}{15}$ Ans.

$$\begin{array}{r} 15 \\ \hline 49 \\ 45 \\ \hline \textit{remainder}\quad 4 \end{array}$$

Or, $15)199.000(13.266+$ Ans.

$$\begin{array}{r} 15 \\ \hline 49 \\ 45 \\ \hline 40 \\ 30 \\ \hline 100 \\ 90 \\ \hline 100 \\ 90 \\ \hline \textit{remainder}\quad 10 \end{array}$$

$13\frac{4}{15} = 13.266+$

$\frac{4}{15} = .266+$

It very frequently happens as in the above example, that the division will never terminate. In such cases, decide to how many decimal places the division is to be carried, and carry the work one place further. If the last figure of the quotient thus obtained is 5 or a greater number, increase the preceding figure by 1, and write after it the minus sign (−), thus indicating that the quotient is not quite as large as indicated; if the figure thus obtained is less than 5, write the plus sign (+) after the quotient, thus indicating that the number is slightly greater than as indicated. In the last example, had it been desired to obtain the answer correct to four decimal places, the work would have been carried to five places, obtaining 13.26666, and the answer would have been given as 13.2667−. This remark applies to any other calculation involving decimals, when it is desired to omit some of the figures in the decimal. Thus, if it is desired to retain three decimal places in the number .2471253, it would be expressed as .247+; if it was desired to retain five decimal places, it would be expressed as .24713 −. Both the + and − signs are frequently omitted; they are seldom used outside of arithmetic, except in exact calculations, when it is desired to call particular attention to the fact that the result obtained is not quite exact.

EXAMPLES FOR PRACTICE.

157. Divide

(*a*) 101.6688 by 2.36.	Ans.	(*a*) 43.08.
(*b*) 187.12264 by 123.107.		(*b*) 1.52.
(*c*) .08 by .008.		(*c*) 10.
(*d*) .0003 by 3.75.		(*d*) .00008.
(*e*) .0144 by .024.		(*e*) .6.
(*f*) .00375 by 1.25.		(*f*) .003.
(*g*) .004 by 400.		(*g*) .00001.
(*h*) .4 by .008.		(*h*) 50.

1. In a steam engine test of an hour's duration, the horsepower developed was found to be as follows, at 10-minute intervals: 25.73, 25.64, 26.13, 25.08, 24.20, 26.7, 26.34. What was the average horsepower? Ans. 25.6886, average.

NOTE.—Add the different horsepowers together and divide by the number of tests, or 7.

2. There are 31.5 gallons in a barrel. How many barrels are there in 2,787.75 gallons? Ans. 88.5 barrels.

3. A car load of 18.75 tons of coal cost $60.75. How much was it worth per ton? Ans. $3.24 per ton.

4. A keg of $\frac{9}{16}$ by $1\frac{5}{8}$-inch boiler rivets weighs 100 pounds and contains 595 rivets. What is the weight of one of the rivets? Ans. .168 pound.

TO REDUCE A FRACTION TO A DECIMAL.

158. Example.—$\frac{3}{4}$ equals what decimal?

Solution.— 4) 3.0 0
.7 5, or $\frac{3}{4}$ = .75. Ans.

Example.—What decimal is equal to $\frac{7}{8}$?

Solution.— 8) 7.0 0 0 (.8 7 5
6 4
6 0
5 6 or $\frac{7}{8}$ = .875. Ans.
4 0
4 0
0

159. Rule 17.—*Annex ciphers to the numerator and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed.*

EXAMPLES FOR PRACTICE.

160. Reduce the following common fractions to decimals:

	Ans.
(*a*) $\frac{15}{32}$.	(*a*) .46875.
(*b*) $\frac{7}{8}$.	(*b*) .875.
(*c*) $\frac{21}{32}$.	(*c*) .65625.
(*d*) $\frac{51}{64}$.	(*d*) .796875.
(*e*) $\frac{4}{25}$.	(*e*) .16.
(*f*) $\frac{5}{8}$.	(*f*) .625.
(*g*) $\frac{10}{200}$.	(*g*) .05.
(*h*) $\frac{4}{1000}$.	(*h*) .004.

161. To reduce inches to decimal parts of a foot:

Example.—What decimal part of a foot is 9 inches?

Solution.—Since there are 12 inches in one foot, 1 inch is $\frac{1}{12}$ of a foot, and 9 inches is 9 × $\frac{1}{12}$ or $\frac{9}{12}$ of a foot. This reduced to a decimal by the above rule, shows what decimal part of a foot 9 inches is.

1 2) 9.0 0 (.7 5 of a foot. Ans.
8 4
6 0
6 0
0

162. Rule 18.—(*a*) *To reduce inches to decimal parts of a foot, divide the number of inches by 12.*

(*b*) *Should the resulting decimal be an unending one and it is desired to terminate the division at some point, say the fourth decimal place, carry the division one place farther, and if the fifth figure is 5 or greater, increase the fourth figure by one and omit the sign +.*

EXAMPLES FOR PRACTICE.

163. Reduce to the decimal part of a foot:

	Ans.
(*a*) 3 in.	(*a*) .25.
(*b*) 4½ in.	(*b*) .375.
(*c*) 5 in.	(*c*) .4167.
(*d*) 6⅝ in.	(*d*) .5521.
(*e*) 11 in.	(*e*) .9167.

1. The lengths of belting required to connect three countershafts with the main line shaft were found with a tape measure to be 27 ft. 4 in., 23 ft. 8 in., and 38 ft. 6 in. How many feet of belting were necessary? Ans. 89.5 ft.

2. The stroke of an engine is 14 inches. What is the length of the crank measured from the center of shaft to center of crank-pin in feet? Ans. .5833 + foot.

3. A steam pipe fitted with an expansion joint was found to expand 1.668 inches when steam was admitted to it. How much was its expansion in decimal parts of a foot? Ans. .139 foot.

TO REDUCE A DECIMAL TO A FRACTION.

164. Example.—Reduce .125 to a fraction.

Solution.—$.125 = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8}$. Ans.

Example.—Reduce .875 to a fraction.

Solution.—$.875 = \frac{875}{1000} = \frac{35}{40} = \frac{7}{8}$. Ans.

165. Rule 19.—*Under the figures of the decimal, place the digit 1 with as many ciphers at its right as there are decimal places in the decimal, and reduce the resulting fraction to its lowest terms by dividing both numerator and denominator by the same number.*

EXAMPLES FOR PRACTICE.

166. Reduce the following to common fractions:

(*a*) .125.	Ans.	(*a*) $\frac{1}{8}$.
(*b*) .625.		(*b*) $\frac{5}{8}$.
(*c*) .3125.		(*c*) $\frac{5}{16}$.
(*d*) .04.		(*d*) $\frac{1}{25}$.
(*e*) .06.		(*e*) $\frac{3}{50}$.
(*f*) .75.		(*f*) $\frac{3}{4}$.
(*g*) .15625.		(*g*) $\frac{5}{32}$.
(*h*) .875.		(*h*) $\frac{7}{8}$.

TO EXPRESS A DECIMAL APPROXIMATELY AS A FRACTION HAVING A GIVEN DENOMINATOR.

167. EXAMPLE.—Express .5827 in 64ths.

SOLUTION.—$.5827 \times \frac{64}{64} = \frac{37.2928}{64}$, say $\frac{37}{64}$.

Hence, $.5827 = \frac{37}{64}$, nearly. Ans.

EXAMPLE.—Express .3917 in 12ths.

SOLUTION.—$.3917 \times \frac{12}{12} = \frac{4.7004}{12}$, say $\frac{5}{12}$.

Hence, $.3917 = \frac{5}{12}$, nearly. Ans.

168. Rule 20.—*Reduce 1 to a fraction having the given denominator. Multiply the given decimal by the fraction so obtained, and the result will be the fraction required.*

EXAMPLES FOR PRACTICE.

169. Express

(*a*) .625 in 8ths.	Ans.	(*a*) $\frac{5}{8}$.
(*b*) .3125 in 16ths.		(*b*) $\frac{5}{16}$.
(*c*) .15625 in 32ds.		(*c*) $\frac{5}{32}$.
(*d*) .77 in 64ths.		(*d*) $\frac{49}{64}$.
(*e*) .81 in 48ths.		(*e*) $\frac{39}{48}$.
(*f*) .923 in 96ths.		(*f*) $\frac{89}{96}$.

170. The sign for dollars is $. It is read dollars. $25 is read 25 dollars.

Since there are 100 cents in a dollar, one cent is 1-one-hundredth of a dollar; the first two figures of a decimal part of a dollar represent *cents*. Since a mill is $\frac{1}{10}$ of a cent, or $\frac{1}{1000}$ of a dollar, the third figure represents mills.

Thus, $25.16 is read twenty-five dollars and sixteen cents; $25.168 is read twenty-five dollars, sixteen cents and eight mills.

171. The **vinculum** ——, **parenthesis** ( ), **bracket** [ ], and **brace** { } are called **symbols of aggregation,** and are used to include numbers which are to be considered together; thus, $13 \times \overline{8 - 3}$, or $13 \times (8 - 3)$, shows that 3 is to be taken from 8 before multiplying by 13.

$$13 \times (8 - 3) = 13 \times 5 = 65. \quad \text{Ans.}$$

$$13 \times \overline{8 - 3} = 13 \times 5 = 65. \quad \text{Ans.}$$

When the vinculum or parenthesis is not used, we have

$$13 \times 8 - 3 = 104 - 3 = 101. \quad \text{Ans.}$$

172. In any series of numbers connected by the signs +, −, ×, and ÷, the operations indicated by the signs must be performed in order from left to right, *except* that no addition or subtraction may be performed if a sign of multiplication or division *follows* the number on the *right* of a sign of addition or subtraction, until the indicated multiplication or division has been performed. In all cases the sign of multiplication takes the precedence, the reason being that when two or more numbers or expressions are connected by the sign of multiplication, the numbers thus connected are regarded as factors of the product indicated, and not as separate numbers.

EXAMPLE.—What is the value of $4 \times 24 - 8 + 17$?

SOLUTION.—Performing the operations in order from left to right, $4 \times 24 = 96$; $96 - 8 = 88$; $88 + 17 = 105$. Ans.

173. EXAMPLE.—What is the value of the following expression: $1{,}296 \div 12 + 160 - 22 \times 3\frac{1}{2} = ?$

SOLUTION.—$1{,}296 \div 12 = 108$; $108 + 160 = 268$; here we cannot subtract 22 from 268 because the sign of multiplication *follows* 22; hence, multiplying 22 by $3\frac{1}{2}$, we get 77, and $268 - 77 = 191$. Ans.

Had the above expression been written $1{,}296 \div 12 + 160 - 22 \times 3\frac{1}{2} \div 7 + 25$, it would have been necessary to have divided $22 \times 3\frac{1}{2}$ by 7 before subtracting, and the final result would have been $22 \times 3\frac{1}{2} = 77$; $77 \div 7 = 11$; $268 - 11 = 257$; $257 + 25 = 282$. Ans. In other words, it is necessary to perform *all* of the multiplication or division included between the signs + and −, or − and +, before adding or subtracting. Also, had the expression been written $1{,}296 \div 12 + 160 - 24\frac{1}{2} \div 7 \times 3\frac{1}{2} + 25$, it would have been necessary to have multiplied $3\frac{1}{2}$ by 7 before dividing $24\frac{1}{2}$, since the sign of multiplication takes the precedence, and the final result would have been $3\frac{1}{2} \times 7 = 24\frac{1}{2}$; $24\frac{1}{2} \div 24\frac{1}{2} = 1$; $268 - 1 = 267$; $267 + 25 = 292$. Ans.

It likewise follows that if a succession of multiplication and division signs occurs, the indicated operations must not be performed in order, from left to right—the multiplication must be performed first. Thus, $24 \times 3 \div 4 \times 2 \div 9 \times 5 = \frac{1}{5}$. Ans. In order to obtain the same result that would be obtained by performing the indicated operations in order, from left to right, symbols of aggregation must be used. Thus, by using two vinculums, the last expression becomes $24 \times \overline{3 \div 4} \times \overline{2 \div 9} \times 5 = 20$, the same result that would be obtained by performing the indicated operations in order, from left to right.

EXAMPLES FOR PRACTICE.

174. Find the values of the following expressions:

			Ans.
(*a*)	$(8 + 5 - 1) \div 4$.	(*a*)	3.
(*b*)	$5 \times 24 - 32$.	(*b*)	88.
(*c*)	$5 \times 24 \div 15$.	(*c*)	8.
(*d*)	$144 - 5 \times 24$.	(*d*)	24.
(*e*)	$(1{,}691 - 540 + 559) \div 3 \times 57$.	(*e*)	10.
(*f*)	$2{,}080 + 120 - 80 \times 4 - 1{,}670$.	(*f*)	210.
(*g*)	$(\overline{90 + 60} \div 25) \times 5 - 29$.	(*g*)	1.
(*h*)	$\overline{90 + 60} \div 25 \times 5$.	(*h*)	1.2.

ARITHMETIC.

(CONTINUED.)

PERCENTAGE.

175. Percentage is the process of calculating by *hundredths.*

176. The *term* **per cent.** is an abbreviation of the Latin words *per centum*, which mean *by the hundred.* A certain per cent. of a number is the number of hundreds of that number which is indicated by the number of units in the per cent. Thus, 6 per cent. of 125 is $125 \times \frac{6}{100} = 7.5$; 25 per cent. of 80 is $80 \times \frac{25}{100} = 20$; 43 per cent. of 432 pounds is $432 \times \frac{43}{100} = 185.76$ pounds.

177. The **sign** of per cent. is %, and is read *per cent.* Thus, 6% is read *six per cent.;* $12\frac{1}{2}\%$ is read *twelve and one-half per cent.*, etc.

When expressing the per cent. of a number to use in calculations it is customary to express it decimally instead of fractionally. Thus, instead of expressing 6%, 25%, and 43% as $\frac{6}{100}$, $\frac{25}{100}$, and $\frac{43}{100}$, it is usual to express them as .06, .25, and .43.

The following table will show how any per cent. can be expressed either as a decimal or as a fraction:

TABLE 3.

Per Cent.	Decimal.	Fraction.	Per Cent.	Decimal.	Fraction.
1%.....	.01	$\frac{1}{100}$	150 %....	1.50	$\frac{150}{100}$ or $1\frac{1}{2}$
2%.....	.02	$\frac{2}{100}$ or $\frac{1}{50}$	500 %....	5.00	$\frac{500}{100}$ or 5
5%.....	.05	$\frac{5}{100}$ or $\frac{1}{20}$	$\frac{1}{4}\%$....	.0025	$\frac{\frac{1}{4}}{100}$ or $\frac{1}{400}$
10%.....	.10	$\frac{10}{100}$ or $\frac{1}{10}$	$\frac{1}{2}\%$....	.005	$\frac{\frac{1}{2}}{100}$ or $\frac{1}{200}$
25%.....	.25	$\frac{25}{100}$ or $\frac{1}{4}$	$1\frac{1}{2}\%$....	.015	$\frac{1\frac{1}{2}}{100}$ or $\frac{3}{200}$
50%.....	.50	$\frac{50}{100}$ or $\frac{1}{2}$	$8\frac{1}{3}\%$....	$.08\frac{1}{3}$	$\frac{8\frac{1}{3}}{100}$ or $\frac{1}{12}$
75%.....	.75	$\frac{75}{100}$ or $\frac{3}{4}$	$12\frac{1}{2}\%$....	.125	$\frac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$
100%.....	1.00	$\frac{100}{100}$ or 1	$16\frac{2}{3}\%$....	$.16\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}$ or $\frac{1}{6}$
125%.....	1.25	$\frac{125}{100}$ or $1\frac{1}{4}$	$62\frac{1}{2}\%$....	.625	$\frac{62\frac{1}{2}}{100}$ or $\frac{5}{8}$

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178. The names of the different elements used in percentage are: the *base*, the *rate per cent.*, the *percentage*, the *amount*, and the *difference*.

179. The **base** is the number on which the per cent. is computed.

180. The **rate** is the number of hundredths of the base to be taken.

181. The **percentage** is the part, or number of *hundredths*, of the base indicated by the rate; or the percentage is the result obtained by multiplying the base by the rate.

Thus, when it is stated that 7% of \$25 is \$1.75, \$25 is the base, 7% is the rate, and \$1.75 is the percentage.

182. The **amount** is the sum of the base and percentage.

183. The **difference** is the remainder obtained by subtracting the percentage from the base.

Thus, if a man has \$180, and he earns 6% more, he will have altogether $\$180 + \$180 \times .06$, or $\$180 + \$10.80 = \$190.80$. Here \$180 is the base, 6% the rate, \$10.80 the percentage, and \$190.80 the *amount*.

Again, if an engine of 125 horsepower uses 16% of it in overcoming friction and other resistances, the amount left for performing useful work is $125 - 125 \times .16 = 125 - 20 = 105$ horsepower. Here 125 is the base, 16% the rate, 20 the percentage, and 105 the *difference*.

184. From the foregoing it is evident that, to find the percentage, the base must be multiplied by the rate. Hence the following

Rule 21.—*To find the percentage, multiply the base by the rate, expressed decimally.*

EXAMPLE.—Out of a lot of 300 boiler tubes 76% were used in a boiler. How many tubes were used?

SOLUTION.—76%, the rate, expressed decimally, is .76; the base is 300; hence, the number of tubes used, or the percentage, is by the above rule $300 \times .76 = 228$ tubes. Ans.

Expressing the rule as a formula, we have

$$\textit{percentage} = \textit{base} \times \textit{rate}.$$

185. When the percentage and rate are given, the base may be found by dividing the percentage by the rate. For, suppose that 12 is 6%, or $\frac{6}{100}$, of some number; then, 1%, or $\frac{1}{100}$, of the number is 12 ÷ 6 or 2. Consequently, if 2 = 1%, or $\frac{1}{100}$, 100%, or $\frac{100}{100} = 2 \times 100 = 200$. But, since the same result may be arrived at by dividing 12 by .06, for 12 ÷ .06 = 200 it follows that

Rule 22.—*When the percentage and rate are given, to find the base, divide the percentage by the rate, expressed decimally.*

Formula, *base* = *percentage* ÷ *rate.*

EXAMPLE.—76% of a lot of boiler tubes are used in the construction of a boiler. If the number of tubes used were 228, how many tubes were in the lot?

SOLUTION.—Here 228 is the percentage, and 76%, or .76, is the rate; hence, applying the rule,

228 ÷ .76 = 300 tubes. Ans.

186. When the base and percentage are given to find the rate, the rate may be found, expressed decimally, by dividing the percentage by the base. For suppose that it is desired to find what per cent. 12 is of 200. 1% of 200 is 200 × .01 = 2. Now, if 1% is 2, 12 is evidently as many per cent. as the number of times that 2 is contained in 12, or 12 ÷ 2 = 6%. But the same result may be obtained by dividing 12, the percentage, by 200, the base, since 12 ÷ 200 = .06 = 6%. Hence,

Rule 23.—*When the percentage and base are given, to find the rate, divide the percentage by the base, and the result will be the rate, expressed decimally.*

Formula, *rate* = *percentage* ÷ *base.*

EXAMPLE.—Out of a lot of 300 boiler tubes, 228 were used. What per cent. of the total number was used?

SOLUTION.—Here 300 is the base and 228 the percentage; hence, applying rule,

rate = 228 ÷ 300 = .76 = 76%. Ans.

EXAMPLE.—What per cent. of 875 is 25?

SOLUTION.—Here 875 is the base, and 25 is the percentage; hence, applying rule,

$25 \div 875 = .02\frac{6}{7} = 2\frac{6}{7}\%$. Ans.

PROOF.—$875 \times .02\frac{6}{7} = 25$.

EXAMPLES FOR PRACTICE.

187. What per cent. of

	Ans.
(*a*) 360 is 90?	(*a*) 25%.
(*b*) 900 is 360?	(*b*) 40%.
(*c*) 125 is 25?	(*c*) 20%.
(*d*) 150 is 750?	(*d*) 500%.
(*e*) 280 is 112?	(*e*) 40%.
(*f*) 400 is 200?	(*f*) 50%.
(*g*) 47 is 94?	(*g*) 200%.
(*h*) 500 is 250?	(*h*) 50%.

188. The amount may be found when the base and rate are given, by multiplying the base by 1 plus the rate, expressed decimally. For suppose that it is desired to find the amount when 200 is the base and 6% is the rate. The percentage is $200 \times .06 = 12$, and, according to definition, Art. **182,** the amount is $200 + 12 = 212$. But the same result may be obtained by multiplying 200 by $1 + .06$, or 1.06, since $200 \times 1.06 = 212$. Hence,

Rule 24.—*When the base and rate are given, to find the amount, multiply the base by 1 plus the rate, expressed decimally.*

Formula, $amount = base \times (1 + rate)$.

Example.—If a man earned $725 in a year, and the next year 10% more, how much did he earn the second year?

Solution.—Here 725 is the base and 10% is the rate, and the amount is required. Hence, applying the rule,

$725 \times 1.10 = \$797.50$. Ans.

189. When the base and rate are given, the difference may be found by multiplying the base by 1 minus the rate, expressed decimally. For suppose that it is desired to find the difference when the base is 200 and the rate is 6%. The percentage is $200 \times .06 = 12$; and, according to definition, Art. **183,** the difference $= 200 - 12 = 188$. But the same result may be obtained by multiplying 200 by $1 - .06$, or $.94$, since $200 \times .94 = 188$. Hence,

Rule 25.—*When the base and rate are given, to find the difference, multiply the base by 1 minus the rate, expressed decimally.*

Formula, $difference = base \times (1 - rate)$.

Example.—Out of a lot of 300 boiler tubes all but 24% were used in one boiler; how many tubes were used?

Solution.—Here 300 is the base, 24% is the rate, and it is desired to find the difference. Hence, applying the rule.

$$300 \times (1 - .24) = 228 \text{ tubes. Ans.}$$

190. When the amount and rate are given, the base may be found by dividing the amount by 1 plus the rate. For suppose that it is known that 212 equals some number increased by 6% of itself. Then, it is evident that 212 equals 106% of the number (base) that it is desired to find. Consequently, if $212 = 106\%$, $1\% = \frac{212}{106} = 2$, and $100\% = 2 \times 100 = 200$ = the base. But the same result may be obtained by dividing 212 by $1 + .06$ or 1.06, since $212 \div 1.06 = 200$. Hence,

Rule 26.—*When the amount and rate are given, to find the base, divide the amount by 1 plus the rate, expressed decimally.*

Formula, $base = amount \div (1 + rate)$.

Example.—The theoretical discharge of a certain pump when running at a piston speed of 100 feet per minute is 278,910 gallons per day of 10 hours. Owing to leakage and other defects this value is 25% greater than the actual discharge. What is the actual discharge?

Solution.—Here 278,910 equals the actual discharge (base) increased by 25% of itself. Consequently, 278,910 is the amount, 25% is the rate, and applying rule,

$$\text{actual discharge} = 278{,}910 \div 1.25 = 223{,}128 \text{ gallons. Ans.}$$

191. When the difference and rate are given, the base may be found by dividing the difference by 1 minus the rate. For suppose that 188 equals some number less 6% of itself. Then, 188 evidently equals $100 - 6 = 94\%$ of some number. Consequently, if $188 = 94\%$, $1\% = 188 \div 94 = 2$, and $100\% = 2 \times 100 = 200$. But the same result may be obtained by dividing 188 by $1 - .06$, or .94, since $188 \div .94 = 200$. Hence,

Rule 27.—*When the difference and rate are given, to find the base, divide the difference by 1 minus the rate, expressed decimally.*

Formula, $base = difference \div (1 - rate)$.

Example.—From a lot of boiler tubes 76% were used in the construction of a boiler. If there were 72 tubes unused, how many tubes were in the lot?

Solution.—Here 72 is the difference and 76% is the rate. Applying rule,

72 ÷ (1 − .76) = 300 tubes. Ans.

Example.—The theoretical number of foot-pounds of work per minute required to operate a boiler feed-pump is 127,344. If 30% of the total number actually required be allowed for friction, leakage, etc., how many foot-pounds are actually required to work the pump?

Solution.—Here the number actually required is the base; hence, 127,344 is the difference, and 30% is the rate. Applying the rule,

127,344 ÷ (1 − .30) = 181,920 foot-pounds. Ans.

192. **Example.**—A certain chimney gave a draft of 2.76 inches of water. By increasing the height 20 feet the draft was increased to 3 inches of water. What was the gain per cent.?

Solution.—Here it is evident that 3 inches is the amount and that 2.76 inches is the base. Consequently, 3 − 2.76 = .24 inch is the percentage, and it is required to find the rate. Hence, applying rule **23**,

gain per cent. = .24 ÷ 2.76 = .087 = 8.7%. Ans.

193. **Example.**—A certain chimney gave a draft of 3 inches of water. After an economizer had been put in the draft was reduced to 1.2 inches of water. What was the loss per cent.?

Solution.—Here it is evident 1.2 inches is the difference (since it equals 3 inches diminished by a certain per cent. loss of itself) and 3 inches is the base. Consequently, 3 − 1.2 = 1.8 inches is the percentage. Hence, applying rule **23**,

loss per cent. = 1.8 ÷ 3 = .60 = 60%. Ans.

194. To find the gain or loss per cent.:

Rule 28.—*Find the difference between the initial and final values; divide this difference by the initial value.*

Example.—If a man buys a steam engine for $1,860 and some time afterwards purchases a condenser for 25% of the cost of the engine, does he gain or lose, and how much per cent., if he sells both engine and condenser for $2,100?

Solution.—The cost of the condenser was $1,860 × .25 = $465; consequently, the total initial value, or cost, was $1,860 + $465 = $2,325. Since he sold them for $2,100, he lost $2,325 − $2,100 = $225. Hence, applying rule,

225 ÷ 2,325 = .0968 = 9.68% loss. Ans.

EXAMPLES FOR PRACTICE.

195. Solve the following:

(*a*) What is 12½% of $900?	Ans.	(*a*) $112.50.
(*b*) “ “ ⅘% “ 627?		(*b*) 5.016.
(*c*) “ “ 33⅓% “ 54?		(*c*) 18.
(*d*) 101 is 68¾% of what number?		(*d*) $146\frac{10}{11}$.
(*e*) 784 “ 83⅓% “ “ “		(*e*) 940.8.
(*f*) What % of 960 is 160?		(*f*) 16⅔%.
(*g*) “ “ “ $3,606 is $450¾?		(*g*) 12½%.
(*h*) “ “ “ 280 is 112?		(*h*) 40%.

1. A steam plant consumed an average of 3,640 pounds of coal per day. The engineer made certain alterations which resulted in a saving of 250 pounds per day. What was the per cent. of coal saved? Ans. 7%, nearly.

2. If the speed of an engine running at 126 revolutions per minute should be increased 6½%, how many revolutions per minute would it then make? Ans. 134.19 revolutions.

3. The list price of an engine was $1,400; of a boiler, $1,150, and of the necessary fittings for the two, $340. If 25% discount was allowed on the engine, 22% on the boiler, and 12½% on the fittings, what was the actual cost of the plant? Ans. $2,244.50.

4. If I lend a man $1,100, and this is 18½% of the amount that I have on interest, how much money have I on interest? Ans. $5,945.95.

5. A test showed that an engine developed 190.4 horsepower, 15% of which was consumed in friction. How much power was available for use? Ans. 161.84 H. P.

6. By adding a condenser to a steam engine, the power was increased 14%, and the consumption of coal per horsepower per hour was decreased 20%. If the engine could originally develop 50 horsepower and required 3½ pounds of coal per horsepower per hour, what would be the total weight of coal used in an hour, with the condenser, assuming the engine to run full power? Ans. 159.6 pounds.

DENOMINATE NUMBERS.

196. A **denominate number** is a concrete number, and may be either simple or compound, as 8 quarts, 5 feet, ten inches, etc.

197. A **simple denominate number** consists of units of but one denomination, as 16 cents, 10 hours, 5 dollars, etc.

198. A **compound denominate number** consists of units of two or more denominations of a similar kind, as 3 yards, 2 feet, 1 inch ; 3 pounds, 5 ounces.

199. In **whole numbers** and in **decimals,** the *law* of increase and decrease is on the scale of 10, but in **compound** or **denominate numbers** the scale varies.

MEASURES.

200. A **measure** is a *standard unit* established by *law* or *custom*, by which *quantity* of any kind is measured. The *standard unit* of **dry measure** is the Winchester bushel; of **weight,** the pound; of **liquid measure,** the gallon, etc.

201. Measures are of six kinds:

1.	Extension.	4.	Time.
2.	Weight.	5.	Angles.
3.	Capacity.	6.	Money or value.

MEASURES OF EXTENSION.

202. **Measures of extension** are used in measuring lengths, distances, surfaces, and solids.

LINEAR MEASURE.

TABLE 4.

		Abbreviation.	in.	ft.	yd.	rd.	fur.
12 inches (in.)	= 1 foot . .	ft.					
3 feet	= 1 yard . .	yd.	36 =	3			
5½ yards	= 1 rod . .	rd.	198 =	16½ =	5½		
40 rods	= 1 furlong .	fur.	7,920 =	660 =	220 =	40	
8 furlongs	= 1 mile . .	mi.	63,360 =	5,280 =	1,760 =	320 =	8

SQUARE MEASURE.

TABLE 5.

144 square inches (sq. in.) . .	= 1 square foot	sq. ft.
9 square feet	= 1 square yard	sq. yd.
30¼ square yards	= 1 square rod	sq. rd.
160 square rods	= 1 acre	A.
640 acres	= 1 square mile	sq. mi.

sq. mi.	A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
1	= 640	= 102,400	= 3,097,600	= 27,878,400	= 4,014,489,600

CUBIC MEASURE.

TABLE 6.

1728 cubic inches (cu. in.) . . . . = 1 cubic foot . . . . . . cu. ft.
27 cubic feet . . . . = 1 cubic yard . . . . . . cu. yd.
128 cubic feet . . . . = 1 cord . . . . cd.
24¾ cubic feet . . . . = 1 perch . . . . P.

cu. yd. cu. ft. cu. in.
1 = 27 = 46,656

MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT.

TABLE 7.

16 ounces (oz.) . . . . = 1 pound . . . . lb.
100 pounds . . . . = 1 hundred-weight . . . . cwt.
20 cwt., or 2,000 lb. . . . . = 1 ton . . . . T.

T. cwt. lb. oz.
1 = 20 = 2,000 = 32,000

203. The ounce is divided into halves, quarters, etc. Avoirdupois weight is used for weighing coarse and heavy articles.

LONG TON TABLE.

TABLE 8.

16 ounces . . . . = 1 pound . . . . lb.
112 pounds . . . . = 1 hundred-weight . . . . cwt.
20 cwt., or 2,240 lb. . . . . = 1 ton . . . . T.

In all the calculations throughout this and the succeeding volumes, 2,000 pounds will be considered one ton, unless the long ton (2,240 pounds) is especially mentioned.

TROY WEIGHT.

TABLE 9.

24 grains (gr.) . . . . = 1 pennyweight . . . . pwt.
20 pennyweight . . . . = 1 ounce . . . . oz.
12 ounces . . . . = 1 pound . . . . lb.

lb. oz. pwt. gr.
1 = 12 = 240 = 5,760

204. Troy weight is used in weighing gold and silver ware, jewels, etc. It is used by jewelers.

MEASURES OF CAPACITY.

LIQUID MEASURE.

TABLE 10.

4 gills (gi.) . . . . = 1 pint . . . . pt.
2 pints . . . . = 1 quart. . . . . qt.
4 quarts. . . . . = 1 gallon . . . . gal.
31½ gallons . . . . = 1 barrel . . . . bbl.
63 gallons . . . . = 1 hogshead . . . . hhd.

hhd. bbl. gal. qt. pt. gi.
1 = 2 = 63 = 252 = 504 = 2,016

DRY MEASURE.

TABLE 11.

2 pints (pt.) . . . . = 1 quart. . . . . qt.
8 quarts . . . . = 1 peck . . . . pk.
4 pecks. . . . . = 1 bushel . . . . bu.

bu. pk. qt. pt.
1 = 4 = 32 = 64

MEASURE OF TIME.

TABLE 12.

60 seconds (sec.) . . . . = 1 minute . . . . min.
60 minutes . . . . = 1 hour . . . . hr.
24 hours . . . . = 1 day . . . . da.
7 days . . . . = 1 week . . . . wk.
365 days }
12 months } . . . . = 1 common year . . . yr.
366 days . . . . = 1 leap year.
100 years . . . . = 1 century.

NOTE.—It is customary to consider one month as 30 days.

MEASURE OF ANGLES OR ARCS.

TABLE 13.

60 seconds (″) . . . . = 1 minute . . . . ′.
60 minutes . . . . = 1 degree . . . . °.
90 degrees . . . . = 1 right angle or quadrant ∟.
360 degrees . . . . = 1 circle . . . . cir.

cir.
1 = 360° = 21,600′ = 1,296,000″

MEASURE OF MONEY.

UNITED STATES MONEY.

TABLE 14.

10 mills (m.) . . . . = 1 cent . . . . ct.
10 cents . . . . = 1 dime . . . . d.
10 dimes . . . . = 1 dollar . . . . $.
10 dollars . . . . = 1 eagle . . . . E.

E. $ d. ct. m.
1 = 10 = 100 = 1,000 = 10,000

MISCELLANEOUS TABLE.

TABLE 15.

12 things are 1 dozen.
12 dozen are 1 gross.
12 gross are 1 great gross.
2 things are 1 pair.
20 things are 1 score.
1 league is 3 miles.
1 fathom is 6 feet.

1 meter is nearly 39.37 inches.
1 hand is 4 inches.
1 palm is 3 inches.
1 span is 9 inches.
24 sheets are 1 quire.
20 quires, or 480 sheets, are 1 ream.
1 bushel contains 2,150.4 cubic in.

1 U. S. standard gallon (also called a wine gallon) contains 231 cubic in.
1 U. S. standard gallon of water weighs 8.355 pounds, nearly.
1 cubic foot of water contains 7.481 U. S. standard gallons, nearly.
1 British imperial gallon weighs 10 pounds.

It will be of great advantage to the student to carefully memorize all of the above tables.

REDUCTION OF DENOMINATE NUMBERS.

205. Reduction of denominate numbers is the process of changing their denomination without changing their value. They may be changed from a higher to a lower denomination or from a lower to a higher—either is reduction. As,

2 hours = 120 minutes.
32 ounces = 2 pounds.

206. Principle.—Denominate numbers are changed to *lower* denominations by *multiplying*, and to *higher* denominations by *dividing*.

To reduce denominate numbers to lower denominations:

207. Example.—Reduce 5 yd. 2 ft. 7 in. to inches.

Solution.—

yd.	ft.	in.
5	2	7
3		
15 ft.		
2 ft.		
17 ft.		
12		
34		
17		
204 in.		
7 in.		
211 inches. Ans.		

Explanation.—Since there are 3 feet in 1 yard, in 5 yards there are 5 × 3, or 15 feet, and 15 feet plus 2 feet = 17 feet. There are 12 inches in a foot; therefore, 12 × 17 = 204 inches, and 204 inches plus 7 inches = 211 inches = number of inches in 5 yards 2 feet and 7 inches. Ans.

208. Example.—Reduce 6 hours to seconds.

Solution.—

6 hours.
60
360 minutes.
60
21600 seconds. Ans.

Explanation.—As there are 60 minutes in one hour, in six hours there are 6 × 60, or 360 minutes; as there are no minutes to add, we multiply 360 minutes by 60, to get the number of seconds.

209. In order to avoid mistakes, if any denomination be omitted, represent it by a cipher. Thus, before reducing 3 rods 6 inches to inches, insert a cipher for yards and a cipher for feet; as,

rd.	yd.	ft.	in.
3	0	0	6

210. **Rule.**—*Multiply the number representing the highest denomination by the number of units in the next lower* *required to make one of the higher denomination, and to the product add the number of given units of that lower denomination. Proceed in this manner until the number is reduced to the required denomination.*

EXAMPLES FOR PRACTICE.

211. Reduce

(*a*)	4 rd. 2 yd. 2 ft. to ft.	Ans.	(*a*)	74 ft.
(*b*)	4 bu. 3 pk. 2 qt. to qt.		(*b*)	154 qt.
(*c*)	13 rd. 5 yd. 2 ft. to ft.		(*c*)	231.5 ft.
(*d*)	5 mi. 100 rd. 10 ft. to ft.		(*d*)	28,060 ft.
(*e*)	8 lb. 4 oz. 6 pwt. to gr.		(*e*)	48,144 gr.
(*f*)	52 hhd. 24 gal. 1 pt. to pt.		(*f*)	26,401 pt.
(*g*)	5 cir. 16° 20′ to minutes.		(*g*)	108,980′.
(*h*)	14 bu. to qt.		(*h*)	448 qt.

To reduce lower to higher denominations:

212. Example.—Reduce 211 in. to higher denominations.

Solution.—
12)211 in.
3)17 ft. + 7 in.
5 yd. + 2 ft. Ans.

Explanation.—There are 12 inches in 1 foot; therefore, 211 divided by 12 = 17 feet and 7 inches over. There are 3 ft. in 1 yd.; therefore, 17 ft. divided by 3 = 5 yd. and 2 ft. over. The last quotient and the two remainders constitute the answer, 5 yd. 2 ft. 7 in.

213. Example.—Reduce 14,135 gi. to higher denominations.

Solution.—
4)14135
2)3533 pt. 3 gi.
4)1766 qt. 1 pt.
441 gal. 2 qt.
31.5)441.0(14 bbl.
315
1260
1260

Explanation.—There are 4 gi. in 1 pt., and in 14,135 gi. there are as many pints as 4 is contained in 14,135, or 3,533 pt. and 3 gi. remaining. There are 2 pt. in 1 qt., and in 3,533 pt. there are 1,766 qt. and 1 pt. remaining. There are 4 qt. in 1 gal., and in 1,766 qt. there are 441 gal. and 2 qt. remaining. There are 31½ gal. in 1 bbl., and in 441 gal. there are 14 bbl.

The last quotient and the three remainders constitute the answer, 14 bbl. 2 qt. 1 pt. 3 gi.

214. Rule 30.—*Divide the number representing the denomination given, by the number of units of this denomination required to make one unit of the next higher denomination. The remainder will be of the same denomination, but the quotient will be of the next higher. Divide this quotient by the number of units of its denomination required to make one unit of the next higher. Continue until the highest denomination is reached, or until there is not enough of a denomination left to make one of the next higher. The last quotient and the remainders constitute the required result.*

EXAMPLES FOR PRACTICE.

215. Reduce to units of higher denominations:

(*a*) 7,460 sq. in.; (*b*) 7,580 sq. yd.; (*c*) 148,760 cu. in.; (*d*) 17,651″; (*e*) 8,000 gi.; (*f*) 36,450 lb.

Ans.
(*a*) 5 sq. yd. 6 sq. ft. 116 sq. in.
(*b*) 1 A. 90 sq. rd. 17 sq. yd. 4 sq. ft. 72 sq. in.
(*c*) 3 cu. yd. 5 cu. ft. 152 cu. in.
(*d*) 4° 54′ 11″.
(*e*) 3 hhd. 61 gal.
(*f*) 18 T. 4 cwt. 50 lb.

ADDITION OF DENOMINATE NUMBERS.

216. Example.—Find the sum of 3 cwt. 46 lb. 12 oz.; 8 cwt. 12 lb. 13 oz.; 12 cwt. 50 lb. 13 oz.; 27 lb. 4 oz.

Solution.—

T.	cwt.	lb.	oz.	
0	3	46	12	
0	8	12	13	
0	12	50	13	
0	0	27	4	
1	4	37	10	Ans.

Explanation.—Begin to add at the right-hand column: $4 + 13 + 13 + 12 = 42$ ounces; as 16 ounces make 1 pound, 42 ounces $\div 16 = 2$ and a remainder of 10 ounces, or 2 pounds and 10 ounces. Place 10 ounces under ounce column and add 2 pounds to the next or pound column. Then, $2 + 27 + 50 + 12 + 46 = 137$ pounds; as 100 pounds make a hundred-weight, $137 \div 100 = 1$ hundred-weight and a remainder of 37 pounds. Place the 37 under the pound column, and add 1 hundred-weight to the next or hundred-weight column. Next, $1 + 12 + 8 + 3 = 24$ hundred-weight. 20 hundred-weight make a ton; therefore $24 \div 20 = 1$ ton and 4 hundred-weight remaining. Hence, the sum is 1 ton 4 hundred-weight 37 pounds 10 ounces. Ans.

217. **Example.**—What is the sum of 2 rd. 3 yd. 2 ft. 5 in.; 6 rd. 1 ft. 10 in.; 17 rd. 11 in.; 4 yd. 1 ft.?

Solution.—

	rd.	yd.	ft.	in.	
	2	3	2	5	
	6	0	1	10	
	17	0	0	11	
	0	4	1	0	
	26	3½	0	2	
or	26	3	1	8	Ans.

Explanation.—The sum of the numbers in the first column = 26 inches, or 2 feet and 2 inches remaining. The sum of the numbers in the next column plus 2 feet = 6 feet, or 2 yards and 0 feet remaining. The sum of the next column plus 2 yards = 9 yards, or $9 \div 5\frac{1}{2} = 1$ rod and $3\frac{1}{2}$ yards remaining. The sum of the next column plus 1 rod = 26 rods. To avoid fractions in the sum, the ½ yard is reduced to 1 foot and 6 inches, which added to 26 rods 3 yards 0 feet and 2 inches = 26 rods 3 yards 1 foot 8 inches. Ans.

218. **Example.**—What is the sum of 47 ft. and 3 rd. 2 yd. 2 ft. 10 in.?

Solution.—When 47 ft. is reduced it equals 2 rd. 4 yd. 2 ft., which can be added to 8 rd. 2 yd. 2 ft. 10 in. Thus,

	rd.	yd.	ft.	in.	
	3	2	2	10	
	2	4	2	0	
	6	1½	1	10	
or	6	2	0	4	Ans.

219. Rule 31.—*Place the numbers so that like denominations are under each other. Begin at the right-hand column, and add. Divide the sum by the number of units of this denomination required to make one unit of the next higher. Place the remainder under the column added, and carry the quotient to the next column. Continue in this manner until the highest denomination given is reached.*

EXAMPLES FOR PRACTICE.

220. What is the sum of

(*a*) 25 lb. 7 oz. 15 pwt. 23 gr.; 17 lb. 16 pwt.; 15 lb. 4 oz. 12 pwt.; 18 lb. 16 gr.; 10 lb. 2 oz. 11 pwt. 16 gr.?

(*b*) 9 mi. 13 rd. 4 yd. 2 ft.; 16 rd. 5 yd. 1 ft. 5 in.; 16 mi. 2 rd. 3 in.; 14 rd. 1 yd. 9 in.?

(*c*) 3 cwt. 46 lb. 12 oz.; 12 cwt. 9½ lb.; 2¼ cwt. 21⅝ lb?

(*d*) 10 yr. 8 mo. 5 wk. 3 da.; 42 yr. 6 mo. 7 da.; 7 yr. 5 mo. 18 wk. 4 da.; 17 yr. 17 da.?

(*e*) 17 tons 11 cwt. 49 lb. 14 oz.; 16 tons 47 lb. 13 oz.; 20 tons 13 cwt. 14 lb. 6 oz.; 11 tons 4 cwt. 16 lb. 12 oz.?

(*f*) 14 sq. yd. 8 sq. ft. 19 sq. in.; 105 sq. yd. 16 sq. ft. 240 sq. in.; 42 sq. yd. 28 sq. ft. 165 sq. in.?

Ans.
(*a*) 86 lb. 3 oz. 16 pwt. 7 gr.
(*b*) 25 mi. 47 rd. 1 ft. 5 in.
(*c*) 18 cwt. 2 lb. 14 oz.
(*d*) 78 yr. 1 mo. 3 wk. 3 da.
(*e*) 65 tons 9 cwt. 28 lb. 13 oz.
(*f*) 167 sq. yd. 136 sq. in.

SUBTRACTION OF DENOMINATE NUMBERS.

221. Example.—From 21 rd. 2 yd. 2 ft. 6½ in., take 9 rd. 4 yd. 10¼ in.

Solution.—

rd.	yd.	ft.	in.	
21	2	2	6½	
9	4	0	10¼	
11	3½	1	8¼	Ans.

Explanation.—Since 10¼ inches can not be taken from 6½ inches, we must borrow 1 foot, or 12 inches, from the 2 feet in the next column and add it to the 6½. 6½ + 12 = 18½. 18½ inches − 10¼ inches = 8¼ inches. Then, 0 foot from the 1 remaining foot = 1 foot. 4 yards can not be taken from 2 yards; therefore, we borrow 1 rod, or 5½ yards, from 21 rods and add it to 2. 2 + 5½ = 7½; 7½ − 4 = 3½ yards. 9 rods from 20 rods = 11 rods. Hence, the remainder is 11 rods 3½ yards 1 foot 8¼ inches. Ans.

To avoid fractions as much as possible, we reduce the ½ yard to inches, obtaining 18 inches; this added to 8¼ inches, gives 26¼ inches, which equals 2 feet 2¼ inches. Then, 2 feet + 1 foot = 3 feet = 1 yard, and 3 yards + 1 yard = 4 yards. Hence, the above answer becomes 11 rods 4 yards 0 feet 2¼ inches.

222. Example.—What is the difference between 3 rd. 2 yd. 2 ft. 10 in. and 47 ft.?

Solution.—47 ft. = 2 rd. 4 yd. 2 ft.

	rd.	yd.	ft.	in.	
	3	2	2	10	
	2	4	2	0	
	0	3½	0	10	
or		3	2	4	Ans.

To find (approximately) the interval of time between two dates:

223. Example.—How many years, months, days, and hours between 4 o'clock P.M. of June 15, 1868, and 10 o'clock A.M., September 28, 1891?

Solution.—

yr.	mo.	da.	hr.	
1891	8	28	10	
1868	5	15	16	
23	3	12	18	Ans.

Explanation.—Counting 24 hours in 1 day, 4 o'clock P.M. is the 16th hour from the beginning of the day, or midnight. On September 28, 8 months and 28 days have elapsed, and on June 15, 5 months and 15 days. After placing the earlier date under the later date, subtract as in the previous problems. Count 30 days as 1 month.

224. Rule 32.—*Place the smaller quantity under the larger quantity, with like denominations under each other. Beginning at the right, subtract successively the number in the subtrahend in each denomination from the one above, and place the differences underneath. If the number in the minuend of any denomination is less than the number under it in the subtrahend, one must be borrowed from the minuend of the next higher denomination, reduced, and added to it.*

EXAMPLES FOR PRACTICE.

225. From

(*a*) 125 lb. 8 oz. 14 pwt. 18 gr. take 96 lb. 9 oz. 10 pwt. 4 gr.
(*b*) 126 hhd. 27 gal. take 104 hhd. 14 gal. 1 qt. 1 pt.
(*c*) 65 T. 14 cwt. 64 lb. 10 oz. take 16 T. 11 cwt. 14 oz.
(*d*) 148 sq. yd. 16 sq. ft. 142 sq. in. take 132 sq. yd. 136 sq. in.
(*e*) 100 bu. take 28 bu. 2 pk. 5 qt. 1 pt.
(*f*) 14 mi. 34 rd. 16 yd. 13 ft. 11 in. take 3 mi. 27 rd. 11 yd. 4 ft. 10 in.

Ans.
(*a*) 28 lb. 11 oz. 4 pwt. 14 gr.
(*b*) 22 hhd. 12 gal. 2 qt. 1 pt.
(*c*) 49 T. 3 cwt. 63 lb. 12 oz.
(*d*) 16 sq. yd. 16 sq. ft. 6 sq. in.
(*e*) 71 bu. 1 pk. 2 qt. 1 pt.
(*f*) 11 mi. 7 rd. 5 yd. 9 ft. 1 in.

MULTIPLICATION OF DENOMINATE NUMBERS.

226. Example.—Multiply 7 lb. 5 oz. 13 pwt. 15 gr. by 12.

Solution.—

lb.	oz.	pwt.	gr.	
7	5	13	15	
			12	
89	8	3	12	Ans.

Explanation.—15 grains × 12 = 180 grains. 180 ÷ 24 = 7 pennyweights and 12 grains remaining. Place the 12 in the grain column and carry the 7 pennyweights to the next. Now, 13 × 12 + 7 = 163 pennyweights; 163 ÷ 20 = 8 ounces and 3 pennyweights remaining. Then, 5 × 12 + 8 = 68 ounces; 68 ÷ 12 = 5 pounds and 8 ounces remaining. Then, 7 × 12 + 5 = 89 pounds. The entire product is 89 pounds 8 ounces 3 pennyweights 12 grains. Ans.

227. **Rule 33.**—*Multiply the number representing each denomination by the multiplier, and reduce each product to the next higher denomination, writing the remainders under each denomination, and carrying the quotient to the next, as in addition of denominate numbers.*

228. Note.—In multiplication and division of denominate numbers, it is sometimes easier to reduce the number to the lowest denomination given before multiplying or dividing, especially if the multiplier or divisor is a decimal. Thus, in the above example, had the multiplier been 1.2, the easiest way to multiply would have been to reduce the number to grains, then, multiply by 1.2, and reduce the product to higher denominations. For example, 7 lb. 5 oz. 13 pwt. 15 gr. = 43,047 gr. 43,047 × 1.2 = 51,656.4 gr. = 8 lb. 11 oz. 12 pwt. 8.4 gr. Also. 43,047 × 12 = 516,564 gr. = 89 lb. 8 oz. 3 pwt. 12 gr., as above. The student may use either method.

EXAMPLES FOR PRACTICE.

229. Multiply

(*a*) 15 cwt. 90 lb. by 5; (*b*) 12 yr. 10 mo. 4 wk. 3 da. by 14; (*c*) 11 mi. 145 rd. by 20; (*d*) 12 gal. 4 pt. by 9; (*e*) 8 cd. 76 cu. ft. by 15; (*f*) 4 hhd. 3 gal. 1 qt. 1 pt. by 12.

Ans. {
(*a*) 79 cwt. 50 lb.
(*b*) 180 yr. 11 mo. 2 wk.
(*c*) 229 mi. 20 rd.
(*d*) 112 gal. 2 qt.
(*e*) 128 cd. 116 cu. ft.
(*f*) 48 hhd. 40 gal. 2 qt.

DIVISION OF DENOMINATE NUMBERS.

230. Example.—Divide 48 lb. 11 oz. 6 pwt. by 8.

Solution.—

	lb.	oz.	pwt.	gr.	
8)	48	11	6	0	
	6 lb.	1 oz.	8 pwt.	6 gr.	Ans.

Explanation.—After placing the quantities as above, proceed as follows: 8 is contained in 48 six times without a remainder. 8 is contained in 11 ounces once with 3 ounces remaining. 3 × 20 = 60; 60 + 6 = 66 pennyweights; 66 pennyweights ÷ 8 = 8 pennyweights and 2 remaining; 2 × 24 grains = 48 grains; 48 grains ÷ 8 = 6 grains. Therefore, the entire quotient is 6 pounds 1 ounce 8 pennyweights 6 grains. Ans.

Example.—A silversmith melted up 2 lb. 8 oz. 10 pwt. of silver, which he made into 6 spoons; what was the weight of each spoon?

SOLUTION.—

	lb.	oz.	pwt.		
6)	2	8	10		
		5 oz.	8 pwt.	8 gr.	Ans.

EXPLANATION.—Since we can not divide 2 pounds by 6, we reduce it to ounces. 2 pounds = 24 ounces, and 24 ounces + 8 ounces = 32 ounces; 32 ounces ÷ 6 = 5 ounces and 2 ounces over. 2 ounces = 40 pennyweights. 40 pennyweights + 10 pennyweights = 50 pennyweights, and 50 pennyweights ÷ 6 = 8 pennyweights and 2 pennyweights over. 2 pennyweights = 48 grains, and 48 grains ÷ 6 = 8 grains. Hence, each spoon contains 5 ounces 8 pennyweights 8 grains. Ans.

231. **Example.**—Divide 820 rd. 4 yd. 2 ft. by 112.

Solution.—

```
          rd.    yd. ft.  rd. yd. ft.  in.
     112)820     4   2 (  7   1   2  5.143  Ans.
         784
         ---
          36 rd. rem.
         5.5
         ---
         180
        180
        ----
        198.0 yd.
          4
        ---
    112)202 yd. (1 yd.
        112
        ---
         90 yd. rem.
          3
        ---
        270 ft.
          2 ft.
        ---
    112)272 ft. (2 ft.
        224
        ---
         48 ft. rem.
         12
         --
         96
        48
        ---
    112)576 in. (5.1428 + in., or 5.143 in.
        560
        ---
         160
         112
         ---
          480
          448
          ---
           320
           224
           ---
            960
            896
            ---
             64
```

Explanation.—The first quotient is 7 rods with 36 rods remaining. 5.5 × 36 = 198 yards; 198 yards + 4 yards = 202 yards; 202 yards ÷ 112 = 1 yard and 90 yards remaining. 90 × 3 = 270 feet; 270 feet + 2 feet = 272 feet; 272 feet ÷ 112 = 2 feet and 48 feet remaining; 48 × 12 = 576 inches; 576 inches ÷ 112 = 5.143 inches, nearly. Ans.

The preceding example is solved by long division, because the numbers are too large to deal with mentally. Instead of expressing the last result as a decimal, it might have been expressed as a common fraction. Thus, $576 \div 112 = 5\frac{16}{112} = 5\frac{1}{7}$ inches. The chief advantage of using a common fraction is that if the quotient be multiplied by the divisor, the result will always be the same as the original dividend.

232. Rule 34.—*Find how many times the divisor is contained in the first or highest denomination of the dividend. Reduce the remainder (if any) to the next lower denomination, and add to it the number in the given dividend expressing that denomination. Divide this new dividend by the divisor. The quotient will be the next denomination in the quotient required. Continue in this manner until the lowest denomination is reached. The successive quotients will constitute the entire quotient.*

EXAMPLES FOR PRACTICE.

233. Divide

(*a*) 376 mi. 276 rd. by 22; (*b*) 1,187 bu. 3 pk. 4 qt. 1 pt. by 10; (*c*) 84 cwt. 48 lb. 49 oz. by 16; (*d*) 78 sq. yd. 18 sq. ft. 41 sq. in. by 18; (*e*) 148 mi. 64 rd. 24 yd. by 12; (*f*) 100 tons 16 cwt. 18 lb. 11 oz. by 15; (*g*) 36 lb. 18 oz. 18 pwt. 14 gr. by 8; (*h*) 112 mi. 48 rd. by 100.

Ans.
- (*a*) 17 mi. $41\frac{7}{11}$ rd.
- (*b*) 118 bu. 3 pk. 1 qt. $\frac{1}{2}$ pt.
- (*c*) 5 cwt. 28 lb. $3\frac{1}{16}$ oz.
- (*d*) 4 sq.yd. 4 sq.ft. $2\frac{5}{18}$ sq.in.
- (*e*) 12 mi. 112 rd. 2 yd.
- (*f*) 6 tons 14 cwt. 41 lb. $3\frac{14}{15}$ oz.
- (*g*) 4 lb. 8 oz. 7 pwt. $7\frac{3}{4}$ gr.
- (*h*) 1 mi. $38\frac{22}{25}$ rd.

INVOLUTION.

234. Involution is the process of multiplying a number by itself one or more times. The product obtained by multiplying a number by itself is called a **power** of that number.

Thus, the *second* power of 3 is 9. since 3 × 3 are 9.

The *third* power of 3 is 27, since 3 × 3 × 3 are 27.

The *fifth* power of 2 is 32, since 2 × 2 × 2 × 2 × 2 are 32.

235. An **exponent** is a *small figure* placed to the *right* and a little *above* a number to show to what *power* it is to be raised, or how many times the number is to be used as a factor, as the small figures ^{2}, ^{3}, and ^{5} below:

$$3^2 = 3 \times 3 = 9.$$
$$3^3 = 3 \times 3 \times 3 = 27.$$
$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32.$$

236. The **root** of a number is that number which, used the required number of times as a factor, produces the number. In the above cases, 3 is a root of 9, since 3 × 3 are 9. It is also a root of 27, since 3 × 3 × 3 are 27. Also, 2 is a root of 32, since 2 × 2 × 2 × 2 × 2 are 32.

237. The *second* power of a number is called its **square.**

Thus, 5^2 is called the **square** of 5, or 5 *squared*, and its value is 5 × 5 = 25.

238. The *third* power of a number is called its **cube.**

Thus, 5^3 is called the *cube* of 5, or 5 *cubed*, and its value is 5 × 5 × 5 = 125.

To find any power of a number:

239. Example.—What is the third power, or cube, of 35?

Solution.— $35 \times 35 \times 35,$

$$\begin{array}{rr}
\text{or} & 35 \\
 & 35 \\
\cline{2-2}
 & 175 \\
 & 105 \\
\cline{2-2}
 & 1225 \\
 & 35 \\
\cline{2-2}
 & 6125 \\
 & 3675 \\
\cline{2-2}
\textit{cube} = & 42875 \quad \text{Ans.}
\end{array}$$

240. **Example.**—What is the fourth power of 15?

Solution.— $15 \times 15 \times 15 \times 15$,

$$\text{or}\quad \begin{array}{r} 15 \\ 15 \\ \hline 75 \\ 15 \\ \hline 225 \\ 15 \\ \hline 1125 \\ 225 \\ \hline 3375 \\ 15 \\ \hline 16875 \\ 3375 \\ \hline \end{array}$$

$$\textit{fourth power} = 50625 \quad \text{Ans.}$$

241. **Example.**— $1.2^3 =$ what?

Solution.— $1.2 \times 1.2 \times 1.2$,

$$\text{or}\quad \begin{array}{r} 1.2 \\ 1.2 \\ \hline 1.44 \\ 1.2 \\ \hline 288 \\ 144 \\ \hline 1.728 \end{array} \quad \text{Ans.}$$

242. **Example.**—What is the third power, or cube, of $\frac{3}{8}$?

Solution.— $\left(\frac{3}{8}\right)^3 = \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} = \frac{3 \times 3 \times 3}{8 \times 8 \times 8} = \frac{27}{512}$. Ans.

243. **Rule 35.**—(*a*) *To raise a whole number or a decimal to any power, use it as a factor as many times as there are units in the exponent.*

(*b*) *To raise a fraction to any power, raise both the numerator and denominator to the power indicated by the exponent.*

EXAMPLES FOR PRACTICE.

244. Raise the following to the powers indicated:

(*a*)	85^2.	Ans.	(*a*)	7,225.
(*b*)	$\left(\frac{12}{13}\right)^2$.		(*b*)	$\frac{144}{169}$.
(*c*)	6.5^2.		(*c*)	42.25.
(*d*)	14^4.		(*d*)	38,416.

(*e*)	$(\frac{3}{4})^3$.	Ans.	(*e*)	$\frac{27}{64}$.
(*f*)	$(\frac{5}{6})^3$.		(*f*)	$\frac{125}{216}$.
(*g*)	$(\frac{7}{5})^3$.		(*g*)	$\frac{343}{125}$.
(*h*)	1.4^5.		(*h*)	5.37824.

EVOLUTION.

245. **Evolution** is the reverse of involution. It is the process of finding the root of a number which is considered as a power.

246. The **square root** of a number is that number which, when used twice as a factor, produces the number.

Thus, 2 is the square root of 4, since 2 × 2, or 2^2, = 4.

247. The **cube root** of a number is that number which, when used three times as a factor, produces the number.

Thus, 3 is the cube root of 27, since 3 × 3 × 3, or 3^3, =27.

248. The **radical sign** $\sqrt{}$, when placed before a number, indicates that some root of that number is to be found.

249. The **index** of the root is a *small figure* placed *over* and to the *left* of the *radical sign*, to show what root is to be found.

Thus, $\sqrt[2]{100}$ denotes the *square root* of 100.
$\sqrt[3]{125}$ denotes the *cube root* of 125.
$\sqrt[4]{256}$ denotes the *fourth root* of 256, and so on.

250. When the square root is to be extracted, the index is generally omitted. Thus, $\sqrt{100}$ indicates the square root of 100. Also, $\sqrt{225}$ indicates the square root of 225.

SQUARE ROOT.

251. The *largest* number that can be written with *one* figure is 9, and $9^2 = 81$; the *largest* number that can be written with *two* figures is 99, and $99^2 = 9{,}801$; with *three* figures 999, and $999^2 = 998{,}001$; with *four* figures 9,999, and $9{,}999^2 = 99{,}980{,}001$, etc.

In *each* of the above it will be noticed that the square of the number contains just *twice* as many figures as the number.

In order to find the square root of a number, the first step is to find how many figures there will be in the root. This is done by pointing off the number into *periods* of *two* figures each, *beginning at the right.* The number of periods will indicate the number of figures in the root.

Thus, the square root of 83,740,801 must contain 4 figures, since, pointing off the periods, we get 83'74'08'01, or 4 periods; consequently, there must be 4 figures in the root. In like manner, the square root of 50,625 must contain 3 figures, since there are (5'06'25) 3 periods.

252. EXAMPLE.—Find the square root of 31,505,769.

```
                                                root.
SOLUTION.—   (a)    5          31'50'57'69(5613  Ans.
                    5      (b) 25
             (d)  100      (c)  650
                    6           636
                  106      (e)   1457
                    6            1121
                 1120             33669
                    1             33669
                 1121                 0
                    1
                11220
                    3
                11223
```

EXPLANATION.—Pointing off into periods of two figures each, it is seen that there are four figures in the root. Now, find the largest single number whose square is less than or equal to 31, the first period. This is evidently 5, since $6^2 = 36$, which is greater than 31. Write it to the right, as in long division, and also to the left, as shown at (*a*). This is the first figure of the root. Now, multiply the 5 at (*a*) by the 5 in the root, and write the result under the first period, as shown at (*b*). Subtract, and obtain 6 as a remainder.

Bring down the next period 50, and annex it to the remainder 6, as shown at (*c*), which we call the **dividend.** Add the root already found to the 5 at (*a*), getting 10, and annex a cipher to this 10, thus making it 100, which we call the **trial divisor.** Divide the dividend (*c*) by the trial divisor (*d*), and obtain 6, which is *probably* the next figure of the root. Write 6 in the root, as shown, and also add it to 100, the trial divisor, making it 106. This is called the **complete divisor.**

Multiply this by 6, the second figure in the root, and subtract the result from the dividend (*c*). The remainder is 14, to which annex the next period, making it 1,457, as shown at (*e*), which we call the **new dividend.** Add the second figure of the root to the complete divisor 106, and annex a cipher, thus getting 1,120. Dividing 1,457 by 1,120, we get 1 as the next figure of the root. Adding this last figure of the root to 1,120, multiplying the result by it, and subtracting from 1,457, the remainder is 336.

Annexing the next and last period, 69, the result is 33,669. Now, adding the last figure of the root to 1,121, and annexing a cipher as before, the result is 11,220. Dividing 33,669 by 11,220, the result is 3, the fourth figure in the root. Adding it to 11,220, and multiplying the sum by it, the result is 33,669. Subtracting, there is no remainder; hence, $\sqrt{31{,}505{,}769} = 5{,}613$. Ans.

253. The square of any number wholly decimal always contains twice as many figures as the number squared. For example, $.1^2 = .01$; $.13^2 = .0169$; $.751^2 = .564001$, etc.

254. It will also be noticed that the number squared is always less than the decimal. Hence, if it be required to find the square root of a decimal, and the decimal has not an even number of figures in it, annex a cipher. The best way to determine the number of figures in the root of a decimal is to begin at the decimal point, and, going towards the *right*, point off the decimal into periods of two figures each. Then, if the last period contains but one figure, annex a cipher.

255. EXAMPLE.—What is the square root of .000576?

```
                                              root
SOLUTION.—          2            .00'05'76(.024   Ans.
                    2             4
                   --             -
                   40             176
                    4             176
                   --             ---
                   44               0
```

Explanation.—Beginning at the decimal point, and pointing off the number into periods of two figures each, it is seen that the first period is composed of ciphers; hence, the first figure of the root must be a cipher. The remaining portion of the solution should be perfectly clear from what has preceded.

256. If the number is not a perfect power, the root will consist of an interminable number of decimal places. The result may be carried to any required number of decimal places by annexing periods of two ciphers each to the number.

257. **Example.**—What is the square root of 3? Find the result to five decimal places.

```
                                              root
Solution.— 1          3.00'00'00'00'00(1.73205 +   Ans.
           1          1
           ‾          ‾
           20         200
            7         189
           ───        ───
           27          1100
            7          1029
           ───         ────
           340           7100
             3           6924
           ───           ────
           343             1760000
             3             1732025
           ───             ───────
           3460              27975
              2
           ────
           3462
              2
           ──────
           346400
                5
           ──────
           346405
```

Explanation.—Annexing five periods of two ciphers each to the right of the decimal point, the first figure of the root is 1. To get the second figure, we find that, in dividing 200 by 20, it is 10. This is evidently too large.

Trying 9, we add 9 to 20, and multiply 29 by 9, the result is 261, a result which is considerably larger than 200; hence, 9 is too large. In the same way it is found that 8 is also too large. Trying 7, 7 times 27 is 189, a result smaller than 200; therefore, 7 is the second figure of the root. The next two figures, 3 and 2, are easily found. The fifth figure in the root is a cipher, since the trial divisor 34,640 is greater than the new dividend 17,600. In a case of this kind, we annex another cipher to 34,640, thereby making it 346,400, and bring down the next period, making the 17,600, 1,760,000. The next figure of the root is 5, and as we now have five decimal places, we will stop.

The square root of 3 is, then, 1.73205 +. Ans.

258. Example.—What is the square root of .3 to five decimal places?

```
                                              root
Solution.—   5          .30'00'00'00'00( .54772+  Ans.
             5           25
           ---           --
           100           500
             4           416
           ---           ---
           104            8400
             4            7609
          ----            ----
          1080             79100
             7             76629
          ----             -----
          1087              247100
             7              219084
         -----              ------
         10940               28016
             7
         -----
         10947
             7
        ------
        109540
             2
        ------
        109542
```

Explanation.—In the above example, we annex a cipher to .3, making the first period .30, since every period of a decimal, as was mentioned before, must have two figures in it. The remainder of the work should be perfectly clear.

259. If it is required to find the square root of a mixed number, begin at the decimal point, and point off the periods both ways. The manner of finding the root will then be exactly the same as in the previous cases.

260. EXAMPLE.—What is the square root of 258.2449?

```
SOLUTION.—          1        2'58.24'49(16.07  Ans.
                    1        1
                   ——        —
                    20       158
                     6       156
                   ——        ———
                    26          22449
                     6          22449
                   ——           —————
                    3200            0
                       7
                   —————
                    3207
```

EXPLANATION.—In the above example, since 320 is greater than 224, we place a cipher for the third figure of the root, and annex a cipher to 320, making it 3,200. Then, bringing down the next period 49, 7 is found to be the fourth figure of the root. Since there is no remainder, the square root of 258.2449 is 16.07. Ans.

261. Proof.—*To prove square root, square the result obtained. If the number is an exact power, the square of the root will equal it; if it is not an exact power, the square of the root will very nearly equal it.*

262. Rule 36.—(*a*) *Begin at units place, and separate the number into periods of two figures each, proceeding from left to right with the decimal part, if there is any.*

(*b*) *Find the greatest number whose square is contained in the first or left-hand period. Write this number as the first figure in the root; also, write it at the left of the given number.*

Multiply this number at the left by the first figure of the root, and subtract the result from the first period; then annex the second period to the remainder.

(*c*) *Add the first figure of the root to the number in the first column on the left, and annex a cipher to the result; this is the trial divisor. Divide the dividend by the trial divisor* *for the second figure in the root, and add this figure to the trial divisor to form the complete divisor. Multiply the complete divisor by the second figure in the root, and subtract this result from the dividend. (If this result is larger than the dividend, a smaller number must be tried for the second figure of the root.) Now bring down the third period, and annex it to the last remainder for a new dividend. Add the second figure of the root to the complete divisor, and annex a cipher for a new trial divisor.*

(*d*) *Continue in this manner to the last period, after which, if any additional places in the root are required, bring down cipher periods, and continue the operation.*

(*e*) *If at any time the trial divisor is not contained in the dividend, place a cipher in the root, annex a cipher to the trial divisor, and bring down another period.*

(*f*) *If the root contains an interminable decimal, and it is desired to terminate the operation at some point, say the fourth decimal place, carry the operation one place further, and if the fifth figure is 5 or greater, increase the fourth figure by 1 and omit the sign* +.

263. Short Method.—If the number whose root is to be extracted is not an exact square, the root will be an interminable decimal. It is then usual to extract the root to a certain number of decimal places. In such cases, the work may be greatly shortened as follows: Determine to how many decimal places the work is to be carried, say 5, for example; add to this the number of places in the integral part of the root, say 2, for example, thus determining the number of figures in the root, in this case $5 + 2 = 7$. Divide this number by 2 and take the next higher number. In the above case, we have $7 \div 2 = 3\frac{1}{2}$; hence, we take 4, the next higher number. Now extract the root in the usual manner until the same number of figures have been obtained as was expressed by the number obtained above, in this case 4. Then form the trial divisor in the usual manner, but omitting to annex the cipher; divide the last remainder by the trial divisor, as in long division, obtaining as many figures of the quotient as there are remaining figures of the root, in this case 7 − 4 = 3. The remainder so obtained is the remaining figures of the root.

Consider the example in Art. **258.** Here there are 5 figures in the root. We therefore extract the root to 3 places in the usual manner, obtaining .547 for the first three root figures. The next trial divisor is 1,094 (with the cipher omitted), and the last remainder is 791. Then, 791 ÷ 1,094 = .723, and the next two figures of the root are 72, the whole root being .54772 +. Always carry the division one place further than desired, and if the last figure is 5 or greater, increase the preceding figure by 1. This method should not be used unless the root contains five or more figures.

Note.—If the last figure of the root found in the regular manner is a cipher, carry the process one place further before dividing as described above.

EXAMPLES FOR PRACTICE.

264. Find the square root of

(*a*)	186,624.	Ans.	(*a*)	432.
(*b*)	2,050,624.		(*b*)	1,432.
(*c*)	29,855,296.		(*c*)	5,464.
(*d*)	.0116964.		(*d*)	.1081 +.
(*e*)	198.1369.		(*e*)	14.0761.
(*f*)	994,009.		(*f*)	997.
(*g*)	2.375 to four decimal places.		(*g*)	1.5411.
(*h*)	1.625 to three decimal places.		(*h*)	1.275.
(*i*)	.3025.		(*i*)	.55.
(*j*)	.571428.		(*j*)	.7559 +.
(*k*)	.78125.		(*k*)	.8839.

CUBE ROOT.

265. In the same manner as in the case of square root, it can be shown that the periods into which a number is divided, whose cube root is to be extracted, must contain three figures, except that the first or left-hand period of a whole or mixed number may contain one, two, or three figures.

266. Example.—What is the cube root of 375,741,853,696?

Solution.—

(1)	(2)	(3) *root*
7	49	375'741'853'696(7216 Ans.
7	98	343
14	14700	32741
7	424	30248
210	15124	2493853
2	428	1557361
212	1555200	936492696
2	2161	936492696
214	1557361	0
2	2162	
2160	155952800	
1	129816	
2161	156082116	
1		
2162		
1		
21680		
6		
21686		

Explanation.—Write the work in three columns as follows: On the right place the number whose cube root is to be extracted, and point it off into periods of three figures each. Call this column (3). Find the largest number whose cube is less than or equal to the first period, in this case 7. Write the 7 on the right, as shown, for the first figure of the root, and also on the extreme left at the head of column (1). Multiply the 7 in column (1) by the first figure of the root 7, and write the product 49 at the head of column (2). Multiply the number in column (2) by the first figure of the root 7, and write the product 343 under the figures in the first period. Subtract and bring down the next period, obtaining 32,741 for the dividend. Add the first figure of the root to the number in column (1), obtaining 14, which call the *first correction*. Multiply the first correction by the first figure of the root, add the product to the number in column (2), and obtain 147. Add the first figure of the root to the first correction, and obtain 21, which call the *second correction*. Annex *two* ciphers to the number in column (2), and obtain 14,700 for the trial divisor; also, annex *one* cipher to the second correction, and obtain 210. Dividing the dividend by the trial divisor, we obtain $\frac{32,741}{14,700}=2+$, and write the 2 as the second figure of the root. Add the 2 to the second correction, and obtain 212, which, multiplied by the second figure of the root, and added to the trial divisor, gives 15,124, the complete divisor. This last result, multiplied by the second figure of the root and subtracted from the dividend, gives a remainder of 2,493. Annexing the third period, we obtain 2,493,853 for the new dividend. Adding the second figure of the root to the number in column (1), we get 214 as the new first correction; this, multiplied by the second figure of the root and added to the trial divisor, gives 15,552. Adding the second figure of the root to the first new correction gives 216 as the second new correction. Annexing two ciphers to the number in column (2) gives 1,555,200, the new trial divisor. Annexing one cipher to the second new correction gives 2,160. Dividing the new dividend by the new trial divisor, we obtain $\frac{2,493,853}{1,555,200}=1+$, and write 1 as the third figure of the root. The remainder of the work should be perfectly clear from what has preceded.

267. In extracting the cube root of a decimal, proceed as above, taking care that each period contains *three* figures. Begin the pointing off at the decimal point, going towards the right. If the last period does not contain three figures, annex ciphers until it does.

268. **Example.**—What is the cube root of .009129329?

Solution.—

		root
2	4	.009'129'329(.209
2	8	8
4	120000	1129329
2	5481	1129329
600	125481	0
9		
609		

Explanation.—Beginning at the decimal point, and pointing off as shown, the largest number whose cube is less than 9 is seen to be 2; hence, 2 is the first figure of the root. When finding the second figure, it is seen that the trial divisor 1,200 is greater than the dividend; hence, write a cipher for the second figure of the root; bring down the next period to form the new dividend; annex two ciphers to the trial divisor to form a new trial divisor; also, annex one cipher to the 60 in column (1). Dividing the new dividend by the new trial divisor, we get $\frac{1129329}{120000} = 9 +$, and write 9 as the third figure of the root. Complete the work as before.

269. **Example.**—What is the cube root of 78,347.809639?

Solution.—

		root
4	16	78'347.809'639 (42.79
4	32	64
8	4800	14347
4	244	10088
120	5044	4259809
2	248	3766483
122	529200	493326639
2	8869	493326639
124	538069	0
2	8918	
1260	54698700	
7	115371	
1267	54814071	
7		
1274		
7		
12810		
9		
12819		

Explanation.—Since we have a mixed number, begin at the decimal point and point off periods of three figures each, in both directions. The first period contains but two figures, and the largest number whose cube is less than 78 is 4; consequently, 4 is the first figure of the root. The remainder of the work should be perfectly clear. When dividing the dividend by the trial divisor for the third figure of the root, the quotient was 8 +; but, on trying it, it was found that 8 was too large, the complete divisor being considerably larger than the trial divisor. Therefore, 7 was used instead of 8.

270. Example.—What is the cube root of 5 to five decimal places?

Solution.—

		root
1	1	5.000'000'000'000'000(1.70997 +
1	2	1
2	300	4000
1	259	3913
30	559	87000000
7	308	78443829
37	8670000	8556171000
7	45981	7889992299
44	8715981	666178701000
7	46062	614014317973
5100	876204300	52164383027
9	461511	
5109	876665811	
9	461592	
5118	87712740300	
9	3590839	
51270	87716331139	
9		
51279		
9		
51288		
9		
512970		
7		
512977		

Explanation.—In the preceding example we annex five periods of ciphers, of three ciphers each, to the 5 for the decimal part of the root, placing the decimal point between the 5 and the first cipher. Since it is easy to see that the next figure of the root will be 5, we increase the last figure by 1, obtaining 1.70998 for the correct root to 5 decimal places. Ans.

271. **Example.**—What is the cube root of .5 to four decimal places?

Solution.—

```
                                                    root
    7         49           .500'000'000'000(.7937+
    7         98           343
   ---       ----          ----
   14        14700         157000
    7         1971         150039
   ---       ------        ------
  210        16671           6961000
    9         2052           5638257
  ----       ------          -------
  219        1872300           1322743000
    9           7119           1321748953
  ----       -------           ----------
  228        1879419               994047
    9           7128
  ----       -------
  2370       188654700
     3          166579
  -----      ---------
  2373       188821279
     3
  -----
  2376
     3
  -----
  23790
      7
  -----
  23797
```

Explanation.—In the above example we annex two ciphers to the .5 to complete the first period, and three periods of three ciphers each. The cube root of 500 is 7; this we write as the first figure of the root. The remainder of the work should be perfectly plain from the explanations of the preceding examples.

272. Example.—What is the cube root of .05 to four decimal places?

Solution.—

```
                                                   root
   3          9          .050'000'000'000(.3684+
   3         18            27
  ---        --          ----
   6       2700            23000
   3        576            19656
  ---      ----            -----
  90       3276             3344000
   6        612             3180032
  ---      ----             -------
  96     388800              163968000
   6       8704              162685504
 ----    ------              ---------
 102     397504                1282496
   6       8768
 ----    ------
 1080  40627200
    8     44176
 ----  --------
 1088  40671876
    8
 ----
 1096
    8
 -----
 11040
     4
 -----
 11044
```

273. Proof.—*To prove cube root, cube the result obtained. If the given number is an exact power, the cube of the root will equal it; if not an exact power, the cube of the root will very nearly equal it.*

274. Rule 37.—**(a)** *Arrange the work in three columns, placing the number whose cube root is to be extracted in the third or right-hand column. Begin at units place, and separate the number into periods of three figures each, proceeding from the decimal point towards the right with the decimal part, if there is any.*

(b) *Find the greatest number whose cube is not greater than the number in the first period. Write this number as the first figure of the root; also, write it at the head of the first column. Multiply the number in the first column by the first figure in the root, and write the result in the second* *column. Multiply the number in the second column by the first figure of the root; subtract the product from the first period, and annex the second period to the remainder for a new dividend; add the first figure of the root to the number in the first column for the first correction. Multiply the first correction by the first figure of the root, and add the product to the number in the second column. Add the first figure of the root to the first correction to form the second correction. Annex one cipher to the second correction, and two ciphers to the last number in the second column; the last number in the second column is the trial divisor.*

(**c**) *Divide the dividend by the trial divisor to find the second figure of the root. Add the second figure of the root to the number in the first column, multiply the sum by the second figure of the root, and add the result to the trial divisor to form the complete divisor. Multiply the complete divisor by the second figure of the root, subtract the result from the dividend in the third column, and annex the third period to the remainder for a new dividend. Add the second figure of the root to the number in the first column to form the first correction; multiply the first correction by the second figure of the root, and add the product to the complete divisor. Add the second figure of the root to the first correction to form the second correction. Annex one cipher to the second correction, and two ciphers to the last number in the second column to form the new trial divisor.*

(**d**) *If there are more periods to be brought down, proceed as before. If there is a remainder after the root of the last period has been found, annex cipher periods, and proceed as before. The figures of the root thus obtained will be decimals.*

(**e**) *If the root contains an interminable decimal, and it is desired to terminate the operation at some point, say the fourth decimal place, carry the operation one place further, and if the fifth figure is 5 or greater, increase the fourth figure by 1 and omit the sign* +.

Art. **263** can be applied to cube root (or any other root) as well as to square root. Thus, in the example, Art. **270**, there are to be $5 + 1 = 6$ figures in the root. Extracting the root in the usual manner to $6 \div 2 = 3$, say 4 figures, we get for the first four figures 1,709. The last remainder is 8,556,171, and the next trial divisor with the ciphers omitted is 8,762,043. Hence, the next two figures of the root are $8{,}556{,}171 \div 8{,}762{,}043 = .976$, say .98. Therefore, the root is 1.70998.

ROOTS OF FRACTIONS.

275. If a given number is in the form of a fraction and it is required to find some root of it, the simplest and most exact method is to reduce the fraction to a decimal and extract the required root of the decimal. If, however, the numerator and denominator of the fraction are perfect powers, extract the required root of each separately, and write the root of the numerator for a new numerator, and the root of the denominator for a new denominator.

276. EXAMPLE.—What is the square root of $\frac{9}{64}$?

SOLUTION.—$\sqrt{\frac{9}{64}} = \frac{\sqrt{9}}{\sqrt{64}} = \frac{3}{8}$.

277. EXAMPLE.—What is the square root of $\frac{5}{8}$?

SOLUTION.—$\sqrt{\frac{5}{8}} = \sqrt{.625} = .7906$.

278. EXAMPLE.—What is the cube root of $\frac{27}{64}$?

SOLUTION.—$\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$.

279. EXAMPLE.—What is the cube root of $\frac{1}{4}$?

SOLUTION.—Since $\frac{1}{4} = .25$, $\sqrt[3]{\frac{1}{4}} = \sqrt[3]{.25} = .62996+$.

280. Rule 38.—*Extract the required root of the numerator and denominator separately; or, reduce the fraction to a decimal and extract the root of the decimal.*

EXAMPLES FOR PRACTICE.

281. Find the cube root of

(*a*) $\frac{27}{512}$.
(*b*) 2 to four decimal places.
(*c*) 4,180,769,192.462 to three decimal places.
(*d*) $\frac{87}{125}$.
(*e*) $\frac{3}{8}$.
(*f*) 518,229.783302144 to three decimal places.

Ans.
(*a*) $\frac{3}{8}$.
(*b*) 1.2599+.
(*c*) 1,610.962.
(*d*) .8862+.
(*e*) .7211+.
(*f*) 80.064.

TO EXTRACT OTHER ROOTS THAN THE SQUARE AND CUBE ROOTS.

282. Example.—What is the fourth root of 256?

Solution.— $\sqrt{256} = 16.$

$\sqrt{16} = 4.$

Therefore, $\sqrt[4]{256} = 4.$ Ans.

In this example, $\sqrt[4]{256}$, the index is 4, which equals 2 × 2. The root indicated by 2 is the square root; therefore, the square root is extracted twice.

283. Example.—What is the sixth root of 64?

Solution.— $\sqrt{64} = 8.$

$\sqrt[3]{8} = 2.$

Therefore, $\sqrt[6]{64} = 2.$ Ans.

In this example, $\sqrt[6]{64}$, the index is 6, which equals 2 × 3. The root indicated by 3 is the cube root; therefore, the square and cube roots are extracted in succession.

284. **Rule 39.**—*Separate the index of the required root into its factors (2's and 3's), and extract successively the roots indicated by the several factors obtained. The final result will be the required root.*

285. Example.—What is the sixth root of 92,873,580 to two decimal places?

Solution.— $6 = 3 \times 2$. Hence, extract the cube root, and then extract the square root of the result. $\sqrt[3]{92,873,580} = 452.8601$, and $\sqrt{452.8601} = 21.28+$. Ans.

286. It matters not which root is extracted first, but it is probably easier and more exact to extract the cube root first.

EXAMPLES FOR PRACTICE.

287. Extract the

(*a*) Fourth root of 100.	Ans.	(*a*)	3.16227+.
(*b*) Fourth root of 3,049,800,625.		(*b*)	235.
(*c*) Sixth root of 9,474,296,896.		(*c*)	46.

RATIO.

288. Suppose that it is desired to compare two numbers, say 20 and 4. If we wish to know how many times larger 20 is than 4, we divide 20 by 4 and obtain 5 for the quotient; thus, $20 \div 4 = 5$. Hence, we say that 20 is 5 times as large as 4, i. e., 20 contains 5 times as many units as 4. Again, suppose we desire to know what part of 20 is 4. We then divide 4 by 20 and obtain $\frac{1}{5}$; thus, $4 \div 20 = \frac{1}{5}$, or .2. Hence, 4 is $\frac{1}{5}$ or .2 of 20. This operation of comparing two numbers is termed *finding the ratio* of the two numbers. Ratio, then, is a comparison. It is evident that the two numbers to be compared must be expressed in the same unit; in other words, the two numbers must both be abstract numbers or concrete numbers of the same kind. For example, it would be absurd to compare 20 horses with 4 birds, or 20 horses with 4. Hence, **ratio** may be defined as a comparison between two numbers of the same kind.

289. A ratio may be *expressed* in three ways; thus, if it is desired to compare 20 and 4, and express this comparison as a ratio, it may be done as follows: $20 \div 4$; $20 : 4$, or $\frac{20}{4}$. All three are read *the ratio of 20 to 4.* The ratio of 4 to 20 would be expressed thus: $4 \div 20$; $4 : 20$, or $\frac{4}{20}$. The first method of expressing a ratio, although correct, is seldom or never used; the second form is the one oftenest met with, while the third is rapidly growing in favor, and is likely to supersede the second. The third form, called the fractional form, is preferred by modern mathematicians, and possesses great advantages to students of algebra and of higher mathematical subjects. The second form seems to be better adapted to arithmetical subjects, and is one we shall ordinarily adopt. There is still another way of expressing a ratio, though seldom or never used in the case of a simple ratio like that given above. Instead of the colon, a straight vertical line is used; thus, $20 \mid 4$.

290. The **terms** of a ratio are the two numbers to be compared; thus, in the above ratio, 20 and 4 are the terms. When both terms are considered together, they are called a **couplet;** when considered separately, the first term is called the **antecedent,** and the second term the **consequent.** Thus, in the ratio 20 : 4, 20 and 4 form a couplet, and 20 is the antecedent, and 4 the consequent.

291. A ratio may be **direct** or **inverse.** The *direct ratio* of 20 to 4 is 20 : 4, while the *inverse ratio* of 20 to 4 is 4 : 20. The direct ratio of 4 to 20 is 4 : 20, and the inverse ratio is 20 : 4. An inverse ratio is sometimes called a **reciprocal** ratio. The **reciprocal** of a number is 1 divided by the number. Thus, the reciprocal of 17 is $\frac{1}{17}$; of $\frac{3}{8}$ is $1 \div \frac{3}{8} = \frac{8}{3}$; i.e., the reciprocal of a fraction is the fraction inverted. Hence, the inverse ratio of 20 to 4 may be expressed as 4 : 20, or as $\frac{1}{20} : \frac{1}{4}$. Both have equal values; for, $4 \div 20 = \frac{1}{5}$, and $\frac{1}{20} \div \frac{1}{4} = \frac{1}{20} \times \frac{4}{1} = \frac{1}{5}$.

292. The term **vary** implies a ratio. When we say that two numbers vary as some other two numbers, we mean that the ratio between the first two numbers is the same as the ratio between the other two numbers.

293. The **value** of a ratio is the result obtained by performing the division indicated. Thus, the value of the ratio 20:4 is 5, it is the quotient obtained by dividing the antecedent by the consequent.

294. By expressing the ratio in the fractional form, for example, the ratio of 20 to 4 as $\frac{20}{4}$, it is easy to see, from the laws of fractions, that if both terms be multiplied, or both divided by the same number, it will not alter the value of the ratio. Thus,

$$\frac{20}{4} = \frac{20 \times 5}{4 \times 5} = \frac{100}{20}; \text{ and } \frac{20}{4} = \frac{20 \div 4}{4 \div 4} = \frac{5}{1}.$$

295. It is also evident, from the laws of fractions, that multiplying the antecedent or dividing the consequent multiplies the ratio; and dividing the antecedent or multiplying the consequent divides the ratio.

296. When a ratio is expressed in words, as the ratio of 20 to 4, the first number named is always regarded as the antecedent and the second as the consequent, without regard to whether the ratio itself is direct or inverse. *When not otherwise specified, all ratios are understood to be direct.* To express an inverse ratio, the simplest way of doing it is to express it as if it were a direct ratio, with the first number named as the antecedent, and then transpose the antecedent to the place occupied by the consequent and the consequent to the place occupied by the antecedent; or if expressed in the fractional form, invert the fraction. Thus, to express the inverse ratio of 20 to 4, first write it 20 : 4, and then, transposing the terms, as 4 : 20; or as $\frac{20}{4}$, and then inverting as $\frac{4}{20}$. Or, the reciprocals of the numbers may be taken, as explained above. To **invert** a ratio is to transpose its terms.

EXAMPLES FOR PRACTICE.

297. What is the value of the ratio of

			Ans.
(*a*)	98 to 49?	(*a*)	2.
(*b*)	$45 to $9?	(*b*)	5.
(*c*)	6¼ to ½?	(*c*)	12½.
(*d*)	3.5 to 4.5?	(*d*)	.77$\frac{7}{9}$.
(*e*)	The inverse ratio of 76 to 19?	(*e*)	¼.
(*f*)	The inverse ratio of 49 to 98?	(*f*)	2.
(*g*)	The inverse ratio of 18 to 24?	(*g*)	1⅓.
(*h*)	The inverse ratio of 9 to 15?	(*h*)	1⅔.
(*i*)	The ratio of 10 to 3, multiplied by 3?	(*i*)	10.
(*j*)	The ratio of 35 to 49, multiplied by 7?	(*j*)	5.
(*k*)	The ratio of 18 to 64, divided by 9?	(*k*)	$\frac{1}{32}$.
(*l*)	The ratio of 14 to 28, divided by 5?	(*l*)	$\frac{1}{10}$.

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298. Instead of expressing the value of a ratio by a single number, as above, it is customary to express it by means of another ratio in which the consequent is 1. Thus, suppose that it is desired to find the ratio of the weights of two pieces of iron, one weighing 45 pounds and the other weighing 30 pounds. The ratio of the heavier to the lighter is then 45 : 30, an inconvenient expression. Using the fractional form, we have $\frac{45}{30}$. Dividing both terms by 30, the consequent, we obtain $\frac{1\frac{1}{2}}{1}$ or $1\frac{1}{2} : 1$. This is the same result as obtained above, for $1\frac{1}{2} \div 1 = 1\frac{1}{2}$, and $45 \div 30 = 1\frac{1}{2}$.

299. A ratio may be squared, cubed, or raised to any power, or any root of it may be taken. Thus, if the ratio of two numbers is 105 : 63, and it is desired to cube this ratio, the cube may be expressed as $105^3 : 63^3$. That this is correct is readily seen; for, expressing the ratio in the fractional form, it becomes $\frac{105}{63}$, and the cube is $\left(\frac{105}{63}\right)^3 = \frac{105^3}{63^3} = 105^3 : 63^3$. Also, if it is desired to extract the cube root of the ratio $105^3 : 63^3$, it may be done by simply dividing the exponents by 3, obtaining 105 : 63. This may be proved in the same way as in the case of cubing the ratio. Thus, $105^3 : 63^3 = \left(\frac{105}{63}\right)^3$, and $\sqrt[3]{\left(\frac{105}{63}\right)^3} = \frac{105}{63} = 105 : 63$.

300. Since $\left(\frac{105}{63}\right)^3 = \left(\frac{5}{3}\right)^3$, it follows that $105^3 : 63^3 = 5^3 : 3^3$ (this expression is read: the ratio of 105 cubed to 63 cubed equals the ratio of 5 cubed to 3 cubed), it follows that the antecedent and consequent may always be multiplied or divided by the same number, irrespective of any indicated powers or roots, without altering the value of the ratio. Thus, $24^2 : 18^2 = 4^2 : 3^2$. For, performing the operations indicated by the exponents, $24^2 = 576$ and $18^2 = 324$. Hence, $576 : 324 = 1\frac{7}{9}$ or $1\frac{7}{9} : 1$. Also, $4^2 = 16$ and $3^2 = 9$; hence, $16 : 9 = 1\frac{7}{9}$ or $1\frac{7}{9} : 1$, the same result as before. Also, $24^2 : 18^2 = \frac{24^2}{18^2} = \left(\frac{24}{18}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{4^2}{3^2} = 4^2 : 3^2$.

The statement may be proved for roots in the same manner. Thus, $\sqrt[3]{24^3} : \sqrt[3]{18^3} = \sqrt[3]{4^3} : \sqrt[3]{3^3}$. For the $\sqrt[3]{24^3} = 24$ and $\sqrt[3]{18^3} = 18$; and, $24 : 18 = 1\frac{1}{3}$ or $1\frac{1}{3} : 1$. Also, $\sqrt[3]{4^3} = 4$ and $\sqrt[3]{3^3} = 3$; $4 : 3 = 1\frac{1}{3}$ or $1\frac{1}{3} : 1$.

Note.—If the numbers composing the antecedent and consequent have different exponents, or if different roots of those numbers are indicated, the operations described in Art. **300** can not be performed. This is evident; for, consider the ratio $4^2 : 8^3$. When expressed in the fractional form, it becomes $\frac{4^2}{8^3}$, which can not be expressed either as $\left(\frac{4}{8}\right)^2$ or as $\left(\frac{4}{8}\right)^3$, and, hence, can not be reduced as described above.

PROPORTION.

301. **Proportion** is an equality of ratios, the equality being indicated by the double colon (: :) or by the sign of equality (=). Thus, to write in the form of a proportion the two equal ratios, 8 : 4 and 6 : 3, which both have the same value 2, we may employ one of the three following forms:

$$8 : 4 :: 6 : 3 \qquad (1)$$

$$8 : 4 = 6 : 3 \qquad (2)$$

$$\frac{8}{4} = \frac{6}{3} \qquad (3)$$

302. The first form is the one most extensively used, by reason of its having been exclusively employed in all the older works on mathematics. The second and third forms are being adopted by all modern writers on mathematical subjects, and, in time, will probably entirely supersede the first form. In this paper we shall adopt the second form, unless some statement can be made clearer by using the third form.

303. A proportion may be *read* in two ways. The old way to read the above proportion was: *8 is to 4 as 6 is to 3;* the new way is: *the ratio of 8 to 4 equals the ratio of 6 to 3.* The student may read it either way, but we recommend the latter.

304. Each ratio of a proportion is termed **a couplet.** In the above proportion, 8 : 4 is a couplet, and so is 6 : 3.

305. The numbers forming the proportion are called **terms**; and they are numbered consecutively from left to right, thus:

first second third fourth
8 : 4 = 6 : 3

Hence, in any proportion, the ratio of the first term to the second term equals the ratio of the third term to the fourth term.

306. The first and fourth terms of a proportion are called the **extremes,** and the second and third terms the **means.** Thus, in the foregoing proportion, 8 and 3 are the extremes and 4 and 6 are the means.

307. A **direct proportion** is one in which both couplets are direct ratios.

308. An **inverse proportion** is one which requires one of the couplets to be expressed as an inverse ratio. Thus, 8 is to 4 inversely as 3 is to 6 must be written 8 : 4 = 6 : 3; i. e., the second ratio (couplet) must be inverted.

309. Proportion forms one of the most useful sections of arithmetic. In our grandfathers' arithmetics, it was called "The rule of three."

310. **Rule 40.**—*In any proportion, the product of the extremes equals the product of the means.*

Thus, in the proportion,

$$17 : 51 = 14 : 42.$$

$17 \times 42 = 51 \times 14$, since both products equal 714.

311. **Rule 41.**—*The product of the extremes divided by either mean gives the other mean.*

EXAMPLE.—What is the third term of the proportion 17 : 51 =   : 42?

SOLUTION.—Applying the rule, $17 \times 42 = 714$, and $714 \div 51 = 14$. Ans.

312. **Rule 42.**—*The product of the means divided by either extreme gives the other extreme.*

EXAMPLE.—What is the first term of the proportion   : 51 = 14 : 42?

SOLUTION.—Applying the rule, $51 \times 14 = 714$, and $714 \div 42 = 17$. Ans.

313. When stating a proportion in which one of the terms is unknown, represent the missing term by a letter, as x. Thus, the last example would be written,

$$x : 51 = 14 : 42$$

and for the value of x we have $x = \frac{51 \times 14}{42} = 17$.

314. If the same (addition and subtraction excepted) operations be performed upon *all* of the terms of a proportion, the proportion is not thereby destroyed. In other words, if all of the terms of a proportion be (1) multiplied or (2) divided by the same number; (3) if all the terms be raised to the same power; if (4) the same root of all the terms be taken, or (5) if both couplets be inverted, the proportion still holds. We will prove these statements by a numerical example, and the student can satisfy himself by other similar ones. The fractional form will be used, as it is better suited to the purpose. Consider the proportion $8 : 4 = 6 : 3$. Expressing it in the third form, it becomes $\frac{8}{4} = \frac{6}{3}$. What we are to prove is that, if any of the five operations enumerated above be performed upon all of the terms of this proportion, the first fraction will still equal the second fraction.

1. Multiplying all the terms by any number, say 7, $\frac{8 \times 7}{4 \times 7} = \frac{6 \times 7}{3 \times 7}$; or $\frac{56}{28} = \frac{42}{21}$. Now $\frac{56}{28}$ evidently equals $\frac{42}{21}$, since the value of either ratio is 2, and the same is true of the original proportion.

2. Dividing all the terms by any number, say 7, $\frac{8 \div 7}{4 \div 7} = \frac{6 \div 7}{3 \div 7}$; or $\frac{\frac{8}{7}}{\frac{4}{7}} = \frac{\frac{6}{7}}{\frac{3}{7}}$. But $\frac{8}{7} \div \frac{4}{7} = 2$, and $\frac{6}{7} \div \frac{3}{7} = 2$ also, the same as in the original proportion.

3. Raising all the terms to the same power, say the cube, $\frac{8^3}{4^3} = \frac{6^3}{3^3}$. This is evidently true, since $\frac{8^3}{4^3} = \left(\frac{8}{4}\right)^3 = 2^3 = 8$, and $\frac{6^3}{3^3} = \left(\frac{6}{3}\right)^3 = 2^3 = 8$ also.

4. Extracting the same root of all the terms, say the cube root, $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \frac{\sqrt[3]{6}}{\sqrt[3]{3}}$. It is evident that this is likewise true, since $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \sqrt[3]{\frac{8}{4}} = \sqrt[3]{2}$, and $\frac{\sqrt[3]{6}}{\sqrt[3]{3}} = \sqrt[3]{\frac{6}{3}} = \sqrt[3]{2}$ also.

5. Inverting both couplets, $\frac{4}{8} = \frac{3}{6}$, which is true, since both equal ½.

315. If both terms of either couplet be multiplied or both divided by the same number, the proportion is not destroyed. This should be evident from the preceding article, and also from Art. **294.** Hence, in any proportion, equal factors may be canceled from the terms of a couplet, before applying rule **41** or **42.** Thus, the proportion $45 : 9 = x : 7.1$, we may divide both terms of the first couplet by 9 (that is, cancel 9 from both terms), obtaining $5 : 1 = x : 7.1$, whence $x = 7.1 \times 5 \div 1 = 35.5$. (See note in Art. **300.**)

316. The principle of all calculations in proportion is this: *Three of the terms are always given, and the remaining one is to be found.*

317. Example.—If 4 men can earn $25 in one week, how much can 12 men earn in the same time?

Solution.—The required term must bear the same relation to the given term of the same kind as one of the remaining terms bears to the other remaining term. We can then form a proportion by which the required term may be found.

The first question the student must ask himself in every calculation by proportion is:

"What is it I want to find?"

In this case it is dollars. We have two sets of men, one set earning $25, and we want to know how many dollars the other set earns. It is evident that the *amount* 12 men earn bears the same relation to the *amount* that 4 men earn as 12 men bears to 4 men. Hence, we have the proportion, the amount 12 men earn is to $25 as 12 men is to 4 men; or, since either extreme equals the product of the means divided by the other extreme, we have

$$\text{The amount 12 men earn} : \$25 = 12 \text{ men} : 4 \text{ men},$$

or the amount 12 men earn $= \frac{\$25 \times 12}{4} = \$75$. Ans.

Since it matters not which place x or the required term occupies, the problem could be stated as any of the following forms, the value of x being the same in each:

(*a*) $25 : the amount 12 men earn = 4 men : 12 men; or the amount 12 men earn $= \frac{\$25 \times 12}{4}$, or $75, since either mean equals the product of the extremes divided by the other mean.

(*b*) 4 men : 12 men = $25 : the amount 12 men earn; or the amount that 12 men earn $= \frac{\$25 \times 12}{4}$, or $75, since either extreme equals the product of the means divided by the other extreme.

(*c*) 12 men : 4 men = the amount 12 men earn : $25; or the amount that 12 men earn $= \frac{\$25 \times 12}{4}$, or $75, since either mean equals the product of the extremes divided by the other mean.

318. If the proportion is an inverse one, first form it as though it were a direct proportion, and then invert one of the couplets.

EXAMPLES FOR PRACTICE.

319. Find the value of x in each of the following:

		Ans.	
(*a*)	$16 : $64 :: x : $4.	(*a*)	$x =$ $1.
(*b*)	x : 85 :: 10 : 17.	(*b*)	$x = 50$.
(*c*)	24 : x :: 15 : 40.	(*c*)	$x = 64$.
(*d*)	18 : 94 :: 2 : x.	(*d*)	$x = 10\frac{4}{9}$.
(*e*)	$75 : $100 = x : 100.	(*e*)	$x = 75$.
(*f*)	15 pwt. : $x = 21 : 10$.	(*f*)	$x = 7\frac{1}{7}$ pwt.
(*g*)	x : 75 yd. = $15 : $5.	(*g*)	$x = 225$ yd.

1. If 75 pounds of lead cost $2.10, what would 125 pounds cost at the same rate? Ans. $3.50.

2. If A does a piece of work in 4 days and B does it in 7 days, how long will it take A to do what B does in 63 days? Ans. 36 days.

3. The circumferences of any two circles are to each other as their diameters. If the circumference of a circle 7 inches in diameter is 22 inches, what will be the circumference of a circle 31 inches in diameter? Ans. $97\frac{3}{7}$ inches.

INVERSE PROPORTION.

320. In Art. **308**, an inverse proportion was defined as one which required one of the couplets to be expressed as an inverse ratio. Sometimes the word ***inverse*** occurs in the statement of the example; in such cases the proportion can be written directly, merely inverting one of the couplets. But it frequently happens that only by carefully studying the conditions of the example can it be ascertained whether the proportion is direct or inverse. When in doubt, the student can always satisfy himself as to whether the proportion is direct or inverse by first ascertaining what is required, and stating the proportion as a direct proportion. Then, in order that the proportion may be true, if the first term is smaller than the second term, the third term must be smaller than the fourth; or if the first term is larger than the second term, the third term must be larger than the fourth term. Keeping this in mind, the student can always tell whether the required term will be larger or smaller than the other term of the couplet to which the required term belongs. Having determined this, the student then refers to the example, and ascertains from its conditions whether the required term is to be larger or smaller than the other term of the same kind. If the two determinations agree, the proportion is direct; otherwise, it is inverse, and one of the couplets must be inverted.

321. Example.—If A's *rate* of doing work is to B's as 5 : 7, and A does a piece of work in 42 days, in what time will B do it?

Solution.—The required term is the number of days it will take B to do the work. Hence, stating as a direct proportion,

$$5 : 7 = 42 : x.$$

Now, since 7 is greater than 5, x will be greater than 42. But, referring to the statement of the example, it is easy to see that B works faster than A; hence it will take B a less number of days to do the work than A. Therefore, the proportion is an inverse one, and should be stated

$$5 : 7 = x : 42,$$

from which $x = \frac{5 \times 42}{7} = 30$ days. Ans.

Had the example been stated thus: The time that A requires to do a piece of work is to the time that B requires, as 5 : 7; A can do it in 42 days, in what time can B do it? it is evident that it would take B a longer time to do the work than it would A; hence, x would be greater than 42, and the proportion would be direct, the value of x being $\frac{7 \times 42}{5} = 58.8$ days.

EXAMPLES FOR PRACTICE.

322. Solve the following:

1. If a pump which discharges 4 gal. of water per min. can fill a tank in 20 hr., how long will it take a pump discharging 12 gal. per min. to fill it? Ans. 6⅔ hr.

2. The circular seam of a boiler requires 50 rivets when the pitch is 2½ in.; how many would be required if the pitch were 3⅛ in.? Ans. 40.

3. The spring hangers on a certain locomotive are 2½ in. wide and ¾ in. thick; those on another engine are of same sectional area, but are 3 in. wide; how thick are they? Ans. ⅝ in.

4. A locomotive with driving wheels 16 ft. in circumference runs a certain distance in 5,000 revolutions; how many revolutions would it make in going the same distance, if the wheels were 22 ft. in circumference (no allowance for slip being made in either case)? Ans. 3,636$\frac{4}{11}$ rev.

POWERS AND ROOTS IN PROPORTION.

323. It was stated in Art. **299** that a ratio could be raised to any power or any root of it might be taken. A proportion is frequently stated in such a manner that one of the couplets must be raised to some power or some root of it must be taken. In all such cases, both terms of the couplet so affected *must be raised to the same power or the same root of both terms must be taken.*

324. EXAMPLE.—Knowing that the weight of a sphere varies as the cube of its diameter, what is the weight of a sphere 6 inches in diameter if a sphere 8 inches in diameter of the same material weighs 180 pounds?

SOLUTION.—This is evidently a direct proportion. Hence, we write

$$6^3 : 8^3 = x : 180.$$

Dividing both terms of the first couplet by 2^3 (see Art. **300**),

$$3^3 : 4^3 = x : 180, \text{ or } 27 : 64 = x : 180;$$

whence, $x = \frac{27 \times 180}{64} = 75\frac{15}{16}$ pounds. Ans.

EXAMPLE.—A sphere 8 inches in diameter weighs 180 pounds; what is the diameter of another sphere of the same material which weighs $75\frac{15}{16}$ pounds?

SOLUTION.—Since the weights of any two spheres are to each other as the cubes of their diameters, we have the proportion

$$180 : 75\tfrac{15}{16} = 8^3 : x^3.$$

x, the required term. must be cubed, because the other term of the couplet is cubed (see Art. **323**). But, $8^3 = 512$; hence,

$$180 : 75\tfrac{15}{16} = 512 : x^3, \text{ or } x^3 = \frac{75\frac{15}{16} \times 512}{180} = 216;$$

whence, $x = \sqrt[3]{216} = 6$ inches. Ans.

325. Since taking the same root of all of the terms of a proportion does not change its value (Art. **314**), the above example might have been solved by extracting the cube root of all of the numbers, thus obtaining $\sqrt[3]{180} : \sqrt[3]{75\frac{15}{16}} = 8 : x$; whence,

$$x = \frac{8 \times \sqrt[3]{75\frac{15}{16}}}{\sqrt[3]{180}} = 8 \times \sqrt[3]{\frac{75\frac{15}{16}}{180}} = 8\sqrt[3]{\frac{1{,}215}{2{,}880}} = 8\sqrt[3]{\frac{27}{64}} =$$

$8 \times \frac{3}{4} = 6$ inches. The process, however, is longer and is not so direct, and the first method is to be preferred.

326. If two cylinders have *equal* volumes, but different diameters, the diameters are to each other inversely as the square roots of their lengths. Hence, if it is desired to find the diameter of a cylinder that is to be 15 inches long, and which shall have the same volume as one that is 9 inches in diameter and 12 inches long, we write the proportion

$$9 : x = \sqrt{15} : \sqrt{12}.$$

Since neither 12 nor 15 are perfect squares, we square all of the terms (Arts. **325** and **314**) and obtain

$$81 : x^2 = 15 : 12; \text{ whence } x^2 = \frac{81 \times 12}{15} = 64.8,$$

and $x = \sqrt{64.8} = 8.05$ inches = diameter of 15-inch cylinder.

EXAMPLES FOR PRACTICE.

327. Solve the following examples:

1. The intensity of light varies inversely as the square of the distance from the source of light. If a gas jet illuminates an object 30 feet away with a certain distinctness, how much brighter will the object be at a distance of 20 feet? Ans. $2\frac{1}{4}$ times as bright.

2. In the last example, suppose that the object had been 40 feet from the gas jet; how bright would it have been compared with its brightness at 30 feet from the gas jet? Ans. $\frac{9}{16}$ as bright.

3. When comparing one light with another, the intensities of their illuminating powers vary as the squares of their distances from the

source. If a man can just distinguish the time indicated by his watch, 50 feet from a certain light, at what distance could he distinguish the time from a light 3 times as powerful? Ans. 86.6+ feet.

4. The quantity of air flowing through a mine varies directly as the square root of the pressure. If 60,000 cubic feet of air flow per minute when the pressure is 2.8 pounds per square foot, how much will flow when the pressure is 3.6 pounds per square foot?
Ans. 68,034 cu. ft. per min., nearly.

5. In the last example, suppose that 70,000 cubic feet per minute had been required; what would be the pressure necessary for this quantity? Ans. 3.81+ lb. per sq. ft.

CAUSES AND EFFECTS.

328. Many examples in proportion may be more easily solved by using the principle of *cause and effect.* That which may be regarded as producing a change or alteration in something, or as accomplishing something, may be called a **cause,** and the change, or alteration, or thing accomplished, is the **effect.**

329. *Like causes produce like effects.* Hence, when two causes of the same kind produce two effects of the same kind, the ratio of the causes equals the ratio of the effects; in other words, the first cause is to the second cause as the first effect is to the second effect. Thus, in the question, if 3 men can lift 1,400 pounds, how many pounds can 7 men lift? we call 3 men and 7 men the *causes* (since they accomplish something, viz., the lifting of the weight), the number of pounds lifted, viz., 1,400 pounds and x pounds, are the effects. If we call 3 men the first cause, 1,400 pounds is the first effect; 7 men is the second cause, and x pounds is the second effect. Hence, we may write

$$\begin{array}{ccccccc} \textit{1st cause} & & \textit{2d cause} & & \textit{1st effect} & & \textit{2d effect} \\ 3 & : & 7 & = & 1{,}400 & : & x, \end{array}$$

whence $x = \frac{7 \times 1{,}400}{3} = 3{,}266\frac{2}{3}$ pounds.

330. The principle of cause and effect is extremely useful in the solution of examples in compound proportion, as we shall now show.

COMPOUND PROPORTION.

331. All the cases of proportion so far considered have been cases of **simple proportion**; i. e., each term has been composed of but one number. There are many cases, however, in which two or all of the terms have more than one number in them; all such cases belong to **compound proportion.** In all examples in compound proportion, both causes or both effects or all four consist of more than two numbers. We will illustrate this by an

EXAMPLE.—If 40 men earn $1,280 in 16 days, how much will 36 men earn in 31 days?

SOLUTION.—Since 40 men earn something, 40 men is a cause, and since they take 16 days in which to earn something, 16 days is also a cause. For the same reason, 36 men and 31 days are also causes. The effects, that which is earned, are 1,280 dollars and x dollars. Then, 40 men and 16 days make up the first cause, and 36 men and 31 days make up the second cause. $1,280 is the first effect and $$x$ is the second effect. Hence, we write

$$\begin{array}{ccccccc} \textit{1st cause} & & \textit{2d cause} & & \textit{1st effect} & & \textit{2d effect} \\ \begin{matrix}40\\16\end{matrix} & : & \begin{matrix}36\\31\end{matrix} & = & 1{,}280 & : & x \end{array}$$

Now, instead of using the colon to express the ratio, we shall use the vertical line (see Art. **289**), and the above becomes

$$\begin{matrix}40\\16\end{matrix} \left| \begin{matrix}36\\31\end{matrix} = 1{,}280 \right| x.$$

In the last expression, the product of all of the numbers included between the vertical lines must equal the product of all the numbers without them; i. e., $36 \times 31 \times 1{,}280 = 40 \times 16 \times x$.

$$\text{Or } x = \frac{36 \times 31 \times \overset{\overset{2}{\cancel{80}}}{\cancel{1280}}}{\cancel{40} \times \cancel{16}} = \$2{,}232. \text{ Ans.}$$

332. The above might have been solved by canceling factors of the numbers in the original proportion. For if any number within the lines has a factor common to any number without the lines, that factor may be canceled from both numbers. Thus, 16 is contained in

$$\begin{matrix}\cancel{40}\\\cancel{16}\end{matrix} \left| \begin{matrix}36\\31\end{matrix} = \begin{matrix}2\\\cancel{80}\\\cancel{1280}\end{matrix} \right| x,$$

1,280, 80 times. Cancel 16 and 1,280, and write 80 above 1,280. 40 is contained in 80, 2 times. Cancel 40 and 80, and write 2 above 80. Now, since there are no more numbers that can be canceled, $x = 36 \times 31 \times 2 = \$2{,}232$, he same result as was obtained in the last article.

333. Rule 43.—*Write all the numbers forming the first cause in a vertical column, and draw a vertical line; on the other side of this line write in a vertical column all of the numbers forming the second cause. Write the sign of equality to the right of the second column, and on the right of this form a third column of the numbers composing the first effect, drawing a vertical line to the right; on the other side of this line, write, for a fourth column, the numbers composing the second effect. There must be as many numbers in the second cause as in the first cause, and in the second effect as in the first effect; hence, if any term is wanting, write x in its place. Multiply together all of the numbers within the vertical lines, and also all those without the lines (canceling previously, if possible), and divide the product of those numbers which do not contain x by the product of the others in which x occurs, and the result will be the value of x.*

334. Example.—If 40 men can dig a ditch 720 feet long, 5 feet wide and 4 feet deep in a certain time, how long a ditch 6 feet deep and 8 feet wide could 24 men dig in the same time?

Solution.—Here 40 men and 24 men are the causes and the two ditches are the effects. Hence,

$$\begin{array}{c|ccc|l} & & & \cancel{3} & \\ & & & \cancel{18} & \\ & & & \cancel{720} & x \\ \cancel{40} & 24 & = & 5 & \cancel{8} \text{ whence, } x = 24 \times 5 \times 4 = 480 \text{ feet. Ans.} \\ & & & 4 & \cancel{6} \end{array}$$

335. Example.—The volume of a cylinder varies directly as its length and directly as the square of its diameter. If the volume of a cylinder 10 inches in diameter and 20 inches long is 1,570.8 cubic inches, what is the volume of another cylinder 16 inches in diameter and 24 inches long?

Solution.—In this example, either the dimensions or the volumes may be considered the causes; say we take the dimensions for the causes. Then, squaring the diameters,

$$\begin{array}{c|ccc|c} 10^2 & 16^2 & = & 1{,}570.8 & x, \\ 20 & 24 & & & \end{array} \text{ or } \begin{array}{c|ccc|c} 100 & 256 & = & 1{,}570.8 & x; \\ \cancel{20} & \cancel{24} & & & \\ 5 & 6 & & & \end{array}$$

whence, $x = \dfrac{256 \times 6 \times 1{,}570.8}{5 \times 100} = 4{,}825.4976$ cubic inches. Ans.

336. Example.—If a block of granite 8 ft. long, 5 ft. wide and 3 ft. thick weighs 7,200 lb., what will be the weight of a block of granite 12 ft. long, 8 ft. wide and 5 ft. thick?

Solution.—Taking the weights as the effects, we have

$$\begin{array}{c|c|l} & 4 & \\ \cancel{8} & \cancel{12} & \\ \cancel{5} & \cancel{8} = 7{,}200 & x, \text{ or } x = 4 \times 7{,}200 = 28{,}800 \text{ pounds. Ans.} \\ \cancel{3} & \cancel{5} & \end{array}$$

337. Example.—If 12 compositors in 30 days of 10 hours each set up 25 sheets of 16 pages each, 32 lines to the page, in how many days 8 hours long can 18 compositors set up, in the same type, 64 sheets of 12 pages each, 40 lines to the page?

Solution.—Here compositors, days, and hours compose the causes, and sheets, pages, and lines the effects. Hence,

$$\begin{array}{c|ccc|l} 3 & \cancel{3} & & \cancel{5} & 2 \\ \cancel{12} & \cancel{18} & & \cancel{25} & \cancel{64} \\ & & & \cancel{4} & \\ \cancel{30} & x & = & \cancel{16} & \cancel{12}, \text{ or } x = 8 \times 10 \times 2 = 60 \text{ days. Ans.} \\ \cancel{6} & & & & \cancel{4} \\ 10 & \cancel{8} & & \cancel{32} & \cancel{40} \\ & & & & \cancel{5} \end{array}$$

338. In examples stated like that in Art. **335,** should an inverse proportion occur, write the various numbers as in the preceding examples, and then transpose those numbers which are said to vary inversely from one side of the vertical line to the other side.

Example.—The centrifugal force of a revolving body varies directly as its weight, as the square of its velocity and inversely as the radius of the circle described by the center of the body. If the centrifugal force of a body weighing 15 pounds is 187 pounds when the body revolves in a circle having a radius of 12 inches, with a velocity of 20 feet per second, what will be the centrifugal force of the same body when the radius is increased to 18 inches and the speed is increased to 24 feet per second?

Solution.—Calling the centrifugal force the effect, we have,

$$\begin{array}{c|c|l} 15 & 15 & \\ 20^2 & 24^2 = 187 & x. \\ 12 & 18 & \end{array}$$

Transposing 12 and 18 (since the radii are to vary inversely) and squaring 20 and 24,

$$\begin{array}{c|c|l} \cancel{15} & \cancel{15} & \\ & 2 & \\ 25 & \cancel{36} = 187 & x, \text{ or } x = \dfrac{12 \times 2 \times 187}{25} = 179.52 \text{ pounds. Ans.} \\ \cancel{400} & \cancel{576} & \\ \cancel{18} & 12 & \end{array}$$

EXAMPLES FOR PRACTICE.

339. Solve the following by compound proportion:

1. If 12 men dig a trench 40 rods long in 24 days of 10 hours each, how many rods can 16 men dig in 18 days of 9 hours each?

Ans. 36 rods.

2. If a piece of iron 7 ft. long, 4 in. wide, and 6 in. thick weighs 600 lb., how much will a piece of iron weigh that is 16 ft. long, 8 in. wide and 4 in. thick? Ans. 1,828$\frac{4}{7}$ lb.

3. If 24 men can build a wall 72 rods long, 6 feet wide, and 5 feet high in 60 days of 10 hours each, how many days will it take 32 men to build a wall 96 rods long, 4 feet wide and 8 feet high, working 8 hours a day? Ans. 80 days.

4. The horsepower of an engine varies as the mean effective pressure, as the piston speed and as the square of the diameter of the cylinder. If an engine having a cylinder 14 inches in diameter develops 112 horsepower when the mean effective pressure is 48 pounds per square inch and the piston speed is 500 feet per minute, what horsepower will another engine develop if the cylinder is 16 inches in diameter, piston speed is 600 feet per minute, and mean effective pressure is 56 pounds per square inch? Ans. 204.8 horsepower.

5. Referring to the example in Art. **335**, what will be the volume of a cylinder 20 inches in diameter and 24 inches long?

Ans. 7,539.84 cubic inches.

6. Knowing that the product of $3 \times 5 \times 7 \times 9$ is 945, what is the product of $6 \times 15 \times 14 \times 36$? Ans. 45,360.

7. The speed in miles per hour of a locomotive is directly proportional to the diameter of its driving wheels and the number of revolutions they make in one minute. A locomotive with driving wheels 66 inches in diameter runs 29.45 miles in an hour when the wheels make 150 revolutions per minute; how many miles will be run in one hour by a locomotive having wheels 72 inches in diameter running 220 revolutions per minute? Ans. 47.12 miles per hour.

8. The capacity of a cylindrical boiler is proportional to its length and the square of its diameter. A boiler 12 feet long and 48 inches in diameter will hold 1,128 gallons; what is the capacity of a boiler 16 feet long and 42 inches in diameter? Ans. 1,151.5 gallons.

9. The power that may be transmitted by a belt is proportional to its width and the diameter and number of revolutions made by the pulley on which it runs. If a belt 12 inches wide will transmit 10 horsepower when running over a pulley 20 inches in diameter that makes 125 revolutions per minute, how many horsepower may be transmitted by a belt 8 inches wide when running over a pulley 30 inches in diameter that makes 200 revolutions per minute?

Ans. 16 horsepower.

10. The load that a beam supported at the two ends will carry is directly proportional to its width and the square of its depth, and inversely proportional to its length. If an oak beam 8 inches wide, 12 inches deep, and 15 feet long will safely carry a load of 13,824 pounds, what is the safe load that a beam 10 inches wide, 16 inches deep, and 20 feet long will support? Ans. 23,040 pounds.

MENSURATION AND USE OF LETTERS IN FORMULAS.

FORMULAS.

340. The term **formula,** as used in mathematics and in technical books, may be defined as *a rule in which symbols are used instead of words;* in fact, a formula may be regarded as a shorthand method of expressing a rule. Any formula can be expressed in words, and when so expressed it becomes a rule.

Formulas are much more convenient than rules; they show at a glance all the operations that are to be performed; they do not require to be read three or four times, as is the case with most rules, to enable one to understand their meaning; they take up much less space, both in the printed book and in one's note-book, than rules; in short, whenever a rule can be expressed as a formula, the formula is to be preferred.

As the term "quantity" is a very convenient one to use, we will define it. In mathematics, the word **quantity** is applied to anything that it is desired to subject to the ordinary operations of addition, subtraction, multiplication, etc., when we do not wish to be more specific and state exactly what the thing is. Thus, we can say "two or more numbers," or "two or more quantities;" the word quantity is more general in its meaning than the word number.

341. The signs used in formulas are the ordinary signs indicative of operations, and the signs of aggregation. All these signs are explained in arithmetic, but some of them will here be explained in order to refresh the student's memory.

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The signs indicative of operations are six in number, viz.: $+$, $-$, $\times$, $\div$, $|$, $\sqrt{}$.

Division is indicated by the sign $\div$, or by placing a straight line between the two quantities. Thus, $25 \mid 17$, $25 \; / \; 17$, and $\frac{25}{17}$ all indicate that 25 is to be divided by 17. When both quantities are placed on the same horizontal line, the straight line indicates that the quantity on the left is to be divided by that on the right. When one quantity is below the other, the straight line between indicates that the quantity above the line is to be divided by the one below it.

The sign ($\sqrt{}$) indicates that some root of the quantity to the right is to be taken; it is called the **radical sign.** To indicate what root is to be taken, a small figure, called the **index,** is placed within the sign, this being always omitted when the square root is to be indicated. Thus, $\sqrt{25}$ indicates that the square root of 25 is to be taken; $\sqrt[3]{25}$ indicates that the cube root of 25 is to be taken; etc.

The signs of aggregation are four in number; viz., ——, ( ), [ ], and { }, respectively called the **vinculum,** the **parenthesis,** the **brackets,** and the **brace;** they are used when it is desired to indicate that all the quantities included by them are to be subjected to the same operation. Thus, if we desire to indicate that the sum of 5 and 8 is to be multiplied by 7, and we do not wish to actually add 5 and 8 before indicating the multiplication, we may employ any one of the four signs of aggregation as here shown: $\overline{5+8} \times 7$, $(5+8) \times 7$, $[5+8] \times 7$, $\{5+8\} \times 7$. The vinculum is placed above those quantities which are to be treated as one quantity and subjected to the same operations.

While any one of the four signs may be used as shown above, custom has restricted their use somewhat. The vinculum is rarely used except in connection with the radical sign. Thus, instead of writing $\sqrt[3]{}(5+8)$, $\sqrt[3]{}[5+8]$, or $\sqrt[3]{}\{5+8\}$ for the cube root of 5 plus 8, all of which would be correct, the vinculum is nearly always used, $\sqrt[3]{5+8}$.

In cases where but one sign of aggregation is needed (except, of course, when a root is to be indicated), the parenthesis

is always used. Hence, $(5+8)\times 7$ would be the usual way of expressing the product of 5 plus 8, and 7.

If two signs of aggregation are needed, the brackets and parenthesis are used, so as to avoid having a parenthesis within a parenthesis, the brackets being placed outside. For example, $[(20-5)\div 3]\times 9$ means that the difference between 20 and 5 is to be divided by 3, and this result multiplied by 9.

If three signs of aggregation are required, the brace, brackets, and parenthesis are used, the brace being placed outside, the brackets next, and the parenthesis inside. For example, $\{[(20-5)\div 3]\times 9-21\}\div 8$ means that the quotient obtained by dividing the difference between 20 and 5 by 3 is to be multiplied by 9, and that after 21 has been subtracted from the product thus obtained, the result is to be divided by 8.

Should it be necessary to use all four of the signs of aggregation, the brace would be put outside, the brackets next, the parenthesis next, and the vinculum inside. For example, $\{[(\overline{20-5}\div 3)\times 9-21]\div 8\}\times 12$.

As stated in arithmetic, when several quantities are connected by the various signs indicating addition, subtraction, multiplication, and division, the operation indicated by the sign of multiplication must always be performed first. Thus, $2+3\times 4$ equals 14, 3 being multiplied by 4, before adding to 2. Similarly, $10\div 2\times 5$ equals 1, since 2×5 equals 10, and $10\div 10$ equals 1. Hence, in the above case, if the brace were omitted, the result would be $\frac{1}{4}$, whereas, by inserting the brace, the result is 36.

Following the sign of multiplication comes the sign of division in order of importance. For example, $5-9\div 3$ equals 2, 9 being divided by 3 before subtracting from 5. The signs of addition and subtraction are of equal value; that is, if several quantities are connected by plus and minus signs, the indicated operations may be performed in the order in which the quantities are placed.

There is one other sign used, which is neither a sign of aggregation nor a sign indicative of an operation to be performed; it is (=), and is called the sign of *equality;* it means that all on one side of it is exactly equal to all on the other side. For example, $2 = 2$, $5-3 = 2$, $5\times(14-9) = 25$.

342. Having called particular attention to certain signs used in formulas, the formulas themselves will now be explained. First, consider the well-known rule for finding the horsepower of a steam-engine, which may be stated as follows:

Divide the continued product of the mean effective pressure in pounds per square inch, the length of the stroke in feet, the area of the piston in square inches, and the number of strokes per minute, by 33,000; the result will be the horsepower.

This is a very simple rule, and very little, if anything, will be saved by expressing it as a formula, so far as clearness is concerned. The formula, however, will occupy a great deal less space, as we shall show.

An examination of the rule will show that four quantities (viz., the mean effective pressure, the length of the stroke, the area of the piston, and the number of strokes) are multiplied together, and the result is divided by 33,000. Hence, the rule might be expressed as follows:

$$\text{Horsepower} = \begin{matrix}\text{mean effective pressure}\\ \text{(in pounds per square inch)}\end{matrix} \times \begin{matrix}\text{stroke}\\ \text{(in feet)}\end{matrix} \times$$

$$\begin{matrix}\text{area of piston}\\ \text{(in square inches)}\end{matrix} \times \begin{matrix}\text{number of strokes}\\ \text{(per minute)}\end{matrix} \div 33{,}000.$$

This expression could be shortened by representing each quantity by a single letter; thus, representing horsepower by the letter "*H*," the mean effective pressure in pounds per square inch by "*P*," the length of stroke in feet by "*L*," the area of the piston in square inches by "*A*," the number of strokes per minute by "*N*," and substituting these letters for the quantities that they represent, the above expression would reduce to

$$H = \frac{P \times L \times A \times N}{33{,}000},$$

a much simpler and shorter expression. This last expression is called a *formula*.

The formula just given shows, as we stated in the beginning, that a formula is really a shorthand method of expressing a rule. It is customary, however, to omit the sign of multiplication between two or more quantities when they are to be multiplied together, or between a number and a letter representing a quantity, it being always understood that, when two letters are adjacent with no sign between them, the quantities represented by these letters are to be multiplied. Bearing this fact in mind, the formula just given can be further simplified to

$$H = \frac{PLAN}{33{,}000}.$$

The sign of multiplication, evidently, can not be omitted between two or more numbers, as it would then be impossible to distinguish the numbers. A near approach to this, however, may be attained by placing a dot between the numbers which are to be multiplied together, and this is frequently done in works on mathematics when it is desired to economize space. In such cases it is usual to put the dot higher than the position occupied by the decimal point. Thus, $2 \cdot 3$ means the same as 2×3; $542 \cdot 749 \cdot 1{,}006$ indicates that the numbers 542, 749, and 1,006 are to be multiplied together.

It is also customary to omit the sign of multiplication in expressions similar to the following: $a \times \sqrt{b+c}$, $3 \times (b+c)$, $(b+c) \times a$, etc., writing them $a\sqrt{b+c}$, $3\,(b+c)$, $(b+c)\,a$, etc. The sign is not omitted when several quantities are included by a vinculum, and it is desired to indicate that the quantities so included are to be multiplied by another quantity. For example, $3 \times \overline{b+c}$, $\overline{b+c} \times a$, $\sqrt{b+c} \times a$, etc., are always written as here printed.

343. Before proceeding further, we will explain one other device that is used by formula makers and which is apt to puzzle one who encounters it for the first time—it is the use of what mathematicians call *primes* and *subs.*, and what printers call *superior* and *inferior* characters. As a rule, formula makers designate quantities by the initial letters of the names of the quantities. For example, they represent volume by v, pressure by p, height by h, etc. This practice is to be commended, as the letter itself serves in many cases to identify the quantity which it represents. Some authors carry the practice a little further, and represent all quantities of the same nature by the same letter throughout the book, always having the same letter represent the same thing. Now, this practice necessitates the use of the primes and subs. above mentioned, when two quantities have the same name but represent different things. Thus, consider the word *pressure* as applied to steam, at different stages between the boiler and the condenser. First, there is *absolute* pressure, which is equal to the gauge pressure in pounds per square inch plus the pressure indicated by the barometer reading (usually assumed in practice to be 14.7 pounds per square inch, when a barometer is not at hand). If this be represented by p, how shall we represent the gauge pressure? Since the absolute pressure is always greater than the gauge pressure, suppose we decide to represent it by a capital letter, and the gauge pressure by a small (lower-case) letter. Doing so, P represents absolute pressure, and p, gauge pressure. Further, there is usually a "drop" in pressure between the boiler and the engine, so that the initial pressure, or pressure at the beginning of the stroke, is less than the pressure at the boiler. How shall we represent the initial pressure? We may do this in one of three ways and still retain the letter p or P to represent the word pressure: First, by the use of the prime mark; thus, p' or P' (read *p prime* and *P major prime*) may be considered to represent the initial gauge pressure, or the initial absolute pressure. Second, by the use of sub. figures; thus, p_1 or P_1 (read *p sub. one* and *P major sub. one*). Third, by the use of sub. letters; thus, p_i or P_i (read *p sub. i* and *P major sub. i*). In the same manner p'' (read *p second*), p_2, or p_r might be used to represent the gauge pressure at release, etc. The sub. letters have the advantage of still further identifying the quantity represented; in many instances, however, it is not convenient to use them, in which case primes and subs. are used instead. The prime notation may be continued as follows: p''', p^{iv}, p^{v}, etc.; it is inadvisable to use superior figures, for example, p^1, p^2, p^3, p^n, etc., as they are liable to be mistaken for exponents.

The main thing to be remembered by the student is that *when a formula is given in which the same letters occur several times, all like letters having the same primes or subs. represent the same quantities, while those which differ in any respect represent different quantities.* Thus, in the formula

$$t = \frac{w_1 s_1 t_1 + w_2 s_2 t_2 + w_3 s_3 t_3}{w_1 s_1 + w_2 s_2 + w_3 s_3},$$

w_1, w_2, and w_3 represent the weights of three different bodies; s_1, s_2, and s_3, their specific heats; and t_1, t_2, and t_3, their temperatures; while t represents the final temperature after the bodies have been mixed together. It should be noted that those letters having the *same* subs. refer to the same bodies. Thus, w_1, s_1, and t_1 all refer to one of the three bodies; w_2, s_2, t_2, to another body; etc.

It is very easy to apply the above formula when the values of the quantities represented by the different letters are known. All that is required is to substitute the numerical values of the letters, and then perform the indicated operations. Thus, suppose that the values of w_1, s_1, and t_1 are, respectively, 2 pounds, .0951, and 80°; of w_2, s_2, and t_2, 7.8 pounds, 1, and 80°; and of w_3, s_3, and t_3, 3¼ pounds, .1138, and 780°; then, the final temperature t is, substituting these values for their respective letters in the formula,

$$t = \frac{2 \times .0951 \times 80 + 7.8 \times 1 \times 80 + 3\tfrac{1}{4} \times .1138 \times 780}{2 \times .0951 + 7.8 \times 1 + 3\tfrac{1}{4} \times .1138} =$$

$$\frac{15.216 + 624 + 288.483}{.1902 + 7.8 + .36985} = \frac{927.699}{8.36005} = 110.97^\circ.$$

In substituting the numerical values, the signs of multiplication are, of course, written in their proper places; all the multiplications are performed before adding, according to the rule previously given.

344. The student should now be able to apply any formula involving only algebraic expressions that he may meet with, and which does not require the use of logarithms for its solution. We will, however, call his attention to one or two other facts that he may have forgotten.

Expressions similar to $\dfrac{160}{\frac{660}{25}}$ sometimes occur. the heavy line indicating that 160 is to be divided by the quotient obtained by dividing 660 by 25. If both lines were light it would be impossible to tell whether 160 was to be divided by $\frac{660}{25}$, or whether $\frac{160}{660}$ was to be divided by 25. If this latter result were desired, the expression would be written $\dfrac{\frac{160}{660}}{25}$. In every case the heavy line indicates that all above it is to be divided by all below it.

In an expression like the following, $\dfrac{160}{7+\frac{660}{25}}$, the heavy line is not necessary, since it is impossible to mistake the operation that is required to be performed. But, since $7+\frac{660}{25}=\frac{175+660}{25}$, if we substitute $\frac{175+660}{25}$ for $7+\frac{660}{25}$, the heavy line becomes necessary in order to make the resulting expression clear. Thus,

$$\frac{160}{7+\frac{660}{25}}=\frac{160}{\frac{175+660}{25}}=\frac{160}{\frac{835}{25}}$$

Fractional exponents are sometimes used instead of the radical sign. That is, instead of indicating the square, cube, fourth root, etc., of some quantity, as 37, by $\sqrt{37}$, $\sqrt[3]{37}$, $\sqrt[4]{37}$, etc., these roots are indicated by $37^{\frac{1}{2}}$, $37^{\frac{1}{3}}$, $37^{\frac{1}{4}}$, etc. Should the numerator of the fractional exponent be some quantity other than 1, this quantity, whatever it may be, indicates that the quantity affected by the exponent is to be raised to the power indicated by the numerator; the denominator is *always* the index of the root. Hence, instead of writing $\sqrt[3]{37^2}$ for the cube root of the square of 37, it may be written $37^{\frac{2}{3}}$, the denominator being the index of the root; in other words, $\sqrt[3]{37^2} = 37^{\frac{2}{3}}$. Likewise, $\sqrt[5]{(1 + a^2 b)^3}$ may also be written $(1 + a^2 b)^{\frac{3}{5}}$, a much simpler expression.

345. We will now give several examples showing how to apply some of the more difficult formulas that the student may encounter.

1. The area of any segment of a circle that is less than (or equal to) a semicircle is expressed by the formula

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r - h),$$

in which A = area of segment;
π = 3.1416;
r = radius;
E = angle obtained by drawing lines from the center to the extremities of arc of segment;
c = chord of segment;
and h = height of segment.

EXAMPLE.—What is the area of a segment whose chord is 10 inches long, angle subtended by chord is 83.46°, radius is 7.5 inches, and height of segment is 1.91 inches?

SOLUTION.—Applying the formula just given,

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r - h) = \frac{3.1416 \times 7.5^2 \times 83.46}{360} - \frac{10}{2}(7.5 - 1.91) =$$
$$40.968 - 27.95 = 13.018 \text{ square inches, nearly. Ans.}$$

2. The area of any triangle may be found by means of the following formula, in which A = the area, and a, b, and c represent the lengths of the sides:

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2}.$$

EXAMPLE.—What is the area of a triangle whose sides are 21 feet, 46 feet, and 50 feet long?

SOLUTION.—In order to apply the formula, suppose we let a represent the side that is 21 feet long; b, the side that is 50 feet long; and c, the side that is 46 feet long. Then substituting in the formula,

$$A=\frac{b}{2}\sqrt{a^2-\left(\frac{a^2+b^2-c^2}{2b}\right)^2}=\frac{50}{2}\sqrt{21^2-\left(\frac{21^2+50^2-46^2}{2\times 50}\right)^2}$$

$$=\frac{50}{2}\sqrt{441-\left(\frac{441+2{,}500-2{,}116}{100}\right)^2}=25\sqrt{441-\left(\frac{825}{100}\right)^2}$$

$$=25\sqrt{441-8.25^2}=25\sqrt{441-68.0625}=25\sqrt{372.9375}$$

$$=25\times 19.312=482.8 \text{ square feet, nearly.}\quad \text{Ans.}$$

The operations in the above examples have been extended much farther than was necessary; it was done in order to show the student every step of the process. The last formula is perfectly general, and the same answer would have been obtained had the 50-foot side been represented by *a*, the 46-foot side by *b*, and the 21-foot side by *c*.

3. The Rankine-Gordon formula for determining the least load in pounds that will cause a long column to break is

$$P=\frac{SA}{1+q\frac{l^2}{G^2}},$$

in which P = load (pressure) in pounds;
S = ultimate strength (in pounds per square inch) of the material composing the column;
A = area of cross-section of column in square inches;
q = a factor (multiplier) whose value depends upon the shape of the ends of the column and on the material composing the column;
l = length of column in inches;
and G = least radius of gyration of cross-section of column.

The values of S, q, and G^2 are given in printed tables in books in which this formula occurs.

EXAMPLE.—What is the least load that will break a hollow wrought-iron column whose outside diameter is 14 inches; inside diameter, 11 inches; length, 20 feet, and whose ends are flat?

SOLUTION.—For steel, $S = 150{,}000$, and $q=\frac{1}{25{,}000}$ for flat-ended steel columns; A, the area of the cross-section, $= .7854\,(d_1^2-d_2^2) = .7854\,(14^2-11^2)$, d_1 and d_2 being the outside and inside diameters, respectively; $l = 20 \times 12 = 240$ inches; and $G^2 = \frac{d_1^2 + d_2^2}{16} = \frac{14^2 + 11^2}{16}$. Substituting these values in the formula,

$$P = \frac{S\,A}{1 + q\frac{l^2}{G^2}} = \frac{150{,}000 \times .7854\,(14^2 - 11^2)}{1 + \frac{1}{25{,}000} \times \frac{240^2}{\frac{14^2 + 11^2}{16}}} =$$

$$\frac{150{,}000 \times 58.905}{1 + .1163} = \frac{8{,}835{,}750}{1.1163} = 7{,}915{,}211 \text{ pounds. Ans.}$$

4. **Example.**—When $A = 10$, $B = 8$, $C = 5$, and $D = 4$, what is the value of E in the following:

(*a*) $E = \sqrt[3]{\frac{B\,C\,D}{A\left(2 + \frac{D^2}{C^2}\right)}}$? (*b*) $E = \frac{A - \frac{3}{4}D + \frac{4\,B^2}{A + C}}{A - \sqrt{\frac{2\,B^2}{A + 22}}}$?

Solution.—(*a*) Substituting,

$$E = \sqrt[3]{\frac{8 \times 5 \times 4}{10\left(2 + \frac{4^2}{5^2}\right)}}.$$

To simplify the denominator, square the 4 and 5, add the resulting fraction to 2, and multiply by 10. Simplifying, we have

$$E = \sqrt[3]{\frac{160}{10\left(2 + \frac{16}{25}\right)}} = \sqrt[3]{\frac{160}{10 \times \frac{66}{25}}} = \sqrt[3]{\frac{160}{\frac{660}{25}}} = \sqrt[3]{\frac{200}{33}}.$$

Reducing the fraction to a decimal before extracting the cube root,

$$E = \sqrt[3]{6.0606} = 1.823. \text{ Ans.}$$

(*b*) Substituting,

$$E = \frac{10 - \frac{3}{4} \times 4 + \frac{4 \times 8^2}{10 + 5}}{10 - \sqrt{\frac{2 \times 8^2}{10 + 22}}} = \frac{10 - 3 + \frac{4 \times 64}{15}}{10 - \sqrt{\frac{2 \times 64}{32}}} =$$

$$\frac{7 + 17.066+}{10 - \sqrt{4}} = \frac{24.066+}{8} = 3.008+. \text{ Ans.}$$

345$_1$.—In the preceding pages, the unknown quantity has always been represented by the single letter at the left of the sign of equality, while the letters at the right have represented known values from which the required values could be found. It is possible, however, to find the value of the quantity represented by any letter in a formula, if the values represented by all the others are known. For example, let it be required to find how many strokes per minute an engine having a piston area of 78.54 sq. in. must make in order to develop 60 horsepower, if the mean effective pressure is 40 lb. per sq. in., and the length of stroke 1¼ ft. By substituting the given values in the formula $H = \frac{PLAN}{33,000}$, we have

$$60 = \frac{40 \times 1\frac{1}{4} \times 78.54 \times N}{33,000},$$

in which N, the number of strokes, is to be found.

But it is evident that the expression on the right of the sign of equality is equal to $\frac{40 \times 1\frac{1}{4} \times 78.54}{33,000} \times N$, a fraction whose numerator is composed of 3 factors. Reducing the numerator to a single number by performing the indicated multiplications, we obtain, after canceling,

$$60 = \frac{119}{1,000} \times N = .119\,N.$$

If 60 equals .119 N, then N equals 60 divided by .119; hence,

$$N = \frac{60}{.119} = 504.2 \text{ strokes per minute.}$$

The method of procedure is essentially the same when the unknown quantity occurs in the denominator of a formula. Thus, in the formula $f = \frac{m\,v^2}{r}$, suppose that $f = 375$, $m = 1.25$, and $v = 60$. Then, substituting,

$$375 = \frac{1.25 \times 60^2}{r} = \frac{4,500}{r}.$$

But if 375 equals 4,500 divided by r, then $375 \times r = 4,500$; hence, r must equal 4,500 divided by 375, or $r = \frac{4,500}{375} = 12$.

EXAMPLES FOR PRACTICE.

Find the numerical values of x in the following formulas, when $A = 9$, $B = 8$, $d = 10$, $e = 3$, and $c = 2$:

1. $x = \frac{d + c^2}{d^2 - 40}$. Ans. $x = \frac{7}{30}$.

2. $x = \frac{\frac{3}{4}(A + e)}{c\,e}$. Ans. $x = 1\frac{1}{2}$.

3. $x = \sqrt{\frac{d^2}{2c}} + \sqrt{A B^2}$. Ans. $x = 29$.

4. $x = \frac{A e}{\sqrt{16 B c}} + \frac{5}{16}$. Ans. $x = 2$.

5. $x = (c + 2e)\left(\sqrt[3]{B} - \frac{1}{c}\right) + \frac{e^2 - c^2}{e^2 + c^2}$. Ans. $x = 12\frac{5}{13}$.

6. $x = \sqrt{\frac{B c d}{A\left(2 + \frac{d^2}{e^2}\right)}}$. Ans. $x = .396 +$.

MENSURATION.

346. **Mensuration** treats of the measurement of lines, angles, surfaces, and solids.

LINES AND ANGLES.

347. A straight line is one that does not change its direction throughout its whole length. To distinguish one straight line from another, its two extreme points are designated by letters. The line shown in Fig. 1 would be called the line *A B*.

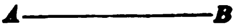


FIG. 1.

348. A **curved line** changes its direction at every point. Curved lines are designated by three or more letters, as the curved line *A B C*, Fig. 2.

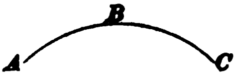


FIG. 2.

349. **Parallel lines** (Fig. 3) are those which are equally distant from each other at all points.

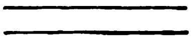


FIG. 3.

350. A line is **perpendicular** to another (see Fig. 4) when it meets that line so as not to incline towards it on either side.

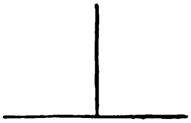


FIG. 4.

351. A **vertical line** is one that points towards the center of the earth, and is also known as a *plumb* line.

352. A **horizontal line** (see Fig. 5) is one that makes a right angle with any vertical line.

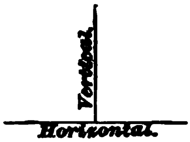


FIG. 5.

353. An **angle** is the opening between two lines which intersect or meet; the point of meeting is called the **vertex** of the angle. Angles are distinguished by naming the vertex and a point on each line. Thus, in Fig. 6, the angle formed by the lines *A B* and *C B* is called the angle *A B C*, or the angle *C B A*; the letter at the vertex is always placed in the middle. When an angle stands alone so that it can not be mistaken for any other angle, only the vertex letter need be used. Thus, the angle referred to might be designated simply as the angle *B*.

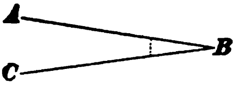


FIG. 6.

354. If one straight line meets another straight line at a point between its ends, as in Fig. 7, two angles, *A B C* and *A B D*, are formed, which are called **adjacent angles.**

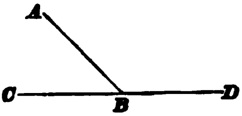


FIG. 7.

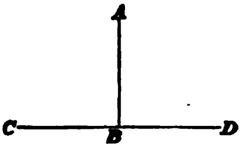


FIG. 8.

355. When these adjacent angles, *A B C* and *A B D*, are equal, as in Fig. 8, they are called **right angles.**

356. An **acute angle** is less than a right angle. *A B C*, Fig. 9, is an acute angle.

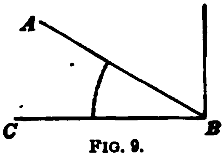


FIG. 9.

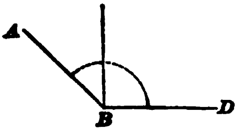


FIG. 10.

357. An **obtuse angle** is greater than a right angle. *A B D* (Fig. 10) is an obtuse angle.

358. A **circle** (see Fig. 11) is a figure bounded by a curved line, called the **circumference,** every point of which is equally distant from a point within, called the **center.**

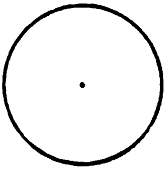


FIG. 11.

359. An **arc** of a circle is any part of its circumference; thus *a e b*, Fig. 12, is an arc of the circle.

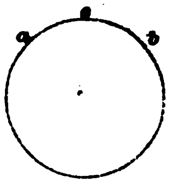


FIG. 12.

The circumference of every circle is considered to be divided into 360 equal parts, or arcs, called **degrees**; every degree is subdivided into 60 equal parts, called **minutes**, and every minute is again divided into 60 equal parts, called **seconds**.

Since 1 degree is $\frac{1}{360}$ of any circumference, it follows that the length of a degree will be different in circles of different sizes, but the proportion of the length of an arc of one degree to the whole circumference will always be the same, viz., $\frac{1}{360}$ of the circumference.

Degrees, minutes, and seconds are denoted by the symbols °, ′, ″. Thus, the expression 37° 14′ 44″ is read 37 degrees, 14 minutes, and 44 seconds.

360. The arcs of circles are used to measure angles. An angle having its vertex at the center of a circle is measured by the arc included between its sides; thus, in Fig. 13, the arc *F B* measures the angle *F O B*. If the arc *F B* contains 20°, or $\frac{20}{360}$ of the circumference, the angle *F O B* would be an angle of 20°; if it contained 20° 14′ 18″, it would be an angle of 20° 14′ 18″, etc.

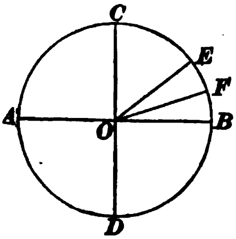


FIG. 13.

In the figure, if the line *C D* be drawn perpendicular to *A B*, the adjacent angles will be equal, and the circle will be divided into four equal angles, each of which will be a right angle. A right angle, therefore, is an angle of $\frac{360^\circ}{4}$, or 90°; two right angles are measured by 180°, or half the circumference, and four right angles by the whole circumference, or 360°. One-half of a right angle, as *E O B*, is an angle of 45°. An *acute* angle may now be defined as an angle of *less* than 90°, and an *obtuse* angle as one of *more* than 90°. These values are important, and should be remembered.

361. From the foregoing it will be evident that if a number of straight lines on the same side of a given straight line meet at the same point, the sum of all the angles formed is equal to two right angles, or 180°. Thus, in Fig. 14, angles $COB + DOC + EOD + FOE + AOF = 2$ right angles, or 180°.

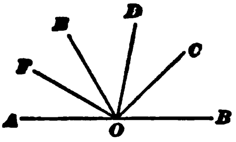


FIG. 14.

362. Also, if through a given point any number of straight lines be drawn, the sum of all the angles formed about the points of intersection equals four right angles, or 360°. Thus, in Fig. 15, angles $HOF + FOC + COA + AOG + GOE + EOD + DOB + BOH =$ four right angles, or 360°.

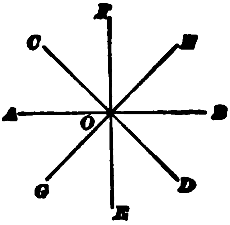


FIG. 15.

EXAMPLE.—In a fly-wheel, with 12 arms, how many degrees are there in the angle included between the center lines of any two arms, the arms being spaced equally?

SOLUTION.—Since there are 12 arms, there are 12 angles, which together equal 360°. Hence, one angle equals $\frac{1}{12}$ of 360°, or $\frac{360°}{12} = 30°$. Ans.

EXAMPLES FOR PRACTICE.

1. How many seconds in 32° 14′ 6″? Ans. 116,046 sec.

2. How many degrees, minutes, and seconds do 38,582 seconds amount to? Ans. 10° 43′ 2″.

3. How many right angles are there in an angle of 170°? Ans. $1\frac{8}{9}$ right angles.

4. In a pulley with five arms, what part of a right angle is included between the center lines of any two arms? Ans. $\frac{4}{5}$ of a right angle.

5. If one straight line meets another so as to form an angle of 20° 10′, what does its adjacent angle equal? Ans. 159° 50′.

6. If a number of straight lines meet a given straight line at a given point, all being on the same side of the given line, so as to form six equal angles, how many degrees are there in each angle? Ans. 30°.

QUADRILATERALS.

363. A **plane figure** is any part of a plane or flat surface bounded by straight or curved lines.

364. A **quadrilateral** is a plane figure bounded by four straight lines.

365. A **parallelogram** is a quadrilateral whose opposite sides are parallel.

There are four kinds of parallelograms: the **square,** the **rectangle,** the **rhombus,** and the **rhomboid.**

366. A **rectangle** (Fig. 16) is a parallelogram whose angles are all right angles.

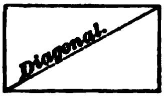


FIG. 16.

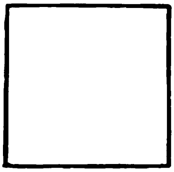


FIG. 17.

367. A **square** (Fig. 17) is a rectangle whose sides are all of the same length.

368. A **rhomboid** (Fig. 18) is a parallelogram whose opposite sides are equal and parallel, and whose angles are not right angles.

FIG. 18.

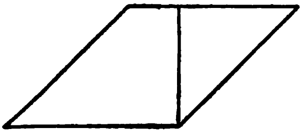


FIG. 19.

369. A **rhombus** (Fig. 19) is a parallelogram having equal sides, and whose angles are not right angles.

370. A **trapezoid** (Fig. 20) is a quadrilateral which has only two of its sides parallel.

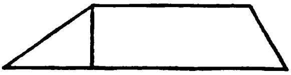


FIG. 20.

371. The **altitude** of a parallelogram or a trapezoid is the perpendicular distance between the parallel lines, as shown by the dotted lines in Figs. 18, 19, and 20.

372. The **base** of *any* plane figure is the side on which it is supposed to stand.

373. The **area** of a surface is expressed by the number of unit squares it will contain.

374. A **unit square** is the square having a unit for its side. For example, if the unit is 1 inch, the unit square is the square each of whose sides measures 1 inch in length, and the area of a surface would be expressed by the number of square inches it would contain. If the unit were 1 foot, the unit square would measure 1 foot on each side, and the area of the given surface would be the number of square feet it would contain, etc.

The square that measures one inch on a side is called a **square inch,** and the one that measures one foot on a side is called a **square foot.** Square inch and square foot are abbreviated to sq. in. and sq. ft.

375. To find the area of any parallelogram:

Rule 44.—*Multiply the base by the altitude.*

NOTE.—Before multiplying, the base and altitude must be reduced to the same kind of units; that is, if the base should be given in feet and the altitude in inches, they could not be multiplied together until either the altitude had been reduced to feet, or the base to inches. This principle holds throughout the subject of mensuration.

EXAMPLE.—The sides of a square piece of sheet iron are each $10\frac{1}{4}$ inches long. How many square inches does it contain?

SOLUTION.—$10\frac{1}{4}$ inches $= 10.25$ inches when reduced to a decimal. The base and altitude are each 10.25 inches. Multiplying them together, $10.25 \times 10.25 = 105.06+$ sq. in. Ans.

EXAMPLE.—What is the area in square rods of a piece of land in the shape of a rhomboid, one side of which is 8 rods long, and whose length, measured on a line perpendicular to this side, is 200 feet?

SOLUTION.—The base is 8 rods and the altitude 200 feet. As the answer is to be in rods, the 200 feet should be reduced to rods. Reducing $200 \div 16\frac{1}{2} = 200 \div \frac{33}{2} = 12.12$ rods. Hence, area $= 8 \times 12.12 = 96.96$ sq. rd. Ans.

376. To find the area of a trapezoid:

Rule 45.—*Multiply one-half the sum of the parallel sides by the altitude.*

EXAMPLE.—A board 14 feet long is 20 inches wide at one end and 16 inches wide at the other. If the ends are parallel, how many square feet does the board contain?

Solution.—One-half the sum of the parallel sides $= \frac{20 + 16}{2} = 18$ inches $= 1\frac{1}{2}$ feet. The length of the board corresponds to the altitude of a trapezoid. Hence, $14 \times 1\frac{1}{2} = 21$ sq. ft. Ans.

377. Having given the area of a parallelogram and one dimension, to find the other dimension:

Rule 46.—*Divide the area by the given dimension.*

Example.—What is the width of a parallelogram whose area is 212 square feet and whose length is $26\frac{1}{2}$ feet?

Solution.—$212 \div 26\frac{1}{2} = 212 \div \frac{53}{2} = 8$ feet. Ans.

The following examples illustrate a few special cases:

Example.—An engine room is 22 feet by 32 feet. The engine-bed occupies a space of 3 feet by 12 feet; the fly-wheel pit, a space of 2 feet by 6 feet, and the outer bearing, a space of 2 feet by 4 feet. How many square feet of flooring will be required for the room?

Solution.—Area of engine-bed $= 3 \times 12 = 36$ sq. ft.
Area of fly-wheel pit $= 2 \times 6 = 12$ sq. ft.
Area of outer bearing $= 2 \times 4 = 8$ sq. ft.
Total, 56 sq. ft.
Area of engine room $= 22 \times 32 = 704$ sq. ft.
$704 - 56 = 648$ square feet of flooring required. Ans.

Example.—How many square yards of plaster will it take to cover the sides and ceiling of a room 16×20 feet and 11 feet high, having four windows, each 7×4 feet, and three doors, each 9×4 feet over all, the baseboard coming 6 inches above the floor?

Solution.—
Area of ceiling $= 16 \times 20 = 320$ sq. ft.
Area of end walls $= 2(16 \times 11) = 352$ sq. ft.
Area of side walls $= 2(20 \times 11) = 440$ sq. ft.
Total area $= 1{,}112$ sq. ft.

From the above must be deducted:
Windows $= 4(7 \times 4) = 112$ sq. ft.
Doors $= 3(9 \times 4) = 108$ sq. ft.

Baseboard less the width of three doors $= (72 - 12) \times \frac{6}{12} = 30$ sq. ft.

Total number of feet to be deducted $= 112 + 108 + 30 = 250$ sq. ft.

Hence, number of square feet to be plastered $= 1{,}112 - 250 = 862$ sq. ft., or $95\frac{7}{9}$ sq. yd. Ans.

Example.—How many acres are contained in a rectangular tract of land 800 rods long and 520 rods wide?

Solution.—$800 \times 520 = 416{,}000$ sq. rd. Since there are 160 square rods in one acre, the number of acres $= 416{,}000 \div 160 = 2{,}600$ acres. Ans.

EXAMPLES FOR PRACTICE.

1. What is the area in square feet of a rhombus whose base is 84 inches, and whose altitude is 3 feet? Ans. 21 sq. ft.

2. A flat roof, 46 feet by 80 feet in size, is covered by tin roofing weighing one-half pound per square foot; what is the total weight of the roof? Ans. 1,840 lb.

3. One side of a room measures 16 ft. If the floor contains 240 square feet, what is the length of the other side? Ans. 15 ft.

4. How many square feet in a board 12 feet long, 18 inches wide at one end, and 12 inches wide at the other end? Ans. 15 sq. ft.

5. How much would it cost to lay a sidewalk a mile long and 8 feet 6 inches wide, at the rate of 20 cents per square foot? How much at the rate of $1.80 per square yard? Ans. $8,976 in each case.

6. How many square yards of plastering will be required for the ceiling and walls of a room 10 × 15 feet and 9 feet high; the room contains one door 3½ × 7 feet, three windows 3½ × 6 feet, and a baseboard 8 inches high? Ans. 53.5 sq. yd.

THE TRIANGLE.

378. A **triangle** is a plane figure having three sides.

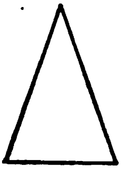


FIG. 21.

379. An **isosceles** triangle is one having two of its sides equal; see Fig. 21.

380. An **equilateral** triangle (Fig. 22) is one having all of its sides of the same length.

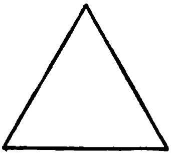


FIG. 22.

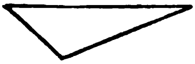


FIG. 23.

381. A **scalene** triangle (Fig. 23) is one having no two of its sides equal.

382. A **right-angled** triangle (Fig. 24) is any triangle having one right angle. The side opposite the right angle is called the **hypotenuse.**

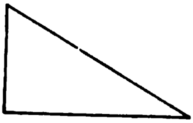


FIG. 24.

In any triangle the sum of the three angles equals two right angles, or 180°. Thus, in Fig. 25, the sum of the angles A, B, and C equals two right angles, or 180°. Hence, if any two angles of a triangle are given and it is required to find the third angle:

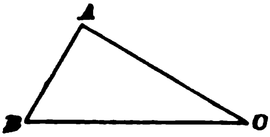


FIG. 25.

Rule 47.—*Add together the two given angles, and subtract their sum from 180°; the result will be the third angle.*

Example.—If two angles of a triangle = 48° 16′ and 47° 50′ respectively, what does the third angle equal?

Solution.—First reduce 48° 16′ and 47° 50′ to minutes, for convenience in adding and subtracting the angles. $48° = 48 \times 60 = 2{,}880'$; $2{,}880' + 16' = 2{,}896'$; hence, $48°\ 16' = 2{,}896'$. In like manner, $47°\ 50' = 47 \times 60' = 2{,}820' + 50' = 2{,}870'$. Adding the two angles together, and subtracting from 180° reduced to minutes, $2{,}896' + 2{,}870' = 5{,}766'$; $180° = 180 \times 60 = 10{,}800'$; $10{,}800 - 5{,}766 = 5{,}034'$. Reducing this last number to degrees and minutes, $\frac{5{,}034}{60} = 83\frac{54}{60}° = 83°\ 54'$. Hence, the third angle in the triangle = 83° 54′. Ans.

383. In any right-angled triangle there can be but one right angle, and since the sum of all the angles is two right angles, it is evident that the sum of the two acute angles must equal one right angle, or 90°. Therefore, if in any right-angled triangle one acute angle is known, to find the other acute angle:

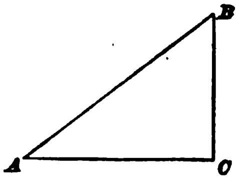


Fig. 26.

Rule 48.—*Subtract the known acute angle from 90°; the result will be the other acute angle.*

Example.—If one acute angle, as A, of the right-angled triangle $A\,B\,C$, Fig. 26, equals 30°, what does the angle B equal?

Solution.—$90° - 30° = 60°$. Ans.

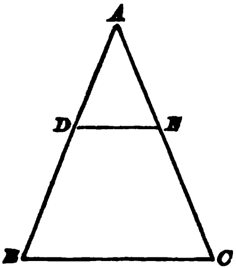


Fig. 27.

384. If a straight line be drawn through two sides of a triangle, parallel to the third side, a second triangle will be formed whose sides will be proportional to the corresponding sides of the first triangle. Thus, in the triangle $A\,B\,C$, Fig. 27, if the line $D\,E$ be drawn parallel to the side $B\,C$, the triangle $A\,D\,E$ will be formed and we shall have

(1) Side $A\,D$: side $D\,E$ = side $A\,B$: side $B\,C$; and,

(2) Side $A\,E$: side $D\,E$ = side $A\,C$: side $B\,C$; also,

(3) Side $A\,D$: side $A\,E$ = side $A\,B$: side $A\,C$.

Example.—In Fig. 27, if $AB = 24$, $BC = 18$, and $DE = 8$, what does AD equal?

Solution.—Writing these values for the sides in (1),

$$AD : 8 = 24 : 18;\ \text{whence}\ AD = \frac{24 \times 8}{18} = 10\tfrac{2}{3}.\quad \text{Ans.}$$

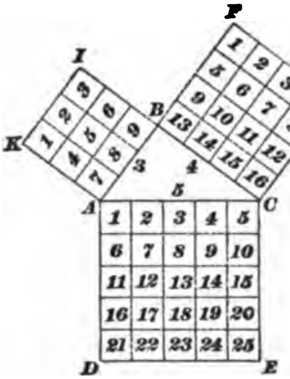


Fig. 28.

385. In any right-angled triangle, the square described on the hypotenuse is equal to the sum of the squares described upon the other two sides. If ABC, Fig. 28, is a right-angled triangle, right-angled at B, then the square described upon the hypotenuse AC is equal to the sum of the squares described upon the sides AB and BC. Hence, having given the two sides forming the right angle in a right-angled triangle, to find the hypotenuse:

Rule 49.—*Square each of the sides forming the right angle; add the squares together, and take the square root of the sum.*

Example.—If $AB = 3$ inches and $BC = 4$ inches, what is the length of the hypotenuse AC?

Solution.—Squaring each of the given sides, $3^2 = 9$ and $4^2 = 16$. Taking the square root of the sum of 9 and 16, the hypotenuse $= \sqrt{9 + 16} = \sqrt{25} = 5$ inches. Ans.

If the hypotenuse and one side are given, the other side can be found as follows:

Rule 50.—*Subtract the square of the given side from the square of the hypotenuse, and extract the square root of the remainder.*

Example.—The side given is 3 inches, the hypotenuse is 5 inches; what is the length of the other side?

Solution.—$3^2 = 9$; $5^2 = 25$. $25 - 9 = 16$, and the $\sqrt{16} = 4$ inches. Ans.

EXAMPLE.—If from a church steeple which is 150 feet high, a rope is to be attached to the top, and to a stake in the ground, which is 85 feet from the center of the base (the ground being supposed to be level), what must be the length of the rope?

SOLUTION.—In Fig. 29, $A\ B$ represents the steeple, 150 feet high; C a stake 85 feet from the foot of the steeple, and $A\ C$ the rope. Here we have a right-angled triangle, right-angled at B, and $A\ C$ is the hypotenuse. The square of $A\ B = 150^2 = 22,500$; of $C\ B$, $85^2 = 7,225$. $22,500 + 7,225 = 29,725$; $\sqrt{29,725} = 172.4$ feet, nearly. Ans.

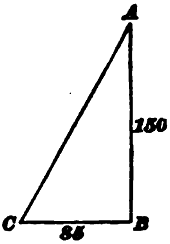


FIG. 29.

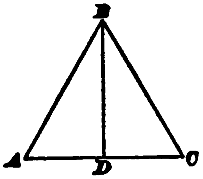


FIG. 30.

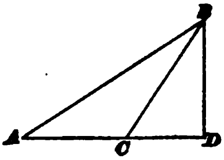


FIG. 31.

386. The **altitude** of any triangle is a line, as $B\ D$, drawn from the vertex B of the angle opposite the base $A\ C$, perpendicular to the base, as in Fig. 30, or to the base extended, as in Fig. 31.

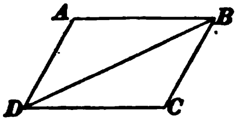


FIG. 32.

If in any parallelogram a straight line, called the **diagonal**, be drawn, connecting two opposite corners it will divide the parallelogram into two equal triangles, as $A\ D\ B$ and $D\ B\ C$ in Fig. 32. The area of each triangle will equal one-half the area of the parallelogram, or one-half the product of the base and the altitude. Hence, to find the area of any triangle:

Rule 51.—*Multiply the base by the altitude, and divide the product by 2.*

EXAMPLE.—What is the area in square feet of a triangle whose base is 18 feet, and whose altitude is 7 feet 9 inches?

SOLUTION.—7 feet 9 inches $= 7\frac{3}{4}$ feet $= \frac{31}{4}$ feet. $18 \times \frac{31}{4} = 139\frac{1}{2}$, and one-half of $139\frac{1}{2} = 69\frac{3}{4}$ square feet. Ans.

To find the altitude or base of a triangle, having given the area and the base or altitude:

Rule 52.—*Multiply the area by 2, and divide by the given dimension.*

EXAMPLE.—What must be the height of a triangular piece of sheet metal to contain 100 square inches, if the base is 10 inches long?

SOLUTION.—$100 \times 2 = 200$; $200 \div 10 = 20$ inches. Ans.

EXAMPLES FOR PRACTICE.

1. What is the area of a triangle whose base is 18 feet long, and whose altitude is 10 feet 6 inches? Ans. 94.5 sq. ft.

2. Two angles of a scalene triangle together equal 100° 4′. What is the size of the third angle? Ans. 79° 56′.

3. One angle of a right-angled triangle equals 20° 10′ 5″. What is the size of the other acute angle? Ans. 69° 49′ 55″.

4. A ladder 65 feet long reaches to the top of a wall when its foot is 25 feet from the wall. How high is the wall? Ans. 60 ft.

5. Draw a triangle, and through two of its sides draw a line parallel to the base. Letter the different lines, and then, without referring to the text, write out the proportions existing between the sides of the two triangles.

6. A triangular piece of sheet metal weighs 24 pounds. If the base of the triangle is 4 feet and its height 6 feet, how much does the metal weigh per square foot? Ans. 2 lb.

7. The area of a triangle is 16 square inches. If the altitude is 4 inches, what does the base measure? Ans. 8 in.

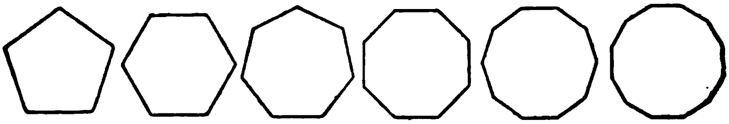
8. Two sides of a right-angled triangle are 92 feet and 69 feet long. How long is the hypotenuse? Ans. 115 ft.

POLYGONS.

387. A **polygon** is a plane figure bounded by straight lines. The term is usually applied to a figure having more than four sides. The bounding lines are called the **sides,** and the sum of the lengths of all the sides is called the **perimeter** of the polygon.

388. A **regular polygon** is one in which all of the sides and all of the angles are equal.

389. A polygon of five sides is called a **pentagon;** one with six sides a **hexagon;** one of seven sides a **heptagon,** etc. Regular polygons having from five to twelve sides are shown in Fig. 33. In any polygon, the sum of all



Pentagon. Hexagon. Heptagon. Octagon. Decagon. Dodecagon.

FIG. 33.

the interior angles, as $A + B + C + D + E$, Fig. 34, equals 180° multiplied by a number which is two less than the number of sides in the polygon. Hence, to find the size of any one of the interior angles of a *regular* polygon:

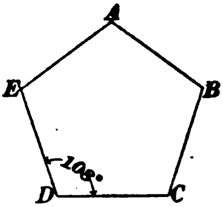


FIG. 34.

Rule 53.—*Multiply 180° by the number of sides less two, and divide the result by the number of sides; the quotient will be the number of degrees in each interior angle.*

EXAMPLE.—If Fig. 34 is a regular pentagon, how many degrees are there in each interior angle?

SOLUTION.—In a pentagon there are five sides; hence, $5 - 2 = 3$ and $180 \times 3 = 540$; $540 \div 5 = 108°$ in each angle. Ans.

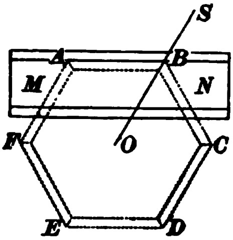


FIG. 35.

EXAMPLE.—It is desired to make a miter-box in which to cut a strip of molding to fit around a column having the shape of a regular hexagon. At what angle should the saw run across the miter-box?

SOLUTION.—In Fig. 35, let $A B$, $B C$, $C D$, etc., represent the pieces of molding as they will fit around the column. First find the size of one of the equal angles of the polygon by the above rule. Number of sides $= 6$; $6 - 2 = 4$; hence, $180 \times 4 = 720$, and $720 \div 6 = 120°$ in each angle. Now, let $M N$ represent the miter-box, and $O S$ the direction in which the saw should run; then, $A B O$ is the angle made by the saw with the side of the miter-box; but, as the polygon is a regular one, this angle is one-half the interior angle $A B C$, which we have found to be 120°. Hence, the saw should run at an angle of $\frac{120}{2} = 60°$ with the side of the miter-box.

390. The area of any regular polygon may be found by drawing lines from the center to each angle, and computing the area of each triangle thus formed. Hence, to find the area of any regular polygon:

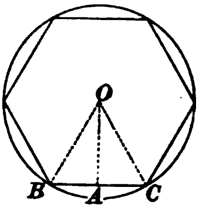


FIG. 36.

Rule 54.—*Multiply the length of a side by half the distance from the side to the center, and that product by the number of sides. The last product will be the area of the figure.*

Example.—In Fig. 86 the side *B C* of the regular hexagon is 12 inches and the distance *A O* is 10.4 inches; required, the area of the polygon.

Solution.—$10.4 + 2 = 5.2$; $12 \times 5.2 \times 6 = 374.4$ sq. in. Ans.

391. To obtain the area of any irregular polygon, draw diagonals dividing the polygon into triangles and quadrilaterals, and compute the areas of these separately; their sum will be the area of the figure.

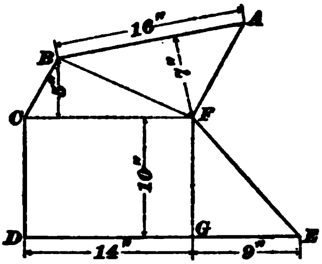


Fig. 87.

Example.—It is required to find the area of the polygon *A B C D E F*, Fig. 87.

Solution.—Draw the diagonals *B F* and *C F* and the line *F G* perpendicular to *D E*, dividing the figure into the triangles *A B F*, *B C F*, and *F G E*, and the rectangle *F C D G*. Let it be supposed that the altitudes of the figures and the lengths of the sides *A B*, *D G*, and *G E* are as indicated in the polygon above. Then,

$$\text{Area } ABF = \frac{16 \times 7}{2} = 56 \text{ sq. in.}$$

$$\text{Area } BCF = \frac{14 \times 5}{2} = 35 \text{ sq. in.}$$

$$\text{Area } FCDG = 14 \times 10 = 140 \text{ sq. in.}$$

$$\text{Area } FGE = \frac{9 \times 10}{2} = 45 \text{ sq. in.}$$

Total area $= 56 + 35 + 140 + 45 = 276$ sq. in. Ans.

EXAMPLES FOR PRACTICE.

1. How many degrees are there in one of the angles of a regular octagon? Ans. 135°.

2. Find the area of the polygon *A B C D E F* (see Fig. 87), supposing each of the given dimensions to be increased to 1½ times the length given in the figure. Ans. 621 sq. in.

3. What is the area of a regular heptagon, whose sides are 4 inches long, the distance from one side to the center being 4.15 inches? Ans. 58.1 sq. in.

4. At what angle should the saw run in a miter-box to cut strips to fit around the edge of a table top, made in the shape of a regular pentagon? Ans. 54°.

THE CIRCLE.

392. A **circle** (Fig. 38) is a figure bounded by a curved line, called the **circumference,** every point of which is equally distant from a point within, called the **center.**

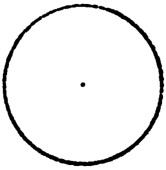


FIG. 38.

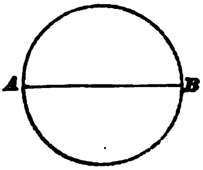


FIG. 39.

393. The **diameter** of a circle is a straight line passing through the center and terminated at both ends by the circumference; thus, *A B* (Fig. 39) is a diameter of the circle.

394. The **radius** of a circle, *A O* (Fig. 40), is a straight line drawn from the center *O* to the circumference. It is equal in length to one-half the diameter. The plural of radius is **radii,** and all radii of a circle are equal.

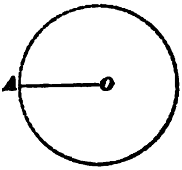


FIG. 40.

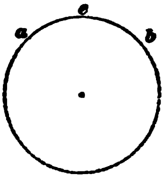


FIG. 41.

395. An **arc** of a circle (see *a e b*, Fig. 41) is any part of its circumference.

396. A **chord** is a straight line joining any two points in a circumference; or it is a straight line joining the extremities of an arc; thus, the straight line *a b*, Fig. 42, is a chord of the circle whose corresponding arc is *a e b*.

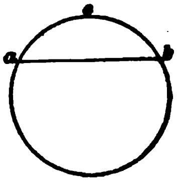


FIG. 42.

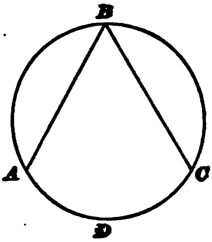


FIG. 43.

397. An **inscribed angle** is one whose vertex lies on the circumference of a circle, and whose sides are chords. It is measured by one-half the intercepted arc. Thus, in Fig. 43, *A B C* is an inscribed angle, and it is measured by one-half the arc *A D C*.

Example.—If in Fig. 43, the arc $A\ D\ C = \frac{2}{5}$ of the circumference, what is the measurement of the inscribed angle $A\ B\ C$?

Solution.—Since the angle is an inscribed angle, it is measured by one-half the intercepted arc, or $\frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$ of the circumference. The whole circumference $= 360°$; hence, $360° \times \frac{1}{5} = 72°$; therefore, angle $A\ B\ C$ is an angle of 72°.

398. If a circle is divided into halves, each half is called a **semicircle,** and each half circumference is called a **semi-circumference.**

Any angle inscribed in a semicircle is a right angle, since it is measured by one-half a semi-circumference, or $180° \div 2 = 90°$. Thus, the angles $A\ D\ C$ and $A\ B\ C$, Fig. 44, are right angles, since they are inscribed in a semicircle.

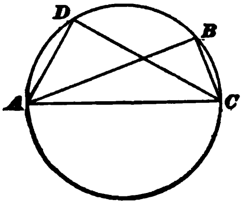


Fig. 44.

399. An **inscribed** polygon is one whose vertexes lie on the circumference of a circle, and whose sides are chords, as $A\ B\ C\ D\ E$, Fig. 45.

The sides of an inscribed regular hexagon have the same length as the radius of the circle.

If, in any circle, a radius be drawn perpendicular to any chord, it bisects (cuts in halves) the chord. Thus, if the radius $O\ C$, Fig. 46, is perpendicular to the chord $A\ B$, $A\ D = D\ B$.

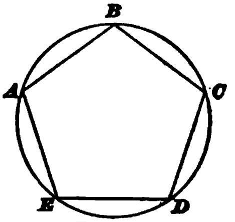


Fig. 45.

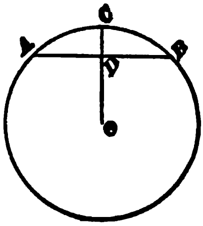


Fig. 46.

Example.—If a regular pentagon be inscribed in a circle, and a radius is drawn perpendicular to one of the sides, what are the lengths of the two parts of the side, the perimeter of the pentagon being 27 inches?

Solution.—A pentagon has five sides, and since it is a regular pentagon, all the sides are of equal lengths; the perimeter of a pentagon, which equals the distance around it, or equals the sum of all the sides, is 27 inches. Therefore, the length of one side $= 27 \div 5 = 5\frac{2}{5}$ inches. Since the pentagon is an inscribed pentagon, its sides are chords, and as a radius perpendicular to a chord bisects it, we have $5\frac{2}{5} \div 2 = 2\frac{7}{10}$ inches, which equals the length of each of the parts of the side, cut by a radius perpendicular to it.

400. If, from any point on the circumference of a circle, a perpendicular be let fall upon a given diameter, it will divide the diameter into two parts, one of which will be in the same ratio to the perpendicular as the perpendicular is to the other. That is, the perpendicular will be a *mean proportional* between the two parts.

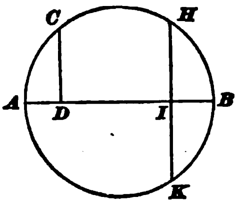


FIG. 47.

If $A\,B$, Fig. 47, is the given diameter, and C any point on the circumference, then $A\,D : C\,D :: C\,D : D\,B$, $C\,D$ being a mean proportional between $A\,D$ and $D\,B$.

Example.—If $H\,K = 30$ feet, and $I\,B = 8$ feet, what is the diameter of the circle, $H\,K$ being perpendicular to $A\,B$?

Solution.—30 feet ÷ 2 feet = 15 feet = $I\,H$. And $B\,I : I\,H :: I\,H : I\,A$, or $8 : 15 :: 15 : I\,A$.

Therefore, $I\,A = \frac{15^2}{8} = \frac{225}{8} = 28\frac{1}{8}$ feet, and $I\,A + I\,B = 28\frac{1}{8} + 8 = 36\frac{1}{8}$ feet $= A\,B$ the diameter of the circle. Ans.

401. When the diameter of a circle and the lengths of the two parts into which it is divided are given, the length of the perpendicular may be found by multiplying the lengths of the two parts together and extracting the square root of the product.

Example.—In Fig. 47, the diameter of the circle $A\,B$ is $36\frac{1}{8}$ feet, and the distance $B\,I$ is 8 feet; what is the length of the line $H\,K$?

Solution.—As the diameter of the circle is $36\frac{1}{8}$ feet, and as $B\,I$ is 8 feet, $I\,A$ is equal to $36\frac{1}{8} - 8 = 28\frac{1}{8}$ feet. The two parts, therefore, are 8 and $28\frac{1}{8}$ feet, and their product $= 8 \times 28\frac{1}{8} = 8 \times \frac{225}{8} = 225$; the square root of their product $= \sqrt{225} = 15$ feet, and as $H\,K = I\,H + I\,K$, or $2\,I\,H$, $H\,K = 15 \times 2 = 30$ feet. Ans.

402. To find the circumference of a circle, the diameter being given:

Rule 55.—*Multiply the diameter by 3.1416.*

Example.—What is the circumference of a circle whose diameter is 15 inches?

Solution.—$15 \times 3.1416 = 47.124$ inches. Ans.

403. To find the diameter of a circle, the circumference being given:

Rule 56.—*Divide the circumference by 3.1416.*

Example.—What is the diameter of a circle whose circumference is 65.973 inches?

Solution.—65.973 ÷ 3.1416 = 21 inches. Ans.

404. To find the length of an arc of a circle:

Rule 57.—*Multiply the length of the circumference of the circle of which the arc is a part by the number of degrees in the arc, and divide by 360.*

Example.—What is the length of an arc of 24°, the radius of the arc being 18 in.?

Solution.—18 × 2 = 36 in. = the diameter of the circle. 36 × 3.1416 = 113.1 in., the circumference of the circle of which the arc is a part.

$113.1 \times \frac{24}{360} = 7.54$ in., or the length of the arc. Ans.

405. To find the area of a circle:

Rule 58.—*Square the diameter, and multiply by .7854.*

Example.—What is the area of a circle whose diameter is 15 inches?

Solution.—$15^2 = 225$; and 225 × .7854 = 176.72 sq. in. Ans.

406. Given the area of a circle, to find its diameter:

Rule 59.—*Divide the area by .7854, and extract the square root of the quotient.*

Example.—The area of a circle = 17,671.5 square inches. What is its diameter in feet?

Solution.—$\sqrt{\frac{17{,}671.5}{.7854}} = 150$ inches.

$\frac{150}{12} = 12\frac{1}{2}$ feet, or the diameter. Ans.

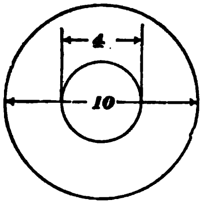


Fig. 48.

Example.—What is the area of a flat circular ring, Fig. 48, whose outside diameter is 10 inches, and whose inside diameter is 4 inches?

Solution.—The area of the large circle = $10^2 \times .7854 = 78.54$ square inches; the area of the small circle = $4^2 \times .7854 = 12.57$ square inches. The area of the ring is the difference between these areas, or 78.54 − 12.57 = 65.97 square inches. Ans.

407. To find the area of a sector (a **sector** of a circle is the area included between two radii and the circumference, as, for example, the area *C O E*, Fig. 13):

Rule 60.—*Divide the number of degrees in the arc of the sector by 360. Multiply the result by the area of the circle of which the sector is a part.*

EXAMPLE.—The number of degrees in the angle formed by drawing radii from the center of a circle to the extremities of the arc of the circle is 75°. The diameter of the circle is 12 inches; what is the area of the sector?

SOLUTION.—$\frac{75}{360} = \frac{5}{24}$; and $12^2 \times .7854 = 113.1$ square inches.

$113.1 \times \frac{5}{24} = 23.56$ square inches, the area. Ans.

408. To find the area of a segment of a circle (a **segment** of a circle is the area included between a chord and its arc; for example, the area $A\ B\ C$, Fig. 49):

Rule 61.—*Divide the diameter by the height of the segment; subtract .608 from the quotient, and extract the square root of the remainder. This result multiplied by 4 times the square of the height of the segment, and then divided by 3, will give the area, very nearly.*

The rule, expressed as a formula, is as follows, where $D =$ the diameter of the circle, and $h =$ the height of the segment, see Fig. 49:

$$\text{Area of } A\,B\,C\,A = \frac{4\,h^2}{3}\sqrt{\frac{D}{h} - .608}.$$

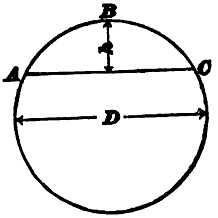


FIG. 49.

EXAMPLE.—What is the area of the segment of a circle whose diameter is 54 inches, the height of the segment being 20 inches?

SOLUTION.—Substituting in the formula,

$$\text{Area} = \frac{4 \times 20^2}{3}\sqrt{\frac{54}{20} - .608}. \quad \frac{4 \times 20^2}{3} = \frac{4 \times 400}{3} = \frac{1{,}600}{3}; \quad \sqrt{\frac{54}{20} - .608} = \sqrt{2.092} = 1.447; \quad \frac{1{,}600}{3} \times 1.447 = 771.7 \text{ sq. in.}$$

Ans.

EXAMPLES FOR PRACTICE.

1. An angle inscribed in a circle intercepts one-third of the circumference. How many degrees are there in the angle? Ans. 60°.

2. Suppose that in Fig. 47, the diameter $A\,B = 15$ feet, and the distance $B\,I = 3$ feet. What is the length of the line $H\,K$?

Ans. 12 ft.

3. The diameter of a fly-wheel is 18 feet. What is the distance around it to the nearest 16th of an inch? Ans. 56 ft. 6$\frac{9}{16}$ in.

4. A carriage wheel was observed to make 71$\frac{5}{8}$ turns while going 300 yards. What was its diameter? Ans. 4 ft., nearly.

5. What is the length of an arc of 64°, the radius of the arc being 30 inches? Ans. 33.51 in.

6. Find the area of a circle 2 feet 3 inches in diameter. Ans. 3.976 sq. ft.

7. What must be the diameter of a circle to contain 100 square inches? Ans. 11.28 in.

8. Compute the area of a segment, whose height is 11 inches, and the radius of whose arc is 21 inches. Ans. 289.04 sq. in.

9. Find the area of a flat circular ring whose outside diameter is 12 inches and whose inside diameter is 6 inches. Ans. 84.82 sq. in.

THE PRISM AND CYLINDER.

409. A **solid,** or body, has three dimensions: length, breadth, and thickness. The sides which enclose it are called the **faces,** and their intersections are called **edges.**

410. A **prism** is a solid whose ends are equal and parallel polygons, and whose sides are parallelograms. Prisms take their names from the form of their bases. Thus, a triangular prism is one having a triangle for its base; a hexagonal prism is one having a hexagon for its base, etc.

411. A **cylinder** is a body of uniform diameter whose ends are equal parallel circles.

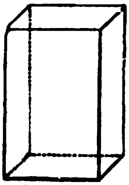


FIG. 50.

412. A **parallelopipedon** (Fig. 50) is a prism whose bases (ends) are parallelograms.

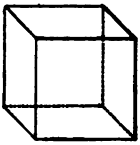


FIG. 51.

413. A **cube** (Fig. 51) is a prism whose faces and ends are squares. All the faces of a cube are equal.

In the case of plane figures, we have had to do with perimeters and areas. In the case of solids, we have to do with the areas of their outside surfaces, and with their contents or volumes.

414. The **entire surface** of any solid is the area of the whole outside of the solid, including the ends.

The **convex surface** of a solid is the same as the entire surface, except that the areas of the ends are not included.

415. A **unit of volume** is a cube each of whose edges is equal in length to the unit. The **volume** is expressed by the number of times it will contain a *unit of volume.*

Thus, if the unit of length is 1 inch, the unit of volume will be the cube whose edges each measure 1 inch, this cube being 1 *cubic inch;* and the number of cubic inches the solid contains will be its volume. If the unit of length is 1 foot, the unit of volume will be 1 *cubic foot*, etc. Cubic inch, cubic foot, and cubic yard are abbreviated to cu. in., cu. ft., and cu. yd., respectively.

Instead of the word *volume*, the expression **cubical contents** is sometimes used.

416. To find the area of the convex surface of a prism or cylinder:

Rule 62.—*Multiply the perimeter of the base by the altitude.*

EXAMPLE.—A block of marble is 24 inches long, and its ends are 9 inches square. What is the area of its convex surface ?

SOLUTION.—$9 \times 4 = 36 =$ the perimeter of the base; $36 \times 24 = 864$ sq. in., the convex area. Ans.

To find the entire area of the outside surface, add the areas of the two ends to the convex area. Thus, the area of the two ends $= 9 \times 9 \times 2 = 162$ sq. in.; $864 + 162 = 1{,}026$ sq. in. Ans.

EXAMPLE.—How many square feet of sheet iron will be required for a pipe 1½ feet in diameter and 10 feet long, neglecting the amount necessary for lapping ?

SOLUTION.—The problem is to find the convex surface of a cylinder 1½ feet in diameter and 10 feet long. The perimeter, or circumference, of the base $= 1\frac{1}{2} \times 3.1416 = 1.5 \times 3.1416 = 4.712$ feet. The convex surface $= 4.712 \times 10 = 47.12$ sq. ft. of metal. Ans.

417. To find the volume of a prism or a cylinder:

Rule 63.—*Multiply the area of the base by the altitude.*

EXAMPLE.—What is the weight of a length of wrought-iron shafting 16 feet long and 2 inches in diameter ? Wrought iron weighs .28 pound per cubic inch.

Solution.—The shaft is a cylinder 16 feet long. The area of one end, or the base, $= 2^2 \times .7854 = 3.1416$ sq. in. Since the weight of the iron is given per cubic inch, the contents of the shaft must be found in cubic inches. The length, 16 feet, reduced to inches $= 16 \times 12 = 192$ in.; $3.1416 \times 192 = 603.19$ cu. in. = the volume. The weight $= 603.19 \times .28 = 168.89$ lb. Ans.

Example.—Find the cubical contents of a hexagonal prism, Fig. 52, 12 inches long, each edge of the base being one inch long.

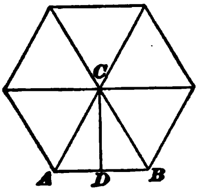


Fig. 52.

Solution.—In order to obtain the area of one end, the distance CD from the center C to one side must be found.

In the right-angled triangle CDA, side $AD = \frac{1}{2} AB$, or one-half inch, and since the polygon is a hexagon, side CA = distance AB, or one inch. Hence, CA being the hypotenuse, the length of side $CD = \sqrt{1^2 - (\frac{1}{2})^2} = \sqrt{1^2 - .5^2} = \sqrt{.75}$, or .866 inch. Area of triangle $ACB = \frac{1 \times .866}{2} = .433$ sq. in.; area of the whole polygon $= .433 \times 6 = 2.598$ sq. in. Hence, the contents of the prism $= 2.598 \times 12 = 31.176$ cu. in. Ans.

Example.—It is required to find the number of cubic feet of steam space in the boiler shown in Fig. 53. The boiler is 16 feet long between heads, 54 inches in diameter, and the mean water line MN is at a distance of 16 inches from the top of the boiler. The volume of the steam outlet casting may be neglected.

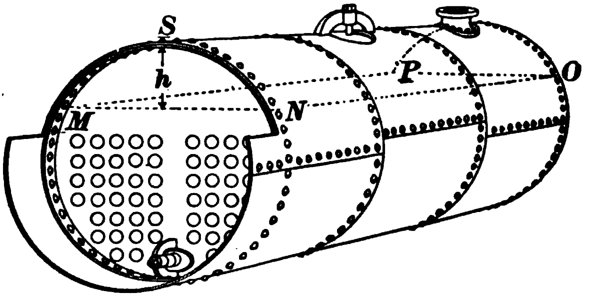


Fig. 53.

Solution.—The volume of the steam space, which is that space within the boiler above the surface $MNOP$ of the water, is found by the rule for finding the volume of a prism or cylinder, the area MNS being the base, and the length NO the altitude. First obtain the area of the segment MNS, whose height h is 16 inches, in square feet; then multiply the result by 16, the length of the boiler.

By the formula previously given, the area of the segment =

$$\frac{4h^2}{3}\sqrt{\frac{D}{h} - .608} = \frac{4 \times 16^2}{3}\sqrt{\frac{54}{16} - .608};$$

$$\frac{4 \times 16^2}{3} = 341.33; \sqrt{\frac{54}{16} - .608} = \sqrt{2.767} = 1.663.$$

Hence, the area = 341.33 × 1.663 = 567.63 sq. in. This, reduced to square feet, = 567.63 ÷ 144 = 3.942 sq. ft., and the volume, therefore, = 3.942 × 16 = 63.07 cu. ft. Ans.

In the above solution, the space occupied by the stays is not considered, for sake of simplicity. They are not shown in the figure.

EXAMPLE.—In the above boiler there are 60 tubes, 3¼ inches outside diameter. How many gallons of water will it take to fill the boiler up to the mean water level, there being 231 cubic inches in a gallon?

SOLUTION.—Find the volume in cubic inches of that part of the boiler below the surface of the water *M N O P*, since the contents of a gallon is given in cubic inches, and from it subtract the volume of the tubes in cubic inches.

This may be done by first finding the *total* area of one end of the boiler in square inches, from it subtracting the area of the segment *M N S*, and the areas of the ends of the tubes in square inches, and then by multiplying the result by the length of the boiler *in inches*.

Total area of one end = 54^2 × .7854 = 2,290.23 sq. in.

Area of segment *M N S*, as found in last example, = 567.63 sq. in.

Area of the end of one tube = 3.25^2 × .7854 = 8.2958 sq. in.

Area of the ends of the 60 tubes = 8.2958 × 60 = 497.75 sq. in.

Hence, the area to be subtracted = 567.63 + 497.75 = 1,065.38 sq. in. Subtracting, 2,290.23 − 1,065.38 = 1,224.85 sq. in. = net area.

The cubical contents = 1,224.85 × 16 × 12 = 235,171.2 cu. in. This, divided by 231, will give the number of gallons; whence, 235,171.2 ÷ 231 = 1,018.06 gallons of water. Ans.

EXAMPLES FOR PRACTICE.

1. Find the area in square inches of the convex surface of a bar of iron 4½ inches in diameter, and 8 feet 5 inches long. Ans. 1,348.53 sq. in.

2. Find the area of the entire surface of the above bar.
Ans. 1,376.9 sq. in.

3. What is the area of the entire surface of the hexagonal prism whose base is shown in Fig. 52? Ans. 77.196 sq. in.

4. A multitubular boiler has the following dimensions: diameter, 50 inches; length between heads, 15 feet; number of tubes, 56; outside diameter of tubes, 3 inches; distance of mean water line from top of boiler, 16 inches. (*a*) Compute the steam space in cubic feet. (*b*) Find the number of gallons of water required to fill the boiler up to the mean water line.

Ans. { (*a*) 56.4 cu. ft.
(*b*) 800 gallons.

THE PYRAMID AND CONE.

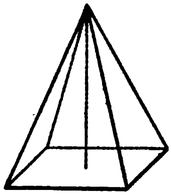


FIG. 54.

418. A **pyramid** (Fig. 54) is a solid whose base is a polygon, and whose sides are triangles uniting at a common point, called the **vertex.**

419. A **cone** (Fig. 55) is a solid whose base is a circle and whose convex surface tapers uniformly to a point called the **vertex.**

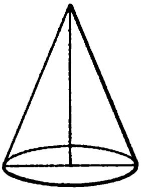


FIG. 55.

420. The **altitude** of a pyramid or cone is the perpendicular distance from the vertex to the base.

421. The **slant height** of a *pyramid* is a line drawn from the vertex perpendicular to one of the sides of the base. The slant height of a *cone* is any straight line drawn from the vertex to the circumference of the base.

422. To find the convex area of a pyramid or cone:

Rule 64.—*Multiply the perimeter of the base by one-half the slant height.*

EXAMPLE.—What is the convex area of a pentagonal pyramid, if one side of the base measures 6 inches, and the slant height = 14 inches?

SOLUTION.—The base of a pentagonal pyramid is a pentagon, and, consequently, has five sides.

$6 \times 5 = 30$ inches, or the perimeter of the base. $30 \times \frac{14}{2} = 210$ sq. in., or the convex area. Ans.

EXAMPLE.—What is the entire area of a right cone whose slant height is 17 inches, and whose base is 8 inches in diameter?

SOLUTION.—The perimeter of the base $= 8 \times 3.1416 = 25.1328$ in.

$$\text{Convex area} = 25.1328 \times \frac{17}{2} = 213.63 \text{ sq. in.}$$

$$\text{Area of base} = 8^2 \times .7854 = \underline{50.27} \text{ sq. in.}$$

$$\text{Entire area} = 263.90 \text{ sq. in.} \quad \textbf{Ans.}$$

423. To find the volume of a pyramid or cone:

Rule 65.—*Multiply the area of the base by one-third of the altitude.*

EXAMPLE.—What is the volume of a triangular pyramid, each edge of whose base measures 6 inches, and whose altitude is 8 inches?

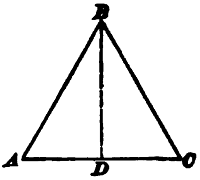


Fig. 56.

Solution.—Draw the base as shown in Fig. 56; it will be an equilateral triangle, all of whose sides are 6 inches long.

Draw a perpendicular BD from the vertex to the base; it will divide the base into two equal parts, since an equilateral triangle is also isosceles, and will be the altitude of the triangle. In order to obtain the area of the base, this altitude must be determined.

In the right-angled triangle BDA, the hypotenuse $BA = 6$ inches, and side $AD = 3$ inches, to find the other side,

$$BD = \sqrt{6^2 - 3^2} = 5.2 \text{ inches, nearly.}$$

Area of the base, or BAC, $= \frac{6 \times 5.2}{2} = 15.6$ sq. in. Hence, the volume $= 15.6 \times \frac{8}{3} = 41.6$ cu. in. Ans.

Example.—What is the volume of a cone whose altitude is 18 inches, and whose base is 14 inches in diameter?

Solution.—Area of the base $= 14^2 \times .7854 = 153.94$ sq. in. Hence, the volume $= 153.94 \times \frac{18}{3} = 923.64$ cu. in. Ans.

EXAMPLES FOR PRACTICE.

1. Find the convex surface of a square pyramid whose slant height is 28 inches, and one edge of whose base is 7½ inches long.
Ans. 420 sq. in.

2. What is the volume of a triangular pyramid, one edge of whose base measures 3 inches, and whose altitude is 4 inches?
Ans. 5.2 cu. in.

3. Find the volume of a cone whose altitude is 12 inches, and the circumference of whose base is 31.416 inches. Ans. 314.16 cu. in.

Note.—Find the diameter of the base and then its area.

THE FRUSTUM OF A PYRAMID OR CONE.

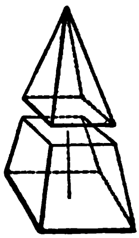


Fig. 57.

424. If a pyramid be cut by a plane, parallel to the base, so as to form two parts, as in Fig. 57, the lower part is called the **frustum** of the pyramid.

If a cone be cut in a similar manner, as in Fig. 58, the lower part is called the **frustum** of the cone.

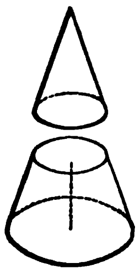


Fig. 58.

425. The upper end of the frustum of a **pyramid or** cone is called the **upper base,** and the lower end the **lower base.** The **altitude** of a frustum is the perpendicular distance between the bases.

426. To find the convex surface of a frustum of a pyramid or cone:

Rule 66.—*Multiply one-half the sum of the perimeters of the two bases by the slant height of the frustum.*

EXAMPLE.—Given, the frustum of a triangular pyramid, in which one side of the lower base measures 10 inches, one side of the upper base measures 6 inches, and whose slant height is 9 inches; find the area of the convex surface.

SOLUTION.—10 in. × 3 = 30 in., the perimeter of the lower base. 6 in. × 3 = 18 in., the perimeter of the upper base. $\frac{30 + 18}{2} = 24$ in., or one-half the sum of the perimeters of the two bases. 24 × 9 = 216 sq. in., the convex area. Ans.

EXAMPLE.—If the diameters of the two bases of a frustum of a cone are 12 inches and 8 inches, respectively, and the slant height is 12 inches, what is the entire area of the frustum?

SOLUTION.—$\frac{(12 \times 3.1416) + (8 \times 3.1416)}{2} \times 12 = 376.99$ sq. in., the area of the convex surface.

Area of the upper base = $8^2 \times .7854 = 50.27$ sq. in.

Area of the lower base = $12^2 \times .7854 = 113.1$ sq. in.

The entire area of the frustum = 376.99 + 50.27 + 113.1 = 540.36 sq. in. Ans.

427. To find the volume of the frustum of a pyramid or cone:

Rule 67.—*Add together the areas of the upper and lower bases, and the square root of the product of the two areas; multiply the sum by one-third of the altitude.*

EXAMPLE.—Given, a frustum of a square pyramid (one whose base is a square); each edge of the lower base is 12 inches, each edge of the upper base is 5 inches, and its altitude is 16 inches; what is its volume?

SOLUTION.—Area of upper base = 5 × 5 = 25 sq. in.; area of lower base = 12 × 12 = 144 sq. in.; the square root of the product of the two areas = $\sqrt{25 \times 144} = 60$. Adding these three results, and multiplying by one-third the altitude, 25 + 144 + 60 = 229; $229 \times \frac{16}{3} = 1{,}221\frac{1}{3}$ cu. in. = the volume. Ans.

Example.—How many gallons of water will a round tank hold, which is 4 feet in diameter at the top, 5 feet in diameter at the bottom, and 8 feet deep?

Solution.—There are 231 cubic inches in a gallon, and the volume of the tank should be found in cubic inches. The tank is in the shape of the frustum of a cone. The upper diameter $= 4 \times 12 = 48$ inches; the lower diameter $= 5 \times 12 = 60$ inches, and the depth $= 8 \times 12 = 96$ inches. Area of upper base $= 48^2 \times .7854 = 1{,}809.56$ sq. in.; area of lower base $= 60^2 \times .7854 = 2{,}827.44$ sq. in.; $\sqrt{1{,}809.56 \times 2{,}827.44} = 2{,}261.95$.

Whence, $1{,}809.56 + 2{,}827.44 + 2{,}261.95 = 6{,}898.95$; $6{,}898.95 \times \frac{96}{3} = 220{,}766.4$ cu. in. = contents. Now, since there are 231 cu. in. in one gallon, the tank will hold $220{,}766.4 \div 231 = 955.7$ gallons, nearly. Ans.

EXAMPLES FOR PRACTICE.

1. Find the convex surface of the frustum of a square pyramid, one edge of whose lower base is 15 inches long, one edge of whose upper base is 14 inches long, and whose slant height is one inch.
Ans. 58 sq. in.

2. Find the volume of the above frustum, supposing its altitude to be 3 inches. Ans. 631 cu. in.

3. Find the volume of the frustum of a cone whose altitude is 12 feet and the diameters of whose upper and lower bases are 8 and 10 feet, respectively. Ans. 766.55 cu. ft.

4. If a tank had the dimensions of example 3, how many gallons would it hold? Ans. 5,734.2 gallons, nearly.

THE SPHERE AND CYLINDRICAL RING.

428. A **sphere** (Fig. 59) is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within, called the center.

The word **ball,** or **globe,** is generally used instead of sphere.

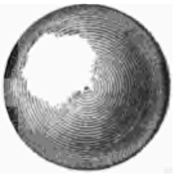


Fig. 59.

429. To find the area of the surface of a sphere:

Rule 68.—*Square the diameter and multiply the result by 3.1416.*

Example.—What is the area of the surface of a sphere whose diameter is 14 inches?

Solution.—Diameter squared $\times 3.1416 = 14^2 \times 3.1416 = 14 \times 14 \times 3.1416 = 615.75$ sq. in. Ans.

From this it will be seen that the surface of a sphere equals the circumference of a great circle multiplied by the diameter, a rule often used; a *great circle* of a sphere is the intersection of its surface with a plane passing through its center; for instance, the *great circle* of a sphere 6 in. diameter is a circle of 6 in. diameter. Any number of *great circles* could be described on a given sphere.

430. To find the volume of a sphere:

Rule 69.—*Cube the diameter and multiply the result by .5236.*

EXAMPLE.—What is the weight of a lead ball 12 inches in diameter, a cubic inch of lead weighing .41 pound?

SOLUTION.—Diameter cubed × .5236 = 12 × 12 × 12 × .5236 = 904.78 cu. in., or the volume of the ball. The weight, therefore, = 904.78 × .41 = 370.96 pounds. Ans.

431. To find the convex area of a cylindrical ring:

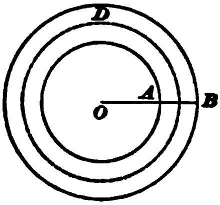


FIG. 60.

A **cylindrical ring** (Fig. 60) is a cylinder bent to a circle. The **altitude** of the cylinder before bending is the same as the length of the dotted center line *D*. The **base** will correspond to a cross-section on the line *A B* drawn from the center *O*. Hence, to find the convex area:

Rule 70.—*Multiply the circumference of an imaginary cross-section on the line A B, by the length of the center line D.*

EXAMPLE.—If the outside diameter of the ring is 12 inches, and the inside diameter is 8 inches, what is its convex area?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters $= \frac{12+8}{2} = 10$, and $10 \times 3.1416 = 31.416$ inches, the length of the center line.

The radius of the inner circle is 4 inches; of the outside circle, 6 inches; therefore, the diameter of the cross-section on the line *A B* is 2 inches. Then, $2 \times 3.1416 = 6.2832$ inches, and $6.2832 \times 31.416 = 197.4$ sq. in., the convex area. Ans.

432. To find the volume of a cylindrical ring:

Rule 71.—*The volume will be the same as that of a cylinder whose altitude equals the length of the dotted center line D, and whose base is the same as a cross-section of the ring on the line A B, drawn from the center O. Hence, to find the volume of a cylindrical ring, multiply the area of an imaginary cross-section on the line A B, by the length of the center line D.*

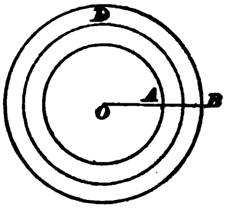


FIG. 61.

EXAMPLE.—What is the volume of a cylindrical ring whose outside diameter is 12 inches, and whose inside diameter is 8 inches?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters $= \frac{12 + 8}{2} = 10.$

$10 \times 3.1416 = 31.416$ inches, the length of the center line.

The radius of the outside circle $= 6$ inches; of the inside circle $=$ 4 inches; therefore, the diameter of the cross-section on the line $A\ B =$ 2 inches.

Then, $2^2 \times .7854 = 3.1416$ sq. in., the area of the imaginary cross-section.

And $3.1416 \times 31.416 = 98.7$ cubic inches, the volume. Ans.

EXAMPLES FOR PRACTICE.

1. What is the volume of a sphere 30 inches in diameter?
Ans. 14,137.2 cu. in.

2. How many square inches in the surface of the above sphere?
Ans. 2,827.44 sq. in.

3. Required the area of the convex surface of a circular ring, the outside diameter of the ring being 10 inches, and the inside diameter 7½ inches. Ans. 107.95 sq. in.

4. Find the cubical contents of the ring in the last example.
Ans. 33.73 cu. in.

5. The surface of a sphere contains 314.16 square inches. What is the volume of the sphere? Ans. 523.6 cu. in.

MECHANICS.

433. Mechanics is that science which treats of the action of forces upon bodies, and the effects which they produce; it treats of the laws which govern the movement and equilibrium of bodies, and shows how they may be utilized.

MATTER AND ITS PROPERTIES.

434. Matter is anything that occupies space. It is the substance of which all bodies consist. Matter is composed of *molecules* and *atoms*.

435. A **molecule** is the smallest portion of matter that can exist without changing its nature.

436. An **atom** is an indivisible portion of matter.

Atoms unite to form molecules, and a collection of molecules forms a mass or body.

A drop of water may be divided and subdivided, until each particle is so small that it can only be seen by the most powerful microscope, but each particle will still be water.

Now, imagine the division to be carried on still further, until a limit is reached beyond which it is impossible to go without changing the nature of the particle. The particle of water is now so small that, if it be divided again, it will cease to be water, and will be something else; we will call this particle a *molecule*.

If a molecule of water be divided, it will yield two atoms of hydrogen gas, and one of oxygen gas. If a molecule of sulphuric acid be divided, it will yield two atoms of hydrogen, one of sulphur, and four of oxygen.

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It has been calculated that the diameter of a molecule is larger than $\frac{1}{1250000000}$ of an inch, and smaller than $\frac{1}{500000000}$ of an inch.

437. **Bodies** are composed of collections of molecules. Matter exists in three conditions or forms: *solid*, *liquid*, and *gaseous*.

438. A **solid body** is one whose molecules change their relative positions with great difficulty; as iron, wood, stone, etc.

439. A **liquid body** is one whose molecules tend to change their relative positions easily. Liquids readily adapt themselves to the shape of vessels which contain them, and their upper surface always tends to become perfectly level. Water, mercury, molasses, etc., are liquids.

440. A **gaseous body,** or gas, is one whose molecules tend to separate from one another; as air, oxygen, hydrogen, etc.

Gaseous bodies are sometimes called **aeriform** (air-like) **bodies.** They are divided into two classes: the so-called "*permanent*" *gases*, and *vapors*.

441. A **permanent gas** is one which remains a gas at ordinary temperatures and pressures.

442. A **vapor** is a body which at ordinary temperatures is a liquid or solid, but when heat is applied, becomes a gas, as steam.

One body may be in all three states; as, for example, mercury, which at ordinary temperatures is a liquid, becomes a solid (freezes) at 40° below zero, and a vapor (gas) at 600° above zero. By means of great cold, all gases, even hydrogen, have been liquefied, and some solidified.

By means of heat, all solids have been liquefied, and a great many vaporized. It is probable that, if we had the means of producing sufficiently great extremes of heat and cold, all solids might be converted into gases, and all gases into solids.

443. Every portion of matter possesses certain qualities called *properties*. Properties of matter are divided into two classes: *general* and *special*.

444. **General properties of matter** are those which are common to all bodies. They are as follows: *Extension*, *impenetrability*, *weight*, *indestructibility*, *inertia*, *mobility*, *divisibility*, *porosity*, *compressibility*, *expansibility*, and *elasticity*.

445. **Extension** is the property of occupying space. Since all bodies must occupy space, it follows that extension is a general property.

446. By **impenetrability** we mean that no two bodies can occupy exactly the same space at the same time.

447. **Weight** is the measure of the earth's attraction upon a body. All bodies have weight. In former times it was supposed that gases had no weight, since, if unconfined, they tend to move away from the earth, but, nevertheless, they will finally reach a point beyond which they can not go, being held in suspension by the earth's attraction. Weight is measured by comparison with a standard. The standard is a bar of platinum owned and kept by the Government; it weighs one pound.

448. **Inertia** means that a body can not put itself in motion nor bring itself to rest. To do either it must be acted upon by some force.

449. **Mobility** means that a body can be changed in position by some force acting upon it.

450. **Divisibility** is that property of matter on account of which a body may be separated into parts.

451. **Porosity** is the term used to denote the fact that there is space between the molecules of a body. The molecules of a body are supposed to be spherical, and, hence, there is space between them, as there would be between peaches in a basket. The molecules of water are larger than those of salt; so that when salt is dissolved in water, its molecules wedge themselves between the molecules of the water, and unless too much salt is added, the water will occupy no more space than it did before. This does not prove that water is penetrable, for the molecules of salt occupy the space that the molecules of water did not.

Water has been forced through iron by pressure, thus proving that iron is porous.

452. Compressibility.—This property is a natural consequence of the preceding one. Since there is space between the molecules, it is evident that by means of force (pressure) they can be brought closer together, and thus the body be made to occupy a smaller space.

453. Expansibility is the term used to denote the fact that the molecules of a body will, under certain conditions (when heated, for example), move farther apart, and so cause the body to *expand*, or occupy a greater space.

454. Elasticity is that property of matter which enables a body when distorted within certain limits to resume its original form when the distorting force is removed. Glass, ivory, and steel are very elastic, clay and putty in their natural state being very slightly so.

455. Indestructibility is the term used to denote the fact that we can not *destroy* matter. A body may undergo thousands of changes, be resolved into its molecules, and its molecules into atoms, which may unite with other atoms to form other molecules and bodies entirely different in appearance and properties from the original body, but the same number of atoms remain. The whole number of atoms in the universe is exactly the same now as it was millions of years ago, and will always be the same. *Matter is indestructible.*

456. Special properties are those which are not possessed by all bodies. Some of the most important are as follows: *hardness*, *tenacity*, *brittleness*, *malleability*, and *ductility*.

457. Hardness.—A piece of copper will scratch a piece of wood, steel will scratch copper, and tempered steel will scratch steel in its ordinary state. We express all this by saying that steel is *harder* than copper, and so on. Emery and corundum are extremely hard, and the diamond is the hardest of all known substances. It can only be polished with its own powder.

458. Tenacity is the term applied to the power with which some bodies resist a force tending to pull them apart. Steel is very tenacious.

459. Brittleness.—Some bodies possess considerable power to resist either a pull or a pressure, but they are easily broken when subjected to shocks or jars; for example, good glass will bear a *greater* compressive force than most woods, but may be easily broken when dropped on to a hard floor; this property is called *brittleness.*

460. Malleability is that property which permits of some bodies being hammered or rolled into sheets. Gold is the most malleable of all substances.

461. Ductility is that property which enables some bodies to be drawn into wire. Platinum is the most ductile of all substances.

MOTION AND VELOCITY.

462. Motion is the opposite of rest, and indicates a changing of position in relation to some object which is for that purpose regarded as being fixed. If a large stone is rolled down hill, it is in motion in relation to the hill.

If a person is on a railway train, and walks in the opposite direction from that in which the train is moving, and with the same speed, he will be in motion as regards the train, but at rest with respect to the earth, since, until he gets to the end of the train, he will be directly over the spot at which he was when he started to walk.

463. The **path** of a body in motion is the line described by a certain point in the body called its center of gravity. No matter how irregular the shape of the body may be, nor how many turns and twists it may make, the line which indicates the direction of this point for every instant that it is in motion is the path of the body.

464. Velocity is rate of motion. It is measured by a unit of space passed over in a unit of time. When equal spaces are passed over in equal times, the velocity is said to be **uniform.** In all other cases it is **variable.**

If the fly-wheel of an engine keeps up a constant speed of a certain number of revolutions per minute, the velocity of any point is uniform. A railway train having a constant speed of 40 miles per hour, moves 40 miles every hour, or $\frac{40}{60} = \frac{2}{3}$ of a mile every minute, and since equal spaces are passed over in equal times, the velocity is uniform.

465. To find the uniform velocity which a body must have to pass over a certain distance or space in a given time:

Rule 72.—*Divide the distance by the time.*

EXAMPLE.—The piston of a steam engine travels 3,000 feet in 5 minutes; what is its velocity in feet per minute?

SOLUTION.—Here 3,000 feet is the distance, and 5 minutes is the time. Applying the rule, 3,000 ÷ 5 = 600 feet per minute. Ans.

CAUTION.—Before applying the above or any of the succeeding rules, care must be taken to reduce the values given to the denominations required in the answer. Thus, in the above example, had the velocity been required in feet per second instead of feet per minute, the 5 minutes would have been reduced to seconds before dividing. The operation would then have been, 5 min. $= 5 \times 60 = 300$ sec. Applying the rule, $3{,}000 \div 300 = 10$ ft. per sec. Ans.

466. Had the velocity been required in inches per second, it would have been necessary to reduce the 3,000 feet to inches and the 5 minutes to seconds, before dividing. Thus, 3,000 ft. $\times 12 = 36{,}000$ in. 5 min. $\times 60 = 300$ sec. Now, applying the rule, $36{,}000 \div 300 = 120$ in. per sec. Ans.

EXAMPLE.—A railroad train travels 50 miles in 1¼ hours; what is its average velocity in feet per second?

Solution.—Reducing the miles to feet, and the hours to seconds, 50 miles × 5,280 = 264,000 ft. $1\frac{1}{2}$ hours × 60 × 60 = 5,400 sec. Applying the rule, 264,000 ÷ 5,400 = $48\frac{8}{9}$ ft. per sec. Ans.

467. If the uniform velocity (or the average velocity) and the time are given, and it is required to find the distance which a body having the given velocity would travel in the given time:

Rule 73.—*Multiply the velocity by the time.*

Example.—The velocity of sound in still air is 1,092 feet per second; how many miles will it travel in 16 seconds?

Solution.—Reducing the 1,092 ft. to miles, $1{,}092 \div 5{,}280 = \frac{1{,}092}{5{,}280}$. Applying the rule, $\frac{1{,}092}{5{,}280} \times 16 = 3.31$ miles, nearly. Ans.

Example.—The piston speed of an engine is 11 ft. per sec., how many miles does the piston travel in 1 hour and 15 minutes?

Solution.—1 hour and 15 minutes reduced to seconds = 4,500 seconds = the time. 11 feet reduced to miles = $\frac{11}{5{,}280}$ mile = velocity in miles per second. Applying the rule, $\frac{11}{5{,}280} \times 4{,}500 = 9.375$ miles. Ans.

468. If the distance through which a body moves is given, and also its average or uniform velocity, and it is desired to know how long it takes the body to move through the given distance:

Rule 74.—*Divide the distance, or space passed over, by the velocity.*

Example.—Suppose that the radius of the crank of a steam engine is 15", and that the shaft makes 120 revolutions per minute, how long will it take the crank-pin to travel 18,849.6 feet?

Solution.—Since the radius, or distance from the center of the shaft to the center of the crank-pin, is 15", the diameter of the circle it moves in is 15" × 2 = 30" = 2.5 ft. The circumference of this circle is 2.5 × 3.1416 = 7.854 ft. 7.854 × 120 = 942.48 ft. = distance that the crank-pin travels in one minute = velocity in feet per minute. Applying the rule, 18,849.6 ÷ 942.48 = 20 minutes. Ans.

Example.—A point on the rim of an engine fly-wheel travels at the rate of 150 feet per second; how long will it take to travel 45,000 feet?

Solution.—Applying the rule,

45,000 ÷ 150 = 300 seconds = 5 minutes. Ans.

EXAMPLES FOR PRACTICE.

1. A locomotive has drivers 80" in diameter. If they make 293 revolutions per minute, what is the velocity of the train in (*a*) feet per second? (*b*) miles per hour?

Ans. { (*a*) 102.277 ft. per sec.
(*b*) 69.784 mi. per hr.

2. Assuming the velocity of steam as it enters the cylinder to be 900 feet per second, how far could it travel, if unobstructed, during the time the fly-wheel of an engine revolved 7 times, if the number of revolutions per minute were 120? Ans. 3,150 ft.

3. The average speed of the piston of an engine is 528 feet per minute, how long will it take the piston to travel 4 miles?

Ans. 40 min.

4. A speed of 40 miles per hour equals how many feet per second?

Ans. 58⅔ ft.

5. The earth turns around once in 24 hours. If the diameter be taken as 8,000 miles, what is the velocity of a point on the earth in miles per minute? Ans. 17.45⅓ mi. per min.

6. The stroke of an engine is 28 inches. If the engine makes 11,400 strokes per hour, (*a*) what is its speed in feet per minute? (*b*) How far will this piston travel in 11 minutes?

Ans. { (*a*) 443⅓ ft. per min.
(*b*) 4,876 ft. 8 in.

FORCE.

469. Force is that which produces, or tends to produce or destroy, motion. Forces are called by various names, according to the effects which they produce upon a body, as *attraction*, *repulsion*, *cohesion*, *adhesion*, *accelerating* force, *retarding* force, *resisting* force, etc., but all are equivalent to a push or pull, according to the direction in which they act upon a body. That the effect of a force upon a body may be compared with another force, it is necessary that three conditions be fulfilled in regard to both bodies. They are as follows:

(1) *The point of application, or point at which the force acts upon the body, must be known.*

(2) *The direction of the force, or, what is the same thing, the straight line along which the force tends to move the point of application, must be known.*

(3) *The magnitude or value of the force, when compared with a given standard, must be known.*

470. *The unit of magnitude of forces will always be taken as one pound,* and all forces will be spoken of as a certain number of pounds.

In practice, force is always regarded as a pressure; that is, a force may always be replaced by an equivalent weight. Thus, a force of 20 lb. acting upon a body is regarded as a pressure of 20 lb. produced by a weight of 20 lb. The tendency of a force is always to produce motion in the direction in which it acts. The resistance may be too great for it to cause motion, but it *always tends* to produce it.

471. The fundamental principles of the relations between force and motion were first stated by Sir Isaac Newton. They are called "Newton's Three Laws of Motion," and are as follows:

(*1*) *All bodies continue in a state of rest, or of uniform motion in a straight line, unless acted upon by some external force that compels a change.*

(*2*) *Every motion or change of motion is proportional to the acting force, and takes place in the direction of the straight line along which the force acts.*

(*3*) *To every action there is always opposed an equal and contrary reaction.*

472. In the first law of motion it is stated that a body once set in motion by any force, no matter how small, will move forever in a straight line, and always with the same velocity, unless acted upon by some other force which compels a change. It is not possible to actually verify this law, on account of the earth's attraction for all bodies, but, from astronomical observations, we are certain that the law is true. This law is often called *the law of inertia.*

473. The word **inertia** is so abused that a full understanding of its meaning is necessary. Inertia is not a force, although it is often so called. If a force acts upon a body and puts it in motion, the effect of the force is stored in the body, and a second body, in stopping the first, will receive a blow equal in every respect to the original force, assuming that there has been no resistance of any kind to the motion of the first body.

It is dangerous for a person to jump from a fast moving train, for the reason that, since his body has the same velocity as the train, it has the same force stored in it that would cause a body of the same weight to take the same velocity as the train, and the effect of a sudden stoppage is the same as the effect of a blow necessary to give the person that velocity.

By "bracing" himself and jumping in the same direction that the train is moving, and running, he brings himself gradually to rest, and thus reduces the danger. If a body is at rest, it must be acted upon by a force in order to be put in motion, and, no matter how great the force may be, it can not be *instantly* put in motion.

The resistance thus offered to being put in motion is commonly, but erroneously, called the "*Resistance of Inertia.*" It should be called the *Resistance due to Inertia.*

474. From the second law, we see that if two or more forces act upon a body, their final effect upon the body will be in proportion to their magnitudes, and to the directions in which they act.

Thus, if the wind be blowing due West, with a velocity of 50 miles per hour, and a ball be thrown due North with the same velocity, or 50 miles per hour, the wind will carry the ball West while the force of the throw is carrying it North, and the combined effect will be to cause it to move Northwest.

The amount of departure from due North will be proportional to the force of the wind, and independent of the velocity due to the force of the throw.

475. In Fig. 62 a ball *e* is supported in a cup, the bottom of which is attached to the lever *o* in such a manner that *o* will swing the bottom horizontally and allow the ball to drop. Another ball *b* rests in a horizontal groove that is provided with a slit in the bottom. A swinging arm is actuated by the spring *d* in such a manner that, when drawn back, as shown, and then released, it will strike the lever *o* and the ball *b* at the same time. This gives *b* an impulse in a horizontal direction, and swings *o* so as to allow *e* to fall.

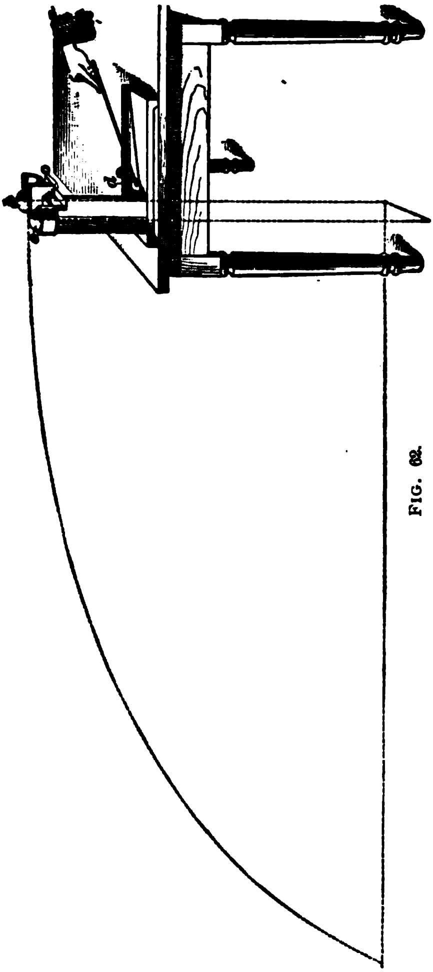

FIG. 62

On trying the experiment, it is found that *b* follows a path shown by the curved dotted line, and reaches the floor at the same instant as *e*, which drops vertically. This shows that the force which gave the first ball its horizontal movement had no effect on the vertical force which compelled both balls to fall to the floor; the vertical force producing the same effect as if the horizontal force had not acted. The second law may also be stated as follows: *A force has the same effect in producing motion, whether it acts upon a body at rest or in motion, and whether it acts alone or with other forces.*

476. The third law states that action and reaction are equal and opposite. A man can not lift himself by his bootstraps, for the reason that he presses downwards with the same force that he pulls upwards; the downward reaction equals the upward action, and is opposite to it.

In springing from a boat we must exercise caution or the reaction will drive the boat from the shore. When we jump from the ground, we tend to push the earth from us, while the earth reacts and pushes us from it.

EXAMPLE.—Two men pull on a rope in opposite directions, each exerting a force of 100 pounds; what is the force which the rope resists?

SOLUTION.—Imagine the rope to be fastened to a tree, and one man to pull with a force of 100 pounds. The rope evidently resists 100 pounds. According to Newton's third law, the reaction of the tree is also 100 pounds. Now, suppose the rope to be slackened, but that one end is still fastened to the tree, and the second man to take hold of the rope near the tree, and pull with a force of 100 pounds, the first man pulling as before. The resistance of the rope is 100 pounds, as before, since the second man merely takes the place of the tree. *He is obliged to exert a force of 100 pounds to keep the rope from slipping through his fingers.* If the rope be passed around the tree, and each man pulls an end with a force of 100 pounds in the same and parallel directions, the stress in the rope is 100 pounds, as before, but the tree must resist the pull of both men, or 200 pounds.

477. A **force** may be represented by a line; thus, in Fig. 63, let A be the *point of application* of the force; let the length of the line $A\ B$ represent its *magnitude*, and let the arrowhead indicate the *direction* in which the force acts, then the line $A\ B$ fulfils the three conditions (see Art. **469**), and the force is fully represented.

FIG. 63.

CENTER OF GRAVITY.

478. *The center of gravity of a body is that point at which the body may be balanced, or it is the point at which the whole weight of a body may be considered as concentrated.*

This point is not always *in* the body; in the case of a horseshoe or a ring it lies outside of the substance of, but within the space enclosed by, the body.

In a moving body, the line described by its center of gravity is always taken as the path of the body. In finding the distance that a body has moved, the distance that the center of gravity has moved is taken.

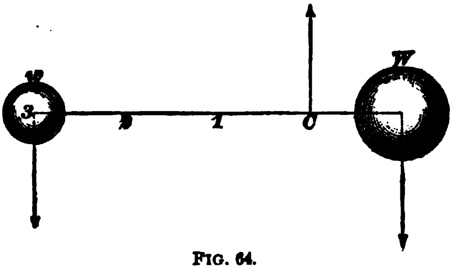

Fig. 64.

The definition of the center of gravity of a body may be applied to a system of bodies if they are considered as being connected at their centers of gravity.

If w and W, Fig. 64, be two bodies of known weight, their center of gravity will be at C. The point C may be readily determined as follows:

Rule 75.—*The distance of the common center of gravity, from the center of gravity of the large weight, is equal to the weight of the smaller body multiplied by the distance between the centers of gravity of the two bodies, and this product divided by the sum of the weights of the two bodies.*

Example.—In Fig. 64, $w = 10$ pounds, $W = 30$ pounds, and the distance between their centers of gravity is 36 inches where is the center of gravity of both bodies situated?

Solution.—Applying the rule, $10 \times 36 = 360$. $10 + 30 = 40$. $360 + 40 = 9'' =$ distance of center of gravity from center of large weight.

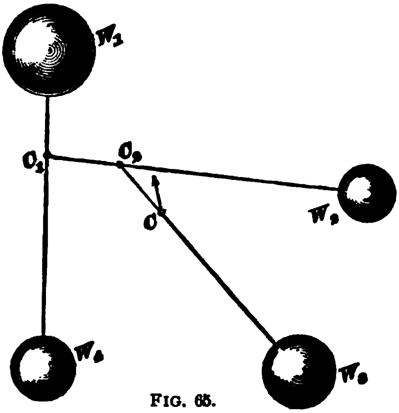

Fig. 65.

479. It is now very easy to extend this principle, to find the center of gravity of any number of bodies when their weights and the distances apart of their centers of gravity are known, by the following rule:

Rule 76.—*Find the center of gravity of two of the bodies as W_1 and W_4, in Fig. 65. Assume that the weight of both* *bodies is concentrated at C_1, and find the center of gravity of this combined weight at C_1, and the weight of W_3; let it be at C_2; then find the center of gravity of the combined weights of W_1, W_4, W_3 (concentrated at C_2), and W_3; let it be at C; then C will be the center of gravity of the four bodies.*

480. To find the center of gravity of **any parallelogram:**

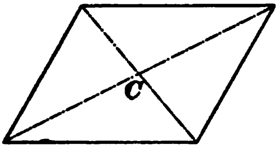

FIG. 66.

Rule 77.—*Draw the two diagonals, Fig. 66, and their point of intersection C will be the center of gravity.*

481. To find the center of gravity of a **triangle,** as *A B C*, Fig. 67:

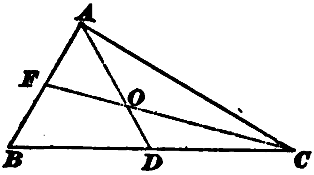

FIG. 67.

Rule 78.—*From any vertex, as A, draw a line to the middle point D of the opposite side B C. From one of the other vertexes, as C, draw a line to F, the middle point of the opposite side A B; the point of intersection O of these two lines is the center of gravity.*

It is also true that the distance $D\,O = \frac{1}{3}\,D\,A$, and that $F\,O = \frac{1}{3}\,F\,C$, and the center of gravity could have been found by drawing from any vertex a line to the middle point of the opposite side, and measuring back from that side $\frac{1}{3}$ of the length of the line.

The center of gravity of **any regular plane figure** is the same as the center of the inscribed or circumscribed circle.

482. To find the center of gravity of **any irregular plane figure,** but of uniform thickness throughout, divide one of the parallel surfaces into triangles, parallelograms, circles, ellipses, etc., according to the shape of the figure; find the area and center of gravity of each part separately, and combine the centers of gravity thus found in the same manner as in rule **76**; in this case, however, dealing with the *area* of each part instead of its weight. See Fig. 68.

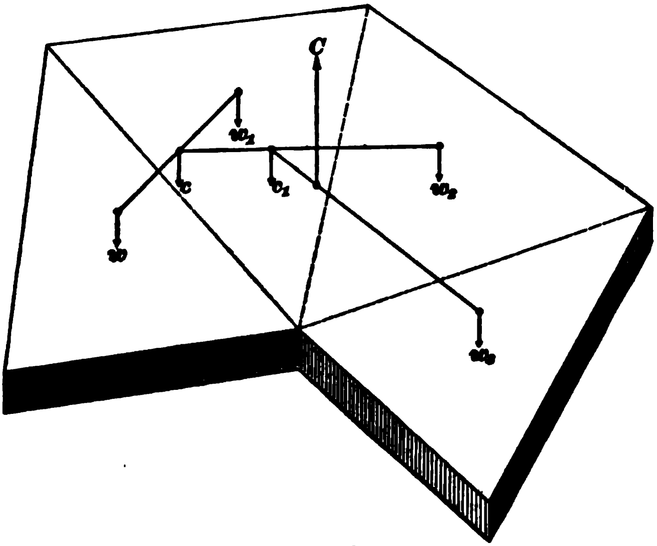

FIG. 68.

EXAMPLE.—Suppose that the two balls shown in Fig. 64 were 5″ and 10″ in diameter, and weighed 10 lb. and 80 lb., respectively. If the distance between their centers were 40″, and they were connected by a steel rod 1″ in diameter, where would the center of gravity be, taking the weight of a cubic inch of steel as .283 lb.?

SOLUTION.—The length of the rod $= 40 - \frac{5}{2} - \frac{10}{2} = 32\frac{1}{2}''$. Its volume is $1^2 \times .7854 \times 32\frac{1}{2} = 25.53$ cu. in. $25.53 \times .283 = 7.22$ lb. The rod being straight, its center of gravity is in the middle at a distance of $\frac{32.5}{2} + \frac{5}{2} = 18\frac{3}{4}''$ from the center of the smaller weight, and $\frac{32.5}{2} + \frac{10}{2} = 21\frac{1}{4}''$ from the center of the larger weight. Now, considering the weight of the rod to be concentrated at its center of gravity, we have three weights of 10, 7.22, and 80 lb., all in a straight line, and the distances between them given to find the center of gravity, or balancing point, of the combination, by rule **76.** We will first find the center of gravity of the two smaller weights by rule **75,** as follows: $7.22 \times 18\frac{3}{4} = 135.38$. $10 + 7.22 = 17.22$. $135.38 \div 17.22 = 7.86''$ = distance from the center of the 10-lb. weight. Considering both of the smaller weights to be concentrated at this point, we find the center of gravity of this combined weight and the large weight as follows. $40 - 7.86 = 32.14''$ = distance between the center of gravity of the two small weights and the center of gravity of the 80-lb. weight. Applying rule **75**, $17.22 \times 32.14 = 553.45$.

$17.22 + 80 = 97.22$. $553.45 \div 97.22 = 5.693''$ = distance from the center of the 80-lb. weight. Ans.

483. Center of Gravity of a Solid.—In a body free to move, the center of gravity will lie in a vertical plumb line drawn through the point of support. Therefore, to find the position of the center of gravity of an irregular solid, as the crank, Fig. 69, suspend it at some point, as *B*, so that it will move freely. Drop a plumb line from the point of suspension, and mark its direction. Suspend the body at another point, as *A*, and repeat the process. The intersection *C* of the two lines will be directly over the center of gravity.

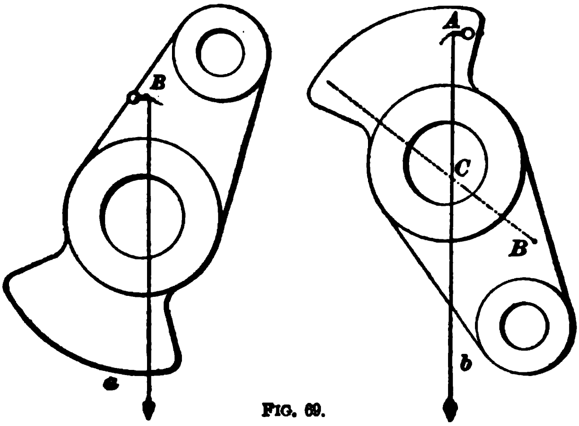

FIG. 69.

Since the center of gravity depends wholly upon the shape and weight of a body, it may be without the body, as in the case of a circular ring whose center of gravity is the same as the center of the circumference of the ring.

EXAMPLES FOR PRACTICE.

1. A spherical shell has a wrought-iron handle attached to it. The shell is 10″ in diameter, and weighs 20 lb. The handle is 1¼″ in diameter, and the distance from the center of the shell to the end of the handle is 4 ft. Where is the center of gravity? Take the weight of a cubic inch of wrought iron as .278 lb. Ans. 13.612″ from center of shell.

2. The distance between the centers of two bodies is 51". The weights of the bodies being 20 and 73 lb., where is the center of gravity?

Ans. 10.968" from the center of large weight.

3. A hollow engine piston weighs 275 lb., and is 3½" thick. Assuming the piston rod to be straight throughout its entire length, and to weigh 140 lb., at what point will the piston and rod balance if the length of the rod is 73" from the face of the piston? Consider the weight of the piston to be concentrated at its center.

Ans. 11.15", nearly, from face of piston.

4. Weights of 5, 9, and 12 lb. lie in one straight line in the order named. Distance from the 5-lb. weight to the 9-lb. weight is 22", and from the 9-lb. weight to the 12-lb. weight is 18". Where is the center of gravity? Ans. 13.923" from 12-lb. weight.

SIMPLE MACHINES.

484. A **lever** is a bar capable of being turned about a pivot, or point, as in Figs. 70, 71, and 72.

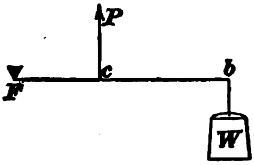

FIG. 70.

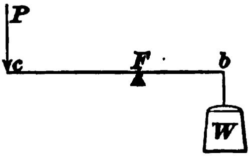

FIG. 71.

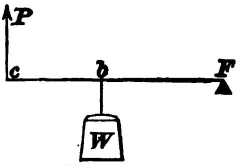

FIG. 72.

The object W to be lifted is called the **weight**; the force employed P is called the **power**; and the point, or pivot, F is called the **fulcrum.**

485. That part of the lever between the fulcrum and the weight, or $F\,b$, is called the **weight arm,** and the part between the fulcrum and the power, or $F\,c$, is called the **power arm.**

In order that the lever shall be in equilibrium (balance), *the power multiplied by the power arm must equal the weight multiplied by the weight arm;* that is, $P \times F\,c$ must equal $W \times F\,b$.

486. If F be taken as the center of a circle, and arcs be described through b and c, it will be seen that if the weight arm be moved through a certain angle, the power arm will move through the same angle. Since, in the same or equal angles, the lengths of the arcs are proportional to the radii with which they were described, it is seen that the power arm is proportional to the distance through which the power moves, and the weight arm is proportional to the distance through which the weight moves.

Hence, instead of writing $P \times Fc = W \times Fb$, we might have written it $P \times$ distance through which P moves $=$ $W \times$ distance through which W moves. This is the general law of all machines, and can be applied to any mechanism from the simple lever up to the most complicated arrangement. Stated in the form of a rule, it is as follows:

Rule 79.—*The power multiplied by the distance through which it moves equals the weight multiplied by the distance through which it moves.*

487. In the above rule, it will be noticed that there are four requirements necessary for a complete knowledge of the lever, viz.: the power (or force), the weight, the power arm (or distance through which the power moves), and the weight arm (or distance through which the weight moves). If any three are given, the fourth may be found by letting x represent the requirement which is to be found, and multiplying the power by the power arm and the weight by the weight arm; then, dividing the product of the two known numbers by the number by which x is multiplied; the result will be the requirement which was to be found.

Example.—If the weight arm of a lever is 6 in. long, and the power arm is 4 ft. long, how great a weight can be raised by a force of 20 lb. at the end of the power arm?

Solution.—In this example, the weight is unknown; hence, representing it by x, we have, after reducing the 4 ft. to inches, $20 \times 48 = 960 =$ power multiplied by the power arm, and $x \times 6 =$ weight multiplied by the weight arm. Dividing the 960 by 6, the result is 160 lb., the weight. Ans.

488. If the distance through which the power or weight moved had been given instead of the power arm or weight arm, and it were required to find the power or weight, the process would have been exactly the same, using the given distance instead of the power arm or weight arm.

Example.—If, in the above example, the weight had moved 2½″, how far would the power have moved?

Solution.—In this example, the distance through which the power moves is required. Let x represent the distance. Then, $20 \times x =$ distance multiplied by power, and $2\frac{1}{2} \times 160 = 400 =$ distance multiplied by the weight. Hence, $x = \frac{400}{20} = 20$ in. = distance through which the power arm moves.

The ratio between the weights and the power is $160 \div 20 = 8$. The ratio between the distance through which the weight moves and the distance through which the power moves is $2\frac{1}{2} \div 20 = \frac{1}{8}$. This shows that while a force of 1 lb. can raise a weight of 8 lb., the 1-lb. weight must move through 8 times the distance that the 8-lb. weight does. It will also be noticed that the ratio of the lengths of the two arms of the lever is also 8, since $48 \div 6 = 8$.

489. The law which governs the straight lever also governs the bent lever, but care must be taken to determine the true lengths of the lever arms, which are in every case *the perpendicular distances from the fulcrum to the line of direction of the weight or power.*

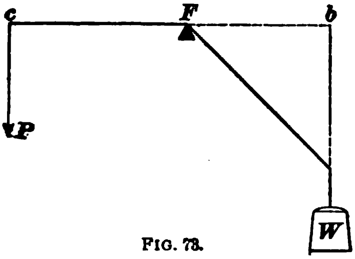

Fig. 73.

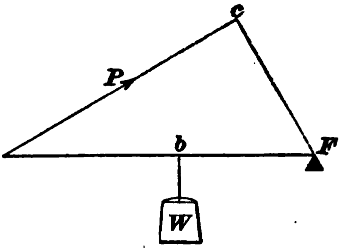

Fig. 74.

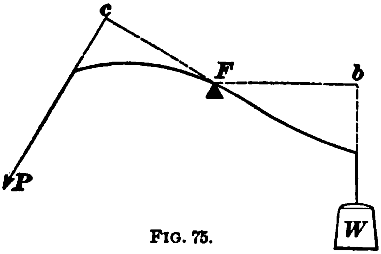

Fig. 75.

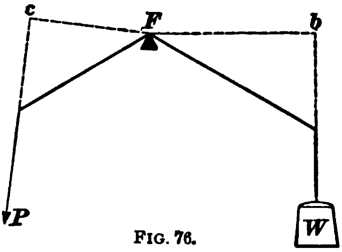

Fig. 76.

Thus, in Figs. 73, 74, 75, and 76, Fc in each case represents the power arm, and Fb the weight arm.

490. A **compound lever** is a series of single levers arranged in such a manner that when a power is applied to the first it is communicated to the second, and from that to the third, and so on.

Fig. 77 shows a compound lever. It will be seen that when a power is applied to the first lever at P it will be communicated to the second lever at P, from this to the third lever at P, and thus raise the weight W.

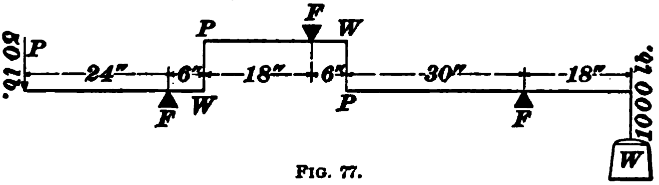

FIG. 77.

The weight which the power of the first lever could raise acts as the power of the second, and the weight which this could raise through the second lever acts as the power of the third lever, and so on, no matter how many single levers make up the compound lever.

In this case, as in every other, the power multiplied by the distance through which it moves equals the weight multiplied by the distance through which it moves.

Hence, if we move the P end of the lever, say 4 inches, and the end carrying the weight W moves $\frac{1}{5}$ of an inch, we know that the ratio between P and W is the same as the ratio between $\frac{1}{5}$ and 4; that is, 1 to 20, and, hence, that 10 pounds at P would balance 200 pounds at W, without measuring the lengths of the different lever arms. If the lengths of the lever arms are known, the ratio between P and W may be readily obtained from the following rule:

Rule 80.—*The continued product of the power and each power arm equals the continued product of the weight and each weight arm.*

EXAMPLE.—If, in Fig. 77, PF = 24 inches, 18 inches, and 30 inches, respectively, and WF = 6 inches, 6 inches, and 18 inches, respectively, how great a force at P would it require to raise 1,000 pounds at W? What is the ratio between W and P?

SOLUTION.—Let x represent the power; then, $x \times 24 \times 18 \times 30 = 12{,}960\,x$ = continued product of the power and each power arm. $1{,}000 \times$

$6 \times 6 \times 18 = 648{,}000$ = continued product of the weight and each weight arm, and, since $12{,}960\,x = 648{,}000$,

$$x = \frac{648{,}000}{12{,}960} = 50 \text{ lb.} = \text{the power. Ans.}$$

$$1{,}000 \div 50 = 20 = \text{ratio between } W \text{ and } P.$$

491. The **wheel and axle** consists of *two cylinders of different diameters, rigidly connected*, so that they turn together about a common axis, as in Fig. 78. Then, as before, $P \times$ distance through which it moves $= W \times$ distance through which it moves; and, since these distances are proportional to the radii of the power cylinder and weight cylinder, $P \times Fc = W \times Fb$.

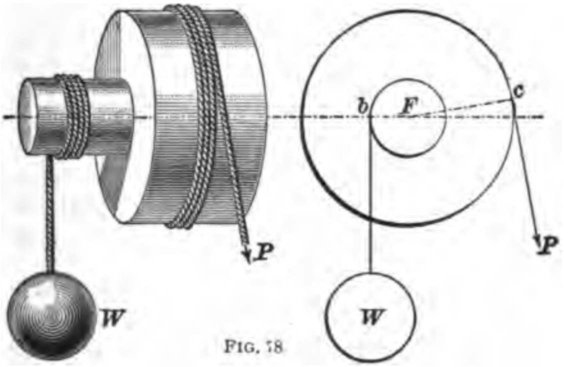

FIG. 78

It is not necessary that an entire wheel be used; an arm, projection, radius, or anything which the power causes to revolve in a circle, may be considered as the wheel. Consequently, if it is desired to hoist a weight with a windlass, Fig. 79, the power is applied to the handle of the crank, and the distance between the center line of the crank handle and the axis of the drum corresponds to the radius of the wheel.

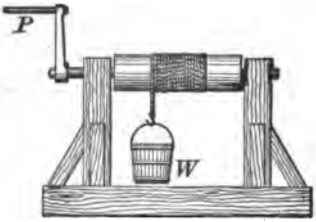

FIG. 79.

EXAMPLE.—If the distance between the center line of the handle and the axis of the drum, in Fig. 79, is 18 inches and the diameter of the drum is 6 inches, what force will be required at P to raise a load of 300 pounds?

SOLUTION.—$P \times (18 \times 2) = 300 \times 6$, or $P = 50$. Ans.

EXAMPLES FOR PRACTICE.

1. The lever of a safety valve is of the form shown in Fig. 70, where the force is applied at a point between the fulcrum and the weight lifted. If the distance from the fulcrum to the valve is $5\frac{1}{2}''$, and from the fulcrum to the weight is 42", what total force is necessary to raise the valve, the weight being 78 lb., and the weight of valve and lever being neglected? Ans. 595.64 lb.

2. If, in Fig. 77, $PF = 10''$, $12''$, $14''$, and $16''$ respectively, and $WF = 2''$, $3''$, $4''$, and $5''$, respectively, (*a*) how great a weight can a force of 20 lb. raise? (*b*) What is the ratio between W and P? (*c*) If P moves 4", how far will W move?

Ans. (*a*) 4,480 lb.
(*b*) 224.
(*c*) $\frac{1}{56}''$.

3. A windlass is used to hoist a weight. If the diameter of the drum on which the rope winds is 4", and the distance from the center of the handle to the axis of the drum is 14", how great a weight can a force of 32 lb. applied to the handle raise? Ans. 224 lb.

PULLEYS.

492. Pulleys for the transmission of power by belts may be divided into two principal classes: 1. The solid pulley shown in Fig. 80, in which the hub, arms, and rim are one entire casting. 2. The split pulley shown in Fig. 81, which is cast in halves.

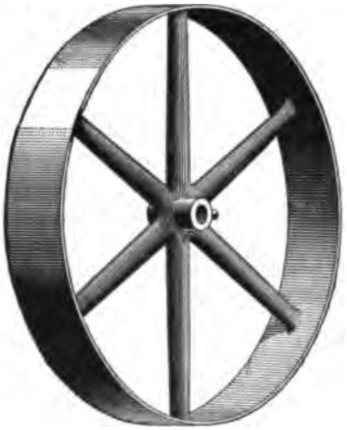

FIG. 80.

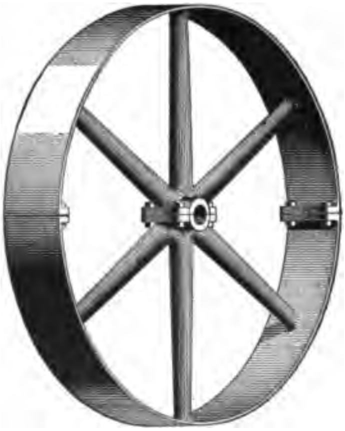

FIG. 81.

The latter style of pulley is more readily placed upon and removed from the shaft than the solid pulley. Pulleys are generally cast in halves or parts when they are more than six feet in diameter; this is done on account of shrinkage strain in large pulley castings, which renders them liable to crack as a result of unequal cooling of the metal.

493. Crowning.—In Fig. 82 is shown a section of the rim of a pulley that has crowning, or, in other words, whose diameter is larger at the center of the face than at its edges. This is done to prevent the belt from running off the pulley. The amount of crowning given to pulleys varies from $\frac{3}{16}$ to $\frac{1}{2}$ an inch per foot of width of the pulley face.

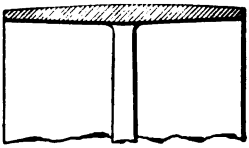

FIG. 82.

494. Balanced Pulleys.—All pulleys which rotate at high speeds should be balanced. If they are not, the centrifugal force which is generated by the pulley's rotation is greater on one side than on the other, and it will cause the pulley shaft to vibrate and shake. Pulleys should run true, so that the strain, or tension, of the belt is equal at all parts of the revolution, thus making the transmitting power equal. The smoother the surface of a pulley, the greater is its driving power.

The transmitting power of a pulley can be increased by covering the face of the pulley with a leather or rubber band; this increases the driving power about one-quarter.

495. The pulley that imparts motion to the belt is called the *driver;* that which receives the motion is called the *driven.*

The revolutions of any two pulleys over which a belt is run vary in an inverse proportion to their diameters; consequently, if a pulley of 20 inches in diameter is driven by one of 10 inches in diameter, the 20-inch pulley will make one revolution while the 10-inch pulley makes two revolutions, or they are in the ratio of 2 to 1. From this the following formulas have been deduced:

Let $D =$ diameter of the driver;
$d =$ diameter of the driven;
$N =$ number of revolutions of the driver;
$n =$ number of revolutions of the driven.

NOTE.—The words revolutions per minute are frequently abbreviated to R. P. M.

496. To find the diameter of the driving pulley when the diameter of the driven pulley and the number of revolutions per minute of each is given:

Rule 81.—*The diameter of the driving pulley equals the product of the diameter and number of revolutions of the driven pulley divided by the number of revolutions of the driving pulley.*

That is, $$D = \frac{dn}{N}.$$

EXAMPLE.—The driving pulley makes 100 revolutions per minute, the driven pulley makes 75 revolutions per minute, and is 18 inches in diameter; what is the diameter of the driving pulley?

SOLUTION.—Formula, $$D = \frac{dn}{N}.$$

Substituting, we have $D = \frac{18 \times 75}{100} = 13\frac{1}{2}$ in. Ans.

497. The diameter and number of revolutions per minute of the driving pulley being given, to find the diameter of the driven pulley, which must make a given number of revolutions per minute:

Rule 82.—*The diameter of the driven pulley equals the product of the diameter and number of revolutions of the driving pulley divided by the number of revolutions of the driven pulley.*

That is, $$d = \frac{DN}{n}.$$

EXAMPLE.—The diameter of the driver is $13\frac{1}{2}$ inches, and makes 100 revolutions per minute; what must be the diameter of the driven to make 75 revolutions per minute?

SOLUTION.—Formula, $$d = \frac{DN}{n}.$$

Substituting, we have $d = \frac{13\frac{1}{2} \times 100}{75} = 18$ in. Ans.

498. To find the number of revolutions per minute of the driven pulley, its diameter and the diameter and number of revolutions per minute of the driving pulley being given:

Rule 83.—*The number of revolutions of the driven pulley equals the product of the diameter and the number of revolutions of the driver divided by the diameter of the driven pulley.*

That is, $n = \frac{D\,N}{d}$.

EXAMPLE.—The driver is 13½ inches in diameter, and makes 100 revolutions per minute; how many revolutions will the driven make in one minute, if it is 18 inches in diameter?

SOLUTION.—Formula, $n = \frac{D\,N}{d}$.

Substituting, we have $n = \frac{13\frac{1}{2} \times 100}{18} = 75$ R. P. M. Ans.

499. To find the number of revolutions per minute of the driving pulley, its diameter and the diameter and number of revolutions per minute of the driven pulley being given:

Rule 84.—*The number of revolutions of the driving pulley equals the product of the diameter and number of revolutions of the driven pulley divided by the diameter of the driving pulley.*

That is, $N = \frac{d\,n}{D}$.

EXAMPLE.—The driven pulley is 18 inches in diameter, and makes 75 revolutions per minute; how many revolutions will the driver make in one minute, if it is 13½ inches in diameter?

SOLUTION.—Formula, $N = \frac{d\,n}{D}$.

Substituting, we have $N = \frac{18 \times 75}{13\frac{1}{2}} = 100$ R. P. M. Ans.

500. Wheelwork.—A combination of wheels and axles, as in Fig. 83, is called a **train.** The wheel in a train, to which motion is imparted from a wheel on another shaft by such means as a belt or gearing, is called the **driven wheel** or **follower;** the wheel which imparts the motion is called the **driver.**

501. It will be seen that the wheel and axle bears the same relation to the train that the simple lever does to the compound lever. Letting D_1, D_2, D_3, etc., represent

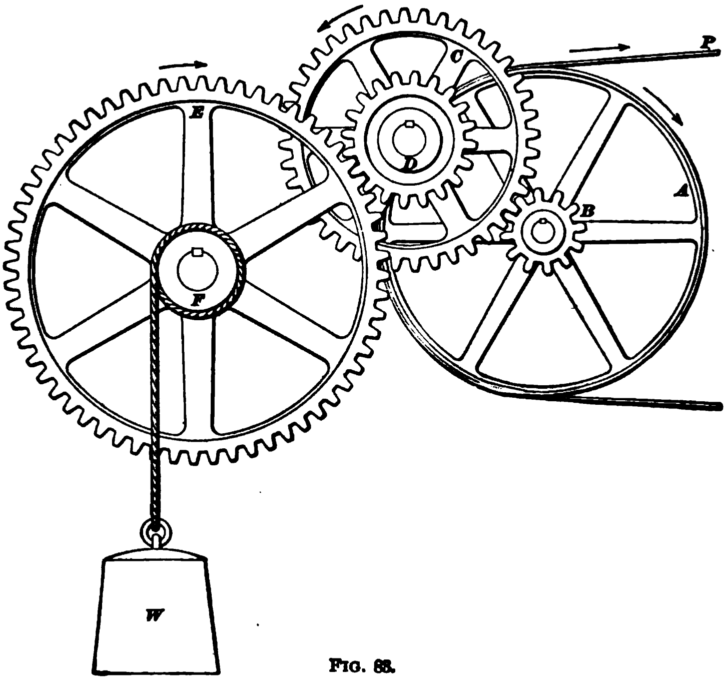

FIG. 88.

the diameters of the driven wheels, and d_1, d_2, d_3, etc., the diameters of the different drivers, we have the following:

Rule 85.—*The continued product of the power and the radii of the driven wheels equals the continued product of the weight, the radius of the drum that moves the weight, and the radii of the drivers.*

That is, $P \times D_1 \times D_2 \times D_3$, etc., $= W \times d_1 \times d_2 \times d_3$, etc.

EXAMPLE.—If the radius of the pulley A is 20 inches; of C, 15 inches, and of E, 24 inches, and the radius of the drum F, is 4 inches; of the pinion D, 5 inches, and of the pinion B, 4 inches, how great a weight will a force of 1 pound at P raise?

SOLUTION.—Formula, $P \times D_1 \times D_2 \times D_3 = W \times d_1 \times d_2 \times d_3$.

Substituting, we have $1 \times 20 \times 15 \times 24 = W \times 4 \times 5 \times 4$, or

$$W = \frac{7{,}200}{80} = 90 \text{ lb.} \quad \text{Ans.}$$

Hence, also, if W were raised 1 inch, P would fall 90 inches, or P would have to move through 90 inches to raise W through 1 inch.

502. It is now clear that another great law has made itself manifest, and that is that *whenever there is a gain in power without a corresponding increase in the initial force, there is a loss in speed;* this is true of any machine.

In the last example if P were to move the entire 90 inches in one second, W would move only 1 inch in one second.

Instead of using the diameter or radius of a gear, the number of teeth may be used when computing the weight which can be raised, or the velocity, as in the last example.

EXAMPLE.—The radius of the pulley A, Fig. 83, is 40″, and that of F is 12″. The number of teeth in B is 9; in C, 27; in D, 12, and in E, 36. If the weight to be lifted is 1,800 lb., how great a force at P is it necessary to apply to the belt?

SOLUTION.—Let P represent the force (power); then, by the rule, **85,** $P \times 40 \times 27 \times 36 = 1{,}800 \times 12 \times 9 \times 12$, or $P \times 38{,}880 = 2{,}332{,}800$.
Hence, $P = \frac{2{,}332{,}800}{38{,}880} = 60$ lb. = force necessary to apply to the belt. Ans.

GEAR-WHEELS.

503. A wheel that is provided with teeth to mesh with similar teeth upon another wheel is called a **gear-wheel,** or **gear.** In Fig. 84 is shown a **spur gear.** On spur gears the teeth are always parallel to the axis of the wheel or to its shaft.

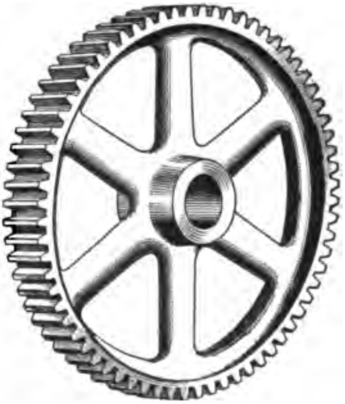

FIG. 84.

504. In Fig. 85 is shown a pair of **bevel gears** in mesh, of which one is smaller than the other.

505. When both are of the same diameter they are called **miter gears.**

In Fig. 86 is shown a pair of miter gears in mesh. It is obvious that the angle which the teeth of these gears make with the axis of the shaft must be 45°.

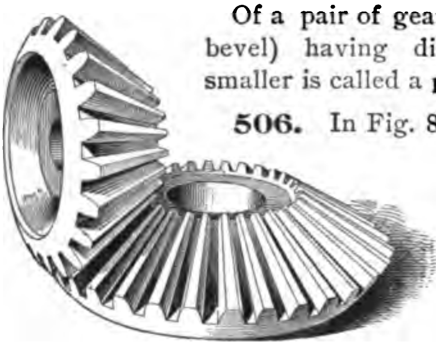

FIG. 85.

Of a pair of gear wheels (either spur or bevel) having different diameters, the smaller is called a **pinion.**

506. In Fig. 87 is shown a revolving screw, or worm, as it is called, in gear; it is used to transmit motion from one shaft to another at right angles to it.

As the worm is nothing else than a screw, each revolution given to the worm will rotate the worm-wheel a distance equal to its pitch, consequently, if there are 40 teeth in the worm-wheel, a single-threaded worm will have to make 40 revolutions in order to turn the wheel once.

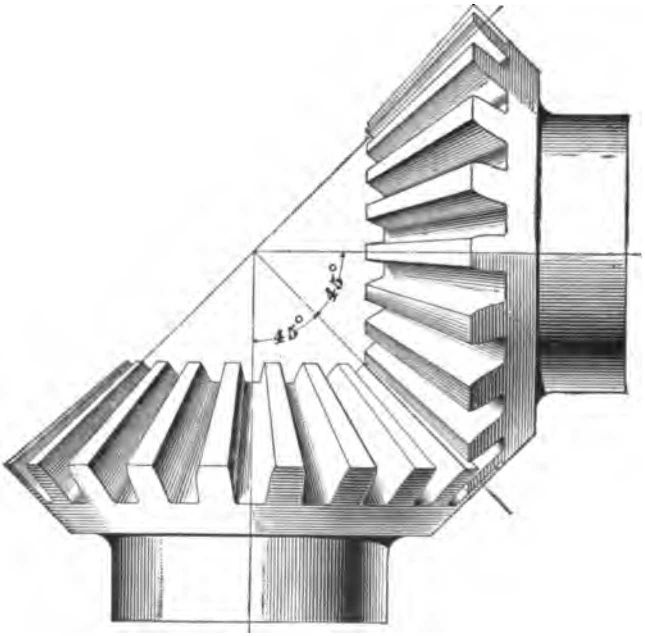

FIG. 86.

507. In Fig. 88 is shown a section of a rack and pinion, both having epicycloidal teeth. The arc *C C* represents part of the **pitch circle;** it is on the pitch circle that all the teeth are laid out. The diameter of a gear or worm-wheel is always taken as the diameter of this circle, unless otherwise specially stated as "diameter over all," or "diameter at the root," etc.

FIG. 87.

The **pitch** of the teeth of the gear-wheel is the distance from the edge of one tooth to the corresponding edge of the following tooth measured on the pitch circle; it is marked *pitch* in the figure.

The length of the tooth of a gear-wheel is .7 of its pitch, .4 of it, called the **root,** being below or within the pitch circle, and .3 of it, called the **addendum,** being above or without the pitch circle. Thus, if the pitch of the teeth of a gear-wheel is 2 inches, the length of a tooth below the pitch circle is 2 × .4 = .8 of an inch; and its length above the pitch circle is 2 × .3 = .6 of an inch. Consequently, we

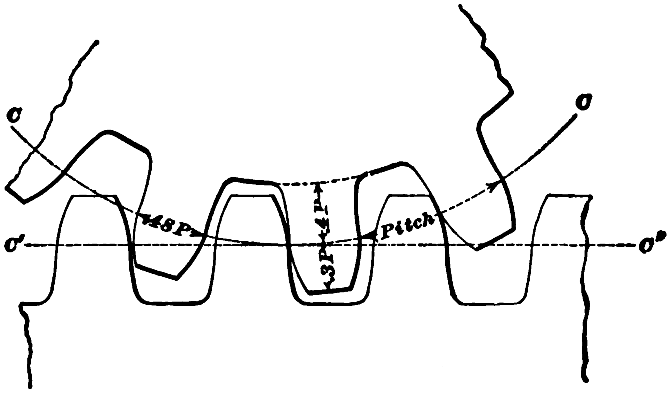

FIG. 88.

have only to multiply the pitch by .4 to obtain the length of the teeth below the pitch circle, and by .3 to obtain the length of the teeth above the pitch circle. The thickness of the teeth of a cast gear-wheel equals .48 × P, that is, .48 of the pitch; therefore, the thickness of the above teeth is .48 × 2, or .96 of an inch.

A rack may be considered as a gear-wheel rolled out so as to make the pitch circle a straight line, as $C' C''$. The teeth of racks are proportioned by the same rules as those of gear-wheels.

508. For the purpose of calculating the pitch-diameter, number of teeth, etc., of the gear-wheels, we have the following rules:

Let $P =$ pitch;
$T =$ number of teeth;
$D =$ pitch diameter of the wheel.

To find the pitch diameter of a gear-wheel in inches, when the pitch and number of teeth are given:

Rule 86.—*The pitch diameter equals the product of the pitch and number of teeth divided by 3.1416.*

That is, $$D = \frac{PT}{3.1416}.$$

Example.—What is the diameter of the pitch circle of a gear-wheel which has 75 teeth, and whose pitch is 1.675 inches?

Solution.—Formula, $D = \frac{PT}{3.1416}$.

Substituting, we have $D = \frac{1.675 \times 75}{3.1416} = 40$ in. Ans.

To find the number of teeth in a gear-wheel when the diameter and pitch are given:

Rule 87.—*The number of teeth equals the product of 3.1416 and the diameter divided by the pitch.*

That is, $$T = \frac{3.1416\,D}{P}.$$

Example.—The diameter of a gear-wheel is 40 inches, and the pitch of the teeth is 1.675 inches; how many teeth are there in the wheel?

SOLUTION.—Formula, $T = \frac{3.1416\,D}{P}$.

Substituting, we have $T = \frac{3.1416 \times 40}{1.675} = 75$ teeth. Ans.

To find the pitch of a gear-wheel when the diameter and the number of teeth are given.

Rule 88.—*The pitch of the teeth equals the product of 3.1416 and the diameter divided by the number of teeth.*

That is, $P = \frac{3.1416\,D}{T}$.

EXAMPLE.—The diameter of a gear-wheel is 40 inches, and it has 75 teeth; what is the pitch of the teeth?

SOLUTION.—Formula, $P = \frac{3.1416\,D}{T}$.

Substituting, we have $P = \frac{3.1416 \times 40}{75} = 1.675$ in. pitch. Ans.

509. The forms of teeth used in ordinary practice are the epicycloidal and involute.

Fig. 88 shows the epicycloidal form, which is composed of two different curves, the curve from the pitch circle to the top of the tooth being an epicycloid, and that from the pitch circle to the bottom of the tooth being a hypocycloid.

In gear-wheels where this form of tooth is employed, their pitch circles must run tangent to one another.

510. In Fig. 89 is shown the involute form of teeth, or teeth having but one curve. The outlines of the teeth shown in the rack are formed of straight lines.

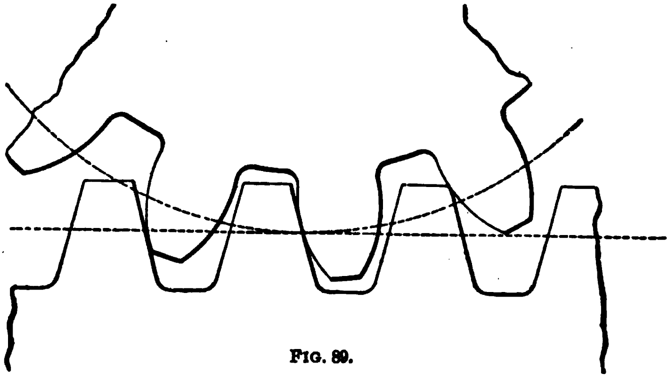

FIG. 89.

Involute teeth have two great advantages over epicycloidal teeth: 1. They are stronger for the same pitch, as they are thicker at the root. 2. They may be spread apart so that their pitch circles do not run tangent to one another without practically affecting the perfect action of the teeth.

511. To calculate the number of teeth or speed of one of two gear-wheels which are to gear together:

Let N = number of revolutions per minute of the driver;
n = number of revolutions per minute of the driven;
T = number of teeth in the driver;
t = number of teeth in the driven.

Rule 89.—*The number of teeth in the driver equals the product of the number of teeth and number of revolutions of the driven divided by the number of revolutions of the driver.*

That is, $$T = \frac{t\,n}{N}.$$

Example.—The driven has 27 teeth, and will make 66 revolutions per minute; if the driver makes 99 revolutions per minute, how many teeth are there in the driver?

Solution.—Formula, $$T = \frac{tn}{N}.$$

Substituting, we have $T = \frac{27 \times 66}{99} = 18$ teeth. Ans.

The number of revolutions per minute of the driver and driven and the number of teeth in the driver being given, to find the number of teeth in the driven:

Rule 90.—*The number of teeth in the driven equals the product of the number of teeth and revolutions per minute of the driver divided by the number of revolutions per minute of the driven.*

That is, $$t = \frac{TN}{n}.$$

Example.—The driver has 18 teeth, and makes 99 revolutions per minute, and the driven must make 66 revolutions per minute; how many teeth must there be in the driven?

Solution.—Formula, $t = \frac{TN}{n}$.

Substituting, we have $t = \frac{18 \times 99}{66} = 27$ teeth. Ans.

The number of teeth in the driver and driven and the number of revolutions per minute of the driver being given, to find the number of revolutions per minute of the driven:

Rule 91.—*The number of revolutions per minute of the driven equals the product of the number of teeth and number of revolutions of the driver divided by the number of teeth of the driven.*

That is, $n = \frac{TN}{t}$.

Example.—There are 18 teeth in the driver, and it makes 99 revolutions per minute; how many revolutions per minute will the driven make if it has 27 teeth?

Solution.—Formula, $n = \frac{TN}{t}$.

Substituting, we have $n = \frac{18 \times 99}{27} = 66$ R. P. M. Ans.

The number of teeth in the driver and driven and the number of revolutions per minute of the driven being given, to find the number of revolutions per minute of the driver:

Rule 92.—*The number of revolutions of the driver equals the product of the number of teeth and revolutions of the driven divided by the number of teeth of the driver.*

That is, $N = \frac{tn}{T}$.

Example.—If there are 27 teeth in the driven, and if it makes 66 revolutions per minute, how many revolutions per minute will the driver make if it has 18 teeth?

Solution.—Formula, $N = \frac{tn}{T}$.

Substituting, we have $N = \frac{27 \times 66}{18} = 99$ R. P. M. Ans.

Example.—In Fig. 90, the crank-shaft makes 60 revolutions per minute; the governor pulley is 4" in diameter, the bevel gear on the governor pulley shaft has 19 teeth; the bevel gear which meshes with it and drives the governor has 30 teeth. The governor is to make 95 revolutions per minute; what should be the size of the pulley on the crank-shaft?

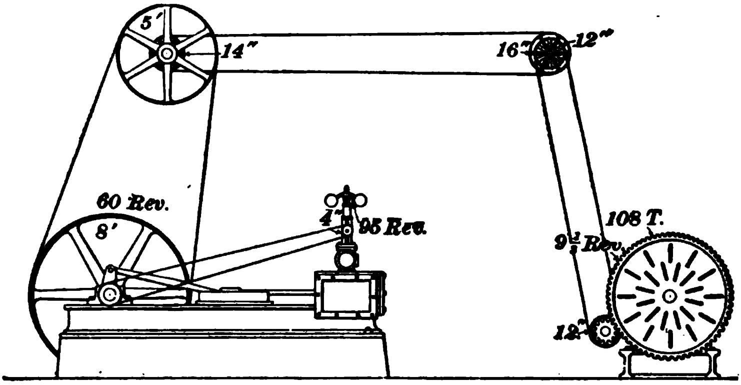

FIG. 90.

SOLUTION.—First determine the number of revolutions of the 4" pulley in order that the governor shall turn 95 times per minute. Applying rule **92**, $N = \frac{tn}{T} = \frac{30 \times 95}{19} = 150$ revolutions of gear on pulley shaft = revolutions of governor pulley. Now, applying rule **81**, the diameter of the pulley on the crank-shaft $= \frac{dn}{N} = \frac{4 \times 150}{60} =$ 10". Ans.

EXAMPLE.—In Fig. 90, the fly-wheel is 8 feet in diameter and drives a 5-foot pulley on the main shaft. A 14" pulley on the main shaft drives a 16" pulley on the countershaft. A 12" pulley on the countershaft drives a 12" pulley on a shaft on which is a pinion that meshes into a large gear attached to the face plate of a large lathe, and which has 108 teeth. How many teeth must the pinion have in order that the face plate may make 9⅓ revolutions per minute?

SOLUTION.—Applying rule **83**, to find the revolutions per minute of the main shaft, $\frac{8 \times 60}{5} = 96$ R. P. M. Applying the same rule again to find the revolutions of the countershaft, $\frac{14 \times 96}{16} = 84$ R. P. M. Applying it once more to find revolutions of the pulley which turns the small gear, $\frac{12 \times 84}{12} = 84$ R. P. M. Applying rule **89**, $\frac{108 \times 9\frac{1}{3}}{84} = 12$ teeth in pinion or driver. Ans.

EXAMPLES FOR PRACTICE.

1. The driving pulley makes 110 R. P. M. and is 21" in diameter; what should be the size of the driven in order to make 385 R. P. M.?
Ans. 6".

2. The main shaft of a certain shop makes 120 R. P. M. It is desired to have the countershaft make 150 R. P. M. There are on hand pulleys of 16", 24", 28", 35", and 38" in diameter. Can two of these be used, or must a new pulley be ordered?
Ans. Use the 28" and 35" pulleys.

3. The pinion (driver) makes 174 R. P. M., and follower makes 24 R. P. M.; how many teeth must the pinion have if the follower has 87 teeth? Ans. 12 teeth.

4. If an engine fly-wheel is 66" in diameter, and makes 160 R. P. M., what must be the diameter of the pulley on the main shaft to make 128 R. P. M.? Ans. 82½".

5. What is the pitch diameter of a gear whose pitch is 1¼", and has 28 teeth? Ans. 11.14".

6. How many teeth are there in a gear whose pitch is .7854", and which is 23" in diameter? Ans. 92 teeth.

7. What is the pitch of a gear whose diameter is 20.372", and which has 128 teeth? Ans. ½".

8. In a train of gears the drivers have 16, 30, 24, and 18 teeth, respectively; the followers have 12, 24, 36, and 40 teeth, respectively. If the first driver makes 80 R. P. M., how many R. P. M. will the last follower make? Ans. 40 R. P. M.

FIXED AND MOVABLE PULLEYS.

512. Pulleys are also used for hoisting or raising loads, in which case the frame which supports the axle of the pulley is called the **block.**

513. A **fixed pulley** is one whose block is not movable, as in Fig. 91. In this case if the weight W be lifted by pulling down P, the other end of the cord W will evidently move the same distance upwards that P moves downwards; hence, P must equal W.

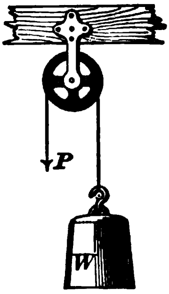

FIG. 91.

514. A **movable pulley** is one whose block is movable, as in Fig. 92. One end of the cord is fastened to the beam, and the weight is suspended from the pulley, the other end of the cord being drawn up by the application of a force P. A little consideration will show that if P moves through a certain distance, say 1 foot, W will move through *half* that distance, or 6 inches; hence, a pull of one pound at P will lift 2 pounds at W.

The same would also be true if the free end of the cord were passed over a *fixed pulley*, as in Fig. 93, in which case the fixed pulley merely changes the direction in which P acts, so that a weight of 1 pound hung on the free end of the cord will balance 2 pounds hung from the *movable pulley*.

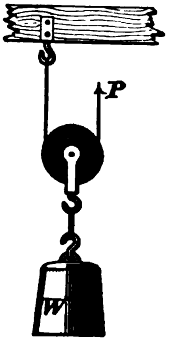

FIG. 92.

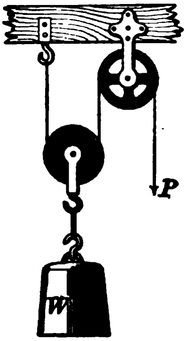

FIG. 93.

515. A combination of pulleys, as shown in Fig. 94, is sometimes used. In this case there are three movable and three fixed pulleys, and the amount of movement of W, owing to a certain movement of P, is readily found.

It will be noticed that there are *six parts* of the rope, not counting the free end; hence, if the movable block be lifted 1 foot, P remaining in the same position, there will be 1 foot of slack in each of the six parts of the rope, or *six feet* in all. Therefore, P would have to move 6 feet in order to take up this slack, or P moves 6 times as far as W. Hence, 1 pound at P will support 6 pounds at W, since the *power multiplied by the distance through which it moves equals the weight multiplied by the distance through which it moves.* It will also be noticed that there are three movable pulleys, and that $3 \times 2 = 6$.

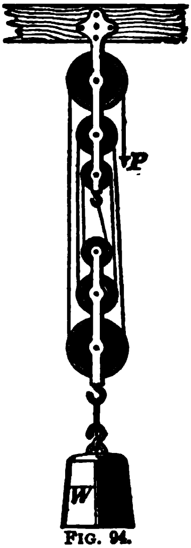

FIG. 94.

516. Law of Combination of Pulleys:

Rule 93.—*In any combination of pulleys where one continuous rope is used, a load on the free end will balance a weight on the movable block as many times as great as itself as there are parts of the rope supporting the load, not counting the free end.*

The above law is good, whether the pulleys are side by side, as in the ordinary block and tackle, or whether they are arranged as in the figure.

EXAMPLE.—In a block and tackle having five movable pulleys, how great a force must be applied to the free end of the rope to raise 1,250 pounds?

SOLUTION.—Since there are five movable pulleys, there must be 10 parts of the rope to support them. Hence, according to the above law, a force applied to the free end will support a load 10 times as great as itself, or the force $= \frac{1,250}{10} = 125$ lb. Ans.

THE INCLINED PLANE.

517. An **inclined plane** is a slope, or a flat surface, making an angle with a horizontal line.

Three cases may arise in practice with the inclined plane:

1. Where the power acts parallel to the plane, as in Fig. 95.

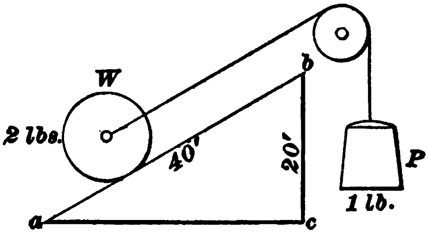

FIG. 95.

2. Where the power acts parallel to the base, as in Fig. 96.

3. Where the power acts at an angle to the plane or to the base, as in Fig. 97.

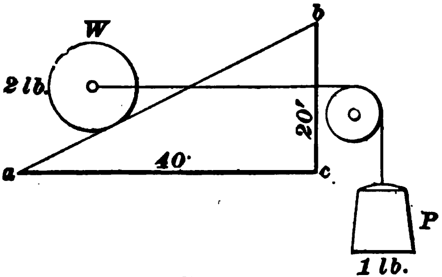

FIG. 96.

518. In Fig. 95 the relation existing between the power and the weight is easily found. The weight ascends a distance equal to $c\,b$, or the height of the inclined plane, while the power descends through a distance equal to $a\,b$, or the length of the inclined plane. Therefore,

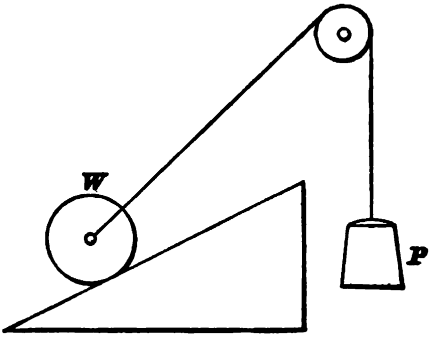

FIG. 97.

Rule 94.—*The power multiplied by the length of the inclined plane equals the weight multiplied by the height of the inclined plane;* or, letting

length of plane $= l$;
base of plane $= b$;
height of plane $= h$;

we have

$$P = \frac{W h}{l} \text{ and } W = \frac{P l}{h}.$$

Example.—The length of an inclined plane is 40 feet, and its height is 5 feet; what force P will sustain a weight W of 100 lb.?

Solution.—$P = \frac{W h}{l} = \frac{100 \times 5}{40} = 12\frac{1}{2}$ lb. Ans.

Example.—The length of an inclined plane is 8 feet, and its height is 15 inches; what weight will 120 lb. support?

Solution.—$W = \frac{P l}{h} = \frac{120 \times 96}{15} = 768$ lb. Ans.

In Fig. 96 the power is supposed to act parallel to the base for any position of W; therefore, while W is moving from the level $a\ c$ to b, or through the height $c\ b$ of the inclined plane, P will move through a distance equal to the length of the base $a\ c$. Hence, when the power acts parallel to the base, we have

Rule 95.—*The power multiplied by the base of the inclined plane equals the weight multiplied by the height of the plane,*

whence, $P = \frac{W h}{b}$ and $W = \frac{P b}{h}$.

Example.—With a base 80 feet long, and a height of 6 feet, what power will sustain a weight of 75 lb.?

Solution.—$P = \frac{75 \times 6}{80} = 15$ lb. Ans.

Example.—A force of 12 lb. sustains a weight on a plane whose base is 6 ft. long, and height 18 inches. Find the weight.

Solution.—$W = \frac{12 \times 6}{1\frac{1}{2}} = 48$ lb. Ans.

For Fig. 97 no rule can be given. The ratio of the power to the weight must be determined by trigonometry for every position of W; for, as W shifts its position, the angle that its cord makes with the face of the plane varies, and the magnitude of the force P depends on this angle; the smaller the angle the less is P required to be to support a given weight on a given plane.

519. The **wedge** is a movable inclined plane, and is used for moving a great weight a short distance. A common method of moving a heavy body is shown in Fig. 98. Simultaneous blows of equal force are struck on the heads of the wedges, thus raising the weight W. The laws for wedges are the same as for Case 2 of the inclined plane.

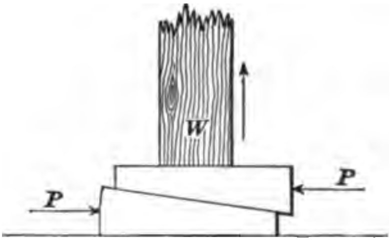

Fig. 98.

THE SCREW.

520. A **screw** is a cylinder with a helical projection winding around its circumference. This helix is called the *thread* of the screw. The distance that a point on the helix is drawn back or advanced in the direction of the length of the screw during one turn, is called the *pitch* of the screw.

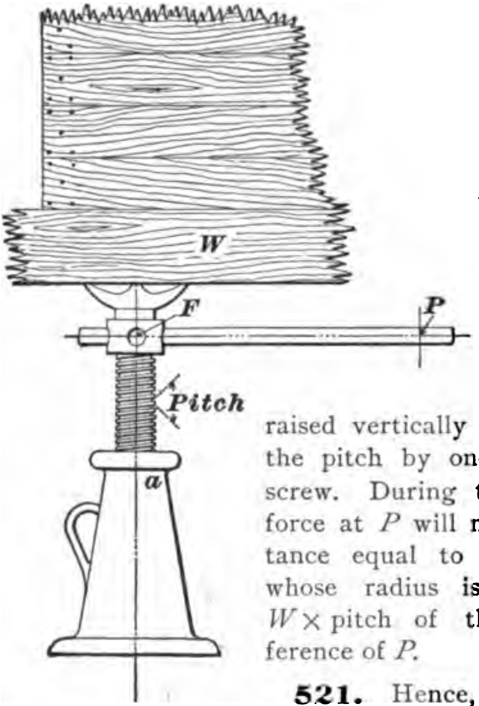

FIG. 99.

The screw in Fig. 99 is turned in a *nut a*, by means of a force applied at the end of the handle *P*. For one complete revolution of the handle, the screw will be advanced lengthwise, *an amount equal to the pitch.* If the nut be fixed, and a weight be placed upon the end of the screw, as shown, it will be raised vertically a distance equal to the pitch by one revolution of the screw. During this revolution, the force at *P* will move through a distance equal to the circumference whose radius is *P F*. Therefore, $W \times$ pitch of thread $= P \times$ circumference of *P*.

521. Hence, to find the weight that a given force will lift, we have

Rule 96.—*Multiply the force by its distance from the axis, or center of screw, and by 6.2832; divide this product by the pitch, and the quotient will be the weight required;*

or

$$W = \frac{P \times 6.2832 \times r}{p},$$

whence,

$$P = \frac{Wp}{6.2832 \times r}.$$

EXAMPLE.—It is desired to raise a weight by means of a screw having 5 threads per inch. The force is 40 pounds, and is applied at a distance of 14 inches from the center of the screw (*FP*, in Fig. 99); how great a weight can be raised?

SOLUTION.—Applying the formula,

$$W = \frac{40 \times 6.2832 \times 14}{\frac{1}{5}} = 17593 \text{ lb., very nearly. Ans.}$$

Example.—The weight to be lifted is 4,000 pounds; the pitch of the screw is $\frac{1}{4}$ (that is, there are 4 threads per inch). The force is applied at a distance of 12 inches from the center of screw; how great must it be?

Solution.—Applying the formula,

$$P = \frac{4,000 \times \frac{1}{4}}{6.2832 \times 12} = \text{rather more than } 13\tfrac{1}{4} \text{ lb., nearly. Ans.}$$

522. Single-threaded screws of less than 1 inch pitch are generally classified by the number of threads they have in 1 inch of their length. In such cases *one inch divided by the number of threads equals the pitch;* thus, the pitch of a screw that has 8 threads per inch is $\frac{1}{8}$; one of 32 threads per inch is $\frac{1}{32}$, etc.

523. Velocity Ratio.—The ratio of the distance through which the power moves to the corresponding distance through which the weight moves is called the **velocity ratio.**

Thus, if the power move through 12 inches while the weight moves through 1 inch, the velocity ratio is 12 to 1, or 12; that is, P moves 12 times as fast as W.

If the velocity ratio be known, the weight which any machine will raise can be found by multiplying the power *by the velocity ratio.* If the velocity ratio is 8.7 to 1, or 8.7, $W = 8.7 \times P$, since $W \times 1 = P \times 8.7$.

Note.—In all of the preceding cases, including the last, the effect of friction has been neglected.

FRICTION.

524. Friction is the resistance that a body meets with from the surface on which it moves.

525. The **ratio** between the *resistance* to the motion of a body due to friction and the *perpendicular* pressure between the surfaces is called the **coefficient of friction.**

If a weight W, as in Fig. 100, rests upon a horizontal plane, and has a cord fastened to it passing over a pulley *a*, from which a weight P is suspended, then, if P is just sufficient to start W, the ratio of P to W, or $\frac{P}{W}$, is the *coefficient of friction* between W and the surface upon which it slides.

The weight W is the perpendicular pressure, and P is the force necessary to overcome the resistance to the motion of W, due to friction.

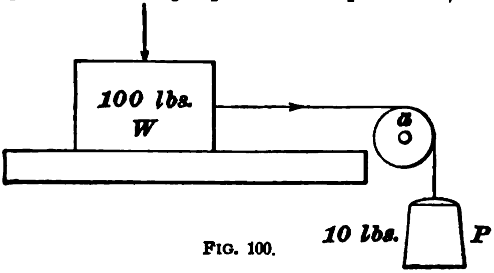

FIG. 100.

If $W = 100$ pounds and $P = 10$ pounds, the coefficient of friction for this particular case would be $\frac{P}{W} = \frac{10}{100} = .1$.

526. Laws of Friction:

1. Friction is directly proportional to the perpendicular pressure between the two surfaces in contact.

2. Friction is independent of the extent of the surfaces in contact when the total perpendicular pressure remains the same.

3. Friction increases with the roughness of the surfaces.

4. Friction is greater between surfaces of the same material than between those of different materials.

5. Friction is greatest at the beginning of motion.

6. Friction is greater between soft bodies than between hard ones.

7. Rolling friction is less than sliding friction.

8. Friction is diminished by polishing or lubricating the surfaces.

527. Law 1 shows why the friction is so much greater on journals after they begin to heat than before. The heat causes the journal to expand, thus increasing the pressure between the journal and its bearing, and, consequently, increasing the friction.

Law 2 states that, no matter how small the surface may be which presses against another, if the perpendicular pressure is the same the friction will be the same. Therefore, large surfaces are used where possible, not to reduce the friction, but to reduce the wear and diminish the liability of heating.

528. For instance, if the perpendicular pressure between a journal and its bearing is 10,000 pounds, and the coefficient of friction is .2, the amount of friction is 10,000 × .2 = 2,000 pounds.

Suppose that one-half the area of the surface of the journal is 80 square inches, then, the amount of friction for each square inch of bearing is 2,000 ÷ 80 = 25 pounds.

If half the area of the surface had been 160 square inches, the friction would have been the same, that is, 2,000 pounds; but the friction per square inch would have been 2,000 ÷ 160 = 12½ pounds, just one-half as much as before, and the wear and liability to heat would be one-half as great also.

TABLE OF COEFFICIENTS OF FRICTION.

TABLE 16.

Description of Surfaces in Contact.	Disposition of Fibers.	State of the Surfaces.	Coefficient of Friction.
Oak on oak.	Parallel	Dry	.48
Oak on oak	Parallel	Soaped	.16
Wrought iron on oak	Parallel	Dry	.62
Wrought iron on oak	Parallel	Soaped	.21
Cast iron on oak	Parallel	Dry	.49
Cast iron on oak	Parallel	Soaped	.19
Wrought iron on cast iron	—	Slightly Unctuous	.18
Wrought iron on bronze	—	Slightly Unctuous	.18
Cast iron on cast iron	—	Slightly Unctuous	.15

529. The power which is required to raise a weight, or overcome an equal resistance in any machine, is thus always *greater than this weight or resistance divided by the velocity ratio of the machine.*

Thus, if there were no friction, a machine whose velocity ratio was 5 would, by an application of 100 pounds of force, raise a weight of 500 pounds.

Now, suppose that the friction in the machine is equivalent to 10 pounds of force; then, it would take 110 pounds of force to raise 500 pounds.

If, in the above illustration, friction were neglected, 110 pounds × 5 = 550 pounds, or the weight that 110 pounds would raise; but, owing to the frictional resistance, it only raised 500 pounds. Therefore, we have for the ratio between the two, $\frac{500}{550} = .91$. That is, 500 : 550 :: .91 : 1.

530. Efficiency.—This ratio between the weight actually raised and the power multiplied by the velocity ratio, is called the **efficiency of the machine.**

For example, if the weight actually raised by a machine, say a screw, is 1,600 pounds, and the power multiplied by the velocity ratio is 2,400 pounds, the efficiency of this machine is

$$\frac{1,600}{2,400} = .66\tfrac{2}{3}, \text{ or } 66\tfrac{2}{3}\%.$$

Example.—In a machine having a combination of pulleys and gears, the velocity ratio of the whole is 9.75. A force of 250 pounds just lifts a weight of 1,626 pounds. What is the efficiency of the machine?

Solution.—Efficiency $= \frac{1,626}{250 \times 9.75} = .6671$, or 66.71%. Ans.

Since the total amount of friction varies with the load, it follows that the efficiency will also vary for different loads.

531. If the efficiency of a machine is known, the force actually required to raise a given load may be found by dividing the load by the product of the velocity ratio of the machine and the efficiency. Thus, if a certain machine has a velocity ratio of 10.6, and its efficiency is 60%, the force which must actually be applied to raise a load of 840 pounds is 840 ÷ 10.6 × .60 = 840 ÷ 6.36 = 132.1 pounds, nearly. If there had been no losses through friction, etc., the force required would have been 840 ÷ 10.6 = 79.25 pounds, nearly.

If the efficiency is known, the weight which a certain force will raise may be found by multiplying together the force, velocity ratio, and the efficiency. Thus, if a certain machine has a velocity ratio of 6½, and an efficiency of 78%, a force of 140 pounds will raise a weight of 140 × 6½ × .78 = 709.8 pounds.

532. When finding the force necessary to overcome the friction, the *perpendicular pressure* on the surface considered must always be taken. In order to find (approximately) the maximum force that is required to overcome the friction between cross-head and guides, we have

Rule 97.—*Multiply the total piston pressure by the length of crank and by the coefficient of friction and divide by the length of main rod;*

or, letting P = total piston pressure;
l = length of main rod;
r = length of crank;
c = coefficient of friction between cross-head and guides;
F = force required to overcome this friction,

we have

$$F = \frac{Prc}{l}.$$

Example.—An engine whose piston is 16" in diameter carries a steam pressure of 80 lb. per sq. in. If the crank is 12" long, and the connecting-rod is 66" long, what is the perpendicular pressure on the guides? The coefficient of friction for this case being 12%, what force will be required to overcome the friction?

Solution.—Pressure on piston = $16^2 \times .7854 \times 80 = 16{,}085$ lb. $\frac{16{,}085 \times 12}{66} = 2{,}924.55$ lb. = perpendicular pressure. Ans. $2{,}924.55 \times .12 = 350.95$ lb. = force required to overcome the friction. Ans.

EXAMPLES FOR PRACTICE.

1. How great a force must be applied to the free end of the rope of a block and tackle which has four movable pulleys, to raise a weight of 746 lb.? Ans. 93¼ lb.

2. An inclined plane is 30 ft. long and 7 ft. high; what force is required to roll a barrel of flour weighing 196 lb. up the plane, friction being neglected? Ans. 45.7⅓ lb.

3. The distance from the axis of a screw to the point on the handle where the force is applied is 12″. The screw has 8 threads per inch. What force is necessary to raise a weight of 1,248 lb.? Ans. 2.07 lb., nearly.

4. In example 3, what should be the length of the handle to raise a weight of 5,400 lb. by the application of a force of 20 lb.? Ans. 5.371″, nearly.

5. What is the velocity ratio (*a*) in example 3? (*b*) In example 4?
Ans. { (*a*) 603, nearly.
(*b*) 270.

6. An engine piston is 24″ in diameter. If the steam pressure is 93 lb. per sq. in.; the length of the connecting-rod, 8 ft. 4″; the length of crank, 20″, and coefficient of friction, 14%, what is (*a*) the greatest perpendicular pressure on the guides? (*b*) The force required to overcome the friction?
Ans. { (*a*) 8,414.46 lb.
(*b*) 1,178 lb.

CENTRIFUGAL FORCE.

533. If a body be fastened to a string and whirled, so as to give it a circular motion, there will be a pull on the string which will be greater or less, according as the velocity increases or decreases. The cause of this pull on the string will now be explained.

Suppose that the body is revolved horizontally, so that the action of gravity upon it will always be the same. According to the first law of motion, a body put in motion tends to move in a straight line, unless acted upon by some other force, causing a change in the direction. When the body moves in a circle, the force that causes it to move in a circle instead of a straight line is exactly equal to the tension of the string. If the string were cut, the pulling

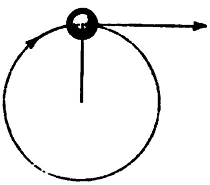

FIG. 101.

force that drew it away from the straight line would be removed, and the body would then "fly off at a tangent;" that is, it would move in a straight line tangent to the circle, as shown in Fig. 101.

Since, according to the third law of motion, every action has an equal and opposite reaction, we call that force which acts as an equal and opposite force to the pull of the string the **centrifugal force,** and it acts *away* from the center of motion.

534. The other force, or tension, of the string is called the **centripetal force,** and it acts *towards* the center of motion. It is evident that these two forces, acting in opposite directions, tend to pull the string apart, and, if the velocity be increased sufficiently, the string will break. It is also evident that no body can revolve without generating centrifugal force.

535. The value of the centrifugal force, expressed in pounds, of any revolving body is calculated by the following rule:

Rule 98.—*The centrifugal force equals the continued product of .00034, the weight of the body in pounds, the radius in feet (taken as the distance between the center of gravity of the body and the center about which it revolves), and the square of the number of revolutions per minute.*

Let F = centrifugal force in pounds;
W = weight of revolving body in pounds;
R = radius in feet of circle described by center of gravity of revolving body;
N = revolutions per minute of revolving body; then

$$F = .00034\ W R N^2.$$

536. In calculating the centrifugal force of fly-wheels, it is the usual practice to consider the rim of the wheel only, and not take the arms and hub of the wheel into account. In this case R would be taken as the *distance between the center of the rim and the center of the shaft.*

Example.—What would be the centrifugal force developed by a cast-iron fly-wheel, whose outside diameter was 10 feet, width of face 20 inches, and thickness of rim 6 inches, turning at the rate of 80 revolutions per minute? Take the weight of a cubic inch of cast iron as .261 lb.

Solution.—First calculate the weight of the rim. The diameter of the rim $= 10 \times 12 = 120$ inches; the diameter of the circle midway between the inside and outside diameters of the rim $= 120 - 6 = 114$ inches. The number of cubic inches in the rim $= 114 \times 3.1416 \times 20 \times 6 = 42{,}977$ cubic inches. $42{,}977 \times .261 = 11{,}217$ pounds = weight.

Radius $= \frac{114}{2} + 12 = 4\frac{3}{4}$ feet. R. P. M. = 80.

Hence, by rule **98**, centrifugal force $= .00034 \times 11{,}217 \times 4\frac{3}{4} \times 80^2 = 115{,}939$ pounds. Ans.

SPECIFIC GRAVITY.

537. The **specific gravity of a body** *is the ratio between its weight and the weight of a like volume of water.*

538. Since gases are so much lighter than water, it is usual to take the specific gravity of a gas as the ratio between the weight of a certain volume of the gas and the weight of the same volume of air.

Example.—A cubic foot of cast iron weighs 450 pounds; what is its specific gravity, a cubic foot of water weighing 62.5 pounds?

Solution.— $\frac{450}{62.5} = 7.2$. Ans.

539. The specific gravities of different bodies are given in printed tables; hence, if it is desired to know the weight of a body that can not be conveniently weighed, *calculate its cubical contents. Multiply the specific gravity of the body by the weight of a like volume of water, remembering that a cubic foot of water weighs 62.5 pounds.*

Example.—How much will 3,214 cubic inches of cast iron weigh? Take its specific gravity as 7.21.

Solution.—Since 1 cubic foot of water weighs 62.5 pounds, 3,214 cubic inches weigh

$$\frac{3{,}214}{1{,}728} \times 62.5 = 116.25 \text{ pounds.}$$

$$116.25 \times 7.21 = 838.16 \text{ pounds. Ans.}$$

Example.—What is the weight of a cubic inch of cast iron?

Solution.— $\frac{62.5}{1,728} \times 7.21 = .2608$ pound. Ans.

Note.—One cubic foot of pure distilled water at a temperature of 39.2° Fahrenheit weighs 62.42 pounds, but the value usually taken in making calculations is 62½ pounds.

Example.—What is the weight in pounds of 7 cubic feet of oxygen?

Solution.—One cubic foot of air weighs .08073 lb., and the specific gravity of oxygen is 1.1056 compared with air; hence,

$$.08073 \times 1.1056 \times 7 = .62479 \text{ pound, nearly.}$$

EXAMPLES FOR PRACTICE.

1. The balls of a steam-engine governor each weigh 5 pounds. If they revolve in a circle whose diameter is 14" at the rate of 80 revolutions per minute, what is the centrifugal force of each ball? Ans. 6.847 lb., nearly.

2. The rim of a cast-iron engine fly-wheel has an outside diameter of 15 feet. If the rim is 8" thick and 12" wide, and the fly-wheel makes 40 R. P. M., what is the centrifugal force of the rim? Ans. 52,785 lb.

3. If a cubic foot of a certain alloy weighs 678 pounds, what is its specific gravity? Ans. 10.848.

4. What is the weight of (*a*) 12.4 cubic inches of lead? (*b*) of steel? (*c*) of aluminum?

$$\text{Ans.} \begin{cases} (a) \ 5.0964 \text{ lb.} \\ (b) \ 3.5216 \text{ lb.} \\ (c) \ 1.116 \text{ lb.} \end{cases}$$

5. The specific gravity of an alloy of lead and zinc is 8.26; what is the weight of a cubic foot? Ans. 516.25 lb.

WORK.

540. Work *is the overcoming of resistance continually occurring along the path of motion.*

Mere motion is not work, but if a body in motion constantly overcomes a resistance, it does work.

541. *The* **measure of work** *is one pound raised vertically one foot*, and is called **one foot-pound.** All work is measured by this standard. A horse going up hill does an amount of work equal to his own weight plus the weight of the wagon and contents plus the frictional resistances reduced to an equivalent weight multiplied by the vertical height of the hill. Thus, if the horse weighs 1,200 pounds, the wagon and contents 1,200 pounds, and the frictional resistances equal 400 pounds, then, if the vertical height of the hill is 100 feet, the work done is equal to $(1,200 + 1,200 + 400) \times 100 = 280,000$ foot-pounds.

Rule 99.—*In all cases the force (or resistance) multiplied by the distance through which it acts equals the work. If a weight be raised, the weight multiplied by the vertical height of the lift equals the work.*

542. The *total* amount of work is independent of time, whether it takes one minute or one year in which to do it; but in order to compare the work done by different machines with a common standard, time must be considered. If one machine does a certain amount of work in 10 minutes, and another machine does exactly the same amount of work in 5 minutes, the second machine can do twice as much work as the first in the same time.

543. The common standard to which all work is reduced is the **horsepower.**

One horsepower *is 33,000 foot-pounds per minute; in other words, it is 33,000 pounds raised vertically one foot in one minute, or 1 pound raised vertically 33,000 feet in one minute, or any combination that will, when multiplied together, give 33,000 foot-pounds in one minute.*

544. Thus, the work done in raising 110 pounds vertically 5 feet in one second is a horsepower; for, since in one minute there are 60 seconds, $110 \times 5 \times 60 = 33,000$ foot-pounds in one minute.

EXAMPLE.—If the coefficient of friction is .3, how many horsepower will it require to draw a load of 10,000 pounds on a level surface, a distance of one mile in one hour?

SOLUTION.—$10,000 \times .3 = 3,000$ pounds = the force necessary to overcome the resistance (resistance of the air is neglected). One mile = 5,280 feet; one hour = 60 minutes. Therefore, $\frac{5,280}{60} = 88$ feet per minute.

Work done = force multiplied by the space = $3,000 \times 88 = 264,000$ foot-pounds per minute.

Horsepower $= \frac{264,000}{33,000} = 8$ horsepower. Ans.

The abbreviation for horsepower is H. P.

545. Energy is a term used to express *the ability of an agent to do work.* Work can not be done without motion, and the work that a moving body is capable of doing in being brought to rest is called the **kinetic energy** of the body.

Kinetic energy means the actual visible energy of a body in motion. The work which a moving body is capable of doing in being brought to rest is exactly the same as the kinetic energy developed by it in falling in a vacuum through a height sufficient to give it the same velocity.

Rule 100.—*The kinetic energy of a moving body in foot-pounds equals its weight in pounds multiplied by the square of its velocity in feet per second, and divided by 64.32.*

If W represents the weight of the body in pounds, and v the velocity in feet per second,

$$\text{kinetic energy} = \frac{Wv^2}{64.32}.$$

If a weight is raised a certain height, a certain amount of work is done equal to the product of the weight and the vertical height. If a weight is suspended at a certain height and allowed to fall, it will do the same amount of work in foot-pounds that was required to raise the weight to the height through which it fell.

EXAMPLE.—If a body weighing 25 pounds falls from a height of 100 feet, how much work can it do?

SOLUTION.—Work $= Wh = 25 \times 100 = 2{,}500$ foot-pounds. Ans.

546. It requires the same amount of work or energy to stop a body in motion within a certain time as it does to give it that velocity in the same time.

EXAMPLE.—A body weighing 50 pounds has a velocity of 100 feet per second; what is its kinetic energy?

SOLUTION.—Kinetic energy $= \frac{Wv^2}{64.32} = \frac{50 \times 100^2}{64.32} = 7{,}773.63$ foot-pounds. Ans.

EXAMPLE.—In the last example, how many horsepower will be required to give the body this amount of kinetic energy in 3 seconds?

SOLUTION.—1 H. P. = 33,000 pounds raised 1 foot in one minute.

If 7,773.63 foot-pounds of work are done in 3 seconds, in one second there would be done $\frac{7,778.65}{3} = 2,591.21$ foot-pounds of work. One horsepower = 33,000 ft.-lb. per min. = 33,000 ÷ 60 = 550 ft.-lb. per sec.

The number of horsepower developed will be

$$\frac{2,591.21}{550} = 4.7113 \text{ H. P.} \quad \text{Ans.}$$

547. Potential energy *is latent energy; it is the energy which a body at rest is capable of giving out under certain conditions.*

If a stone is suspended by a string from a high tower, it has potential energy. If the string is cut, the stone will fall to the ground, and during its fall its potential energy will change into kinetic energy, so that at the instant it strikes the ground its potential energy is wholly changed into kinetic energy.

At a point equal to one-half the height of the fall, the potential and kinetic energies are equal. At the end of the first quarter the potential energy was $\frac{3}{4}$, and the kinetic energy $\frac{1}{4}$; at the end of the third quarter the potential energy was $\frac{1}{4}$, and the kinetic energy $\frac{3}{4}$.

A pound of coal has a certain amount of potential energy. When the coal is burned, the potential energy is liberated and changed into kinetic energy in the form of heat. The kinetic energy of the heat changes water into steam, which thus has a certain amount of potential energy. The steam acting on the piston of an engine causes it to move through a certain space, thus overcoming a resistance, changing the potential energy of the steam into kinetic energy, and thus doing work.

Potential energy *then, is the energy stored within a body which may be liberated and produce motion, thus generating kinetic energy, and enabling work to be done.*

548. The principle of **conservation of energy** teaches that energy, like matter, can never be destroyed. If a clock is put in motion, the potential energy of the spring is changed into kinetic energy of motion, which turns the wheels, thus producing friction.

The friction produces heat, which dissipates into the surrounding air, but still the energy is not destroyed—it merely exists in another form. The potential energy in coal was received from the sun, in the form of heat, ages ago, and has lain dormant for millions of years.

BELTS.

549. A **belt** is a flexible connecting band which drives a pulley by its frictional resistance to slipping at the surface of the pulley. Belts are most commonly made of leather or rubber, and united in long lengths by *cementing*, *riveting*, or *lacing*.

550. Belts are made *single* and *double*. A **single belt** is one composed of a single thickness of leather; a **double belt** is one composed of two thicknesses of leather cemented and riveted together the whole length of the belt.

551. To find the length of a belt:

In practice, the necessary length for a belt to pass around pulleys that are already in their position on a shaft is usually obtained by passing a tape line around the pulleys, the stretch of the tape line being allowed as that necessary for the belt. The lengths of open-running belts for pulleys not in position can be obtained as follows:

Rule 101.—*The length of a belt for open-running pulleys equals $3\frac{1}{4}$ times one-half the sum of the diameters of the pulleys plus 2 times the distance between the centers of the shaft.*

Let $D =$ diameter of one pulley;
$D_1 =$ diameter of other pulley;
$L =$ distance between the centers of the shafts;
$B =$ length of the belt.

Then,
$$B = 3\tfrac{1}{4}\left(\frac{D + D_1}{2}\right) + 2L.$$

Example.—The distance between the centers of two shafts is 9 feet 7 inches; the diameter of the large pulley is 36 inches, and the diameter of the small one is 14 inches; what is the necessary length of the belt?

Solution.—By formula, $B = 3\frac{1}{4}\left(\frac{D + D_1}{2}\right) + 2L.$

Substituting the values given, we have, since 9 ft. 7″ = 115″,

$$B = 3\tfrac{1}{4}\left(\frac{36 + 14}{2}\right) + 2 \times 115 = 311\tfrac{1}{4} \text{ in., or } 25 \text{ ft. } 11\tfrac{1}{4} \text{ in.} \quad \text{Ans.}$$

552. To find the width of a single leather belt that will transmit any given horsepower when equal pulleys are used:

Rule 102.—*The width of the belt in inches equals 800 times the horsepower to be transmitted divided by the speed of the belt in feet per minute.*

Let W = width of belt in inches;
H = horsepower to be transmitted;
S = speed of belt in feet per minute; then,

$$W = \frac{800\,H}{S}.$$

Example.—What width of single leather belt is required to transmit 20 horsepower when equal pulleys are used, and the speed is 1,600 feet per minute?

Solution.—Formula, $W = \frac{800\,H}{S}$.
Substituting, we have

$$W = \frac{800 \times 20}{1{,}600} = 10 \text{ inches. Ans.}$$

553. To find the number of horsepower that a single leather belt will transmit, its width and speed being given:

Rule 103.—*The number of horsepower equals the product of the width in inches and the speed in feet per minute divided by 800.*

Or, $$H = \frac{WS}{800}.$$

Example.—If a 10″ single leather belt is to be run at a speed of 1,600 feet per minute, what horsepower will it transmit?

Solution.—Formula, $H = \frac{WS}{800}$.
Substituting, we have

$$H = \frac{10 \times 1{,}600}{800} = 20 \text{ horsepower. Ans.}$$

554. When the pulleys are of different diameters, the arc of contact must be considered. To find the number of degrees in the arc of contact, *multiply the length of belt in contact on the smaller pulley by 360, and divide the product by the circumference of the pulley, calculating the result to the nearest whole number. The quotient is the arc of contact.*

Having found the arc of contact, *subtract it from 180° and multiply the result by 3. Add this last result to 800; the number thus obtained should be used instead of 800, in rules* ***102*** *and* ***103.***

Example.—What should be the width of a single leather belt to transmit 25.24 horsepower at a speed of 1,500 feet per minute, the diameter of the smaller pulley being 24″, and the belt having 30″ of its length in contact with it?

Solution.—Arc of contact $= \frac{30 \times 360}{24 \times 3.1416} = 143°$. $(180 - 143) \times 3 = 111$. $800 + 111 = 911$. Using rule **102,** and 911 instead of 800,

$$W = \frac{911 \times 25.24}{1{,}500} = 15.33'', \text{ say } 15\tfrac{1}{3}''. \quad \text{Ans.}$$

555. To find the width of a double belt that will transmit the same horsepower as a given single belt, let W_1 represent the width of the double belt; then,

Rule 104.—*Multiply the width of a single belt that will transmit the same horsepower by* $\frac{2}{3}$.

$$\text{Or, } W_1 = \tfrac{2}{3}W.$$

Example.—If a single leather belt is 15″ in width, and transmits 22 horsepower, what must be the width of a double belt to transmit the same horsepower?

Solution.—Applying the rule, $15 \times \frac{2}{3} = 10$ in. = width of double belt. Ans.

556. Lacing Belts.—Many good methods of fastening the ends of belts are employed, but lacing is generally used, as it is flexible, like the belt, and runs noiselessly over the pulleys.

When punching a belt for lacing, use an oval punch, the long diameter of the hole to be parallel with the side of the belt.

In a 3-inch belt there should be four holes in each end, two in each row. In a 6-inch belt, seven holes, four in the row nearest the end. A 10-inch belt should have nine holes, five in the row nearest the end. The edges of the holes should not be nearer than $\frac{3}{4}$ of an inch from the sides, and $\frac{7}{8}$ of an inch from the ends of the belt. The second row should be at least $1\frac{3}{4}$ inches from the end.

Always begin to lace from the center of the belt, and take care to get the ends exactly in line. The lacing should not be crossed on the side of the belt that runs next to the pulley. Always run the hair side of the belt next to the pulley.

HORSEPOWER OF GEARS.

557. To find the horsepower which can be safely transmitted by gears whose face, or breadth of tooth, is from 2½ to 3 times their pitch:

Rule 105.—*The horsepower which can be safely transmitted equals the continued product of the square of the pitch, the velocity in feet per minute and .01.*

Let p = the pitch;

s = circumferential speed of a point on the pitch circle in feet per minute; then,

$$\text{H. P.} = .01\ s p^2.$$

Example.—What horsepower can be safely transmitted by a gear whose pitch diameter is 66.84 in., pitch 1¾ in., and which makes 60 R. P. M.?

Solution.—The velocity which is to be used when applying rule **105** is the circumferential speed of a point on the pitch circle.

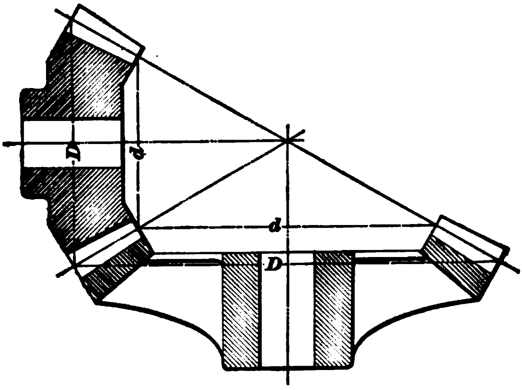

Fig. 102.

Hence, $66.84 \times 3.1416 = 209.98$ in. = circumference of pitch circle = $\frac{209.98}{12}$ ft. $\frac{209.98}{12} \times 60 = 1{,}049.9$ = velocity in ft. per min.

Now, applying formula, $\text{H. P.} = .01 \times 1{,}049.9 \times 1.75^2 = 32.1532$ horsepower. Ans.

558. When measuring bevel gears, the diameter of the largest pitch circle should be taken, as *D*, Fig. 102.

When calculating their horsepower, use the small, or inner, diameter, as *d*, Fig. 102. Either diameter may be used when calculating the revolutions per minute or number of teeth, by rules **86-92,** but if the inner or outer diameter of one gear be used, the corresponding diameter of the other gear which meshes with it must also be used.

EXAMPLES FOR PRACTICE.

1. How many foot-pounds of work are required to overcome for 7 minutes the friction of the cross-head of an engine which has a stroke of 4 ft., and makes 160 strokes per minute, if the coefficient of friction is 8%, and the average perpendicular pressure is 12,460 lb.?
Ans. 4,465,664 ft.-lb.

2. In the above example, what horsepower is required?
Ans. 19.332 H. P.

3. A cannon ball weighing 500 lb. is fired with a velocity of 1,600 ft. per sec.; what is its kinetic energy? Ans. 19,900,497.5 ft.-lb.

4. An open belt drives two pulleys which are, respectively, 42" and 20" in diameter, and 23 ft. apart between their centers; what should be the length of the belt? Ans. 652¼", or 54 ft. 4¼".

5. What width of single leather belting, which has 2 ft. 9" contact on the small pulley, is required to transmit 10 horsepower at a speed of 1,500 ft. per min.? Give width to nearest half inch. Diameter of small pulley 26". Ans. 6".

6. What should be the width of the main belt of a steam engine to transmit 120 horsepower? The engine runs at 80 R. P. M., the band-wheel is 8 ft. in diameter, the belt is double, and has a contact of 6 ft. on the smaller pulley, which is 5 ft. in diameter. Take the speed of the belt the same as that of a point on the circumference of the band-wheel. Ans. 36½".

7. A 26" double belt runs at a speed of 2,830 ft. per min., and has a contact of 5 ft. on the smaller pulley; what horsepower is it transmitting? Diameter of small pulley is 48". Ans. 121.15 H. P.

8. What horsepower can be safely transmitted by a gear whose pitch is 2½", pitch diameter 44.66", and which makes 80 R. P. M.?
Ans. 42.24 H. P.

MECHANICS.

(CONTINUED.)

HYDROSTATICS.

559. **Hydrostatics** treats of liquids at rest under the action of forces.

560. Liquids are very nearly *incompressible.* A pressure of 15 pounds per square inch compresses water less than $\frac{1}{20000}$ of its volume.

561. Fig. 103 represents two cylindrical vessels of exactly the same size. The vessel *a* is fitted with a wooden block of the same size as the cylinder, and can move in it; the vessel *b* is filled with water, whose depth is the same as the length of the wooden block in *a*. Both vessels are fitted with air-tight pistons *P*, whose areas are each 10 sq. in.

Suppose, for convenience, that the weights of the pistons, block, and water be neglected, and that a force of 100 pounds be applied to both pistons. The pressure per square inch will be $\frac{100}{10} = 10$ pounds. In the vessel *a* this pressure will be transmitted to the bottom of the vessel, and will be 10 pounds per square inch; it is easy to see that there will be no pressure on the sides. In the vessel *b* an entirely different result is obtained. The pressure on the bottom will be the same as in the other case, that is, 10 pounds per square inch, but, owing to the fact that the

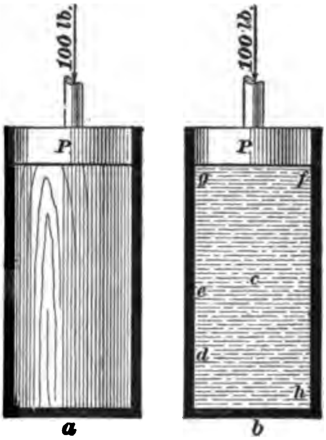

FIG. 103.

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molecules of the water are perfectly free to move, this pressure of 10 pounds per square inch is ***transmitted in every direction with the same intensity;*** that is to say, the pressure at any point *c*, *d*, *e*, *f*, *g*, *h*, etc., due to the force of 100 pounds, is exactly the same, and equals 10 pounds per square inch.

562. This may be easily proven experimentally by means of an apparatus like that shown in Fig. 104. Let the area of the piston *a* be 20 sq. in.; of *b*, 7 sq. in.; of *c*, 1 sq. in.; of *d*, 6 sq. in.; of *e*, 8 sq. in., and of *f*, 4 sq. in.

If the pressure due to the weight of the water be neglected, and a force of 5 pounds be applied at *c* (whose area is 1 sq. in.), a pressure of 5 pounds per square inch will be transmitted in all directions, and in order that there shall be no movement, a force of 6 × 5 = 30 pounds must be applied at *d*, 40 pounds at *e*, 20 pounds at *f*, 100 pounds at *a*, and 35 pounds at *b*.

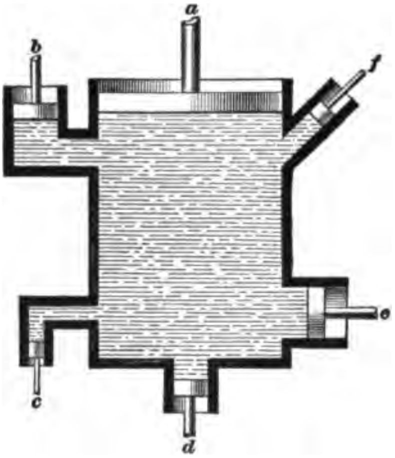


FIG. 104.

If a force of 99 pounds were applied to *a*, instead of 100 pounds, the piston *a* would rise, and the other pistons *b*, *c*, *d*, *e*, and *f*, would move inwards; but, if the force applied to *a* were 100 pounds, they would all be in equilibrium. If 101 pounds were applied at *a*, the pressure per square inch would be $\frac{101}{20} = 5.05$ pounds, which would be transmitted in all directions; and, since the pressure due to the load on *c* is only 5 pounds per square inch, it is now evident that the piston *a* will move downwards, and the pistons *b*, *c*, *d*, *e*, and *f* will be forced outwards.

563. The whole of the preceding may be summed up as follows:

The pressure per unit of area exerted anywhere upon a mass of liquid is transmitted undiminished in all directions, and acts with the same force upon all surfaces, in a direction at **right angles** *to those surfaces.*

This law was first discovered by Pascal, and is the most important in **hydromechanics.** Its meaning should be thoroughly understood.

EXAMPLE.—If the area of the piston *e*, in Fig. 104, were 8.25 sq. in., and a force of 150 pounds were applied to it, what forces would have to be applied to the other pistons to keep the water in equilibrium, assuming that their areas were the same as given before?

SOLUTION.— $\frac{150}{8.25} = 18.182$ pounds per square inch, nearly.

$$\left.\begin{array}{l} 20 \times 18.182 = 363.64 \text{ lb.} = \text{force to balance } a. \\ 7 \times 18.182 = 127.274 \text{ lb.} = \text{force to balance } b. \\ 1 \times 18.182 = 18.182 \text{ lb.} = \text{force to balance } c. \\ 6 \times 18.182 = 109.092 \text{ lb.} = \text{force to balance } d. \\ 4 \times 18.182 = 72.728 \text{ lb.} = \text{force to balance } f. \end{array}\right\} \text{Ans.}$$

564. The pressure due to the weight of a liquid *may be downwards, upwards, or sideways.*

565. Downward Pressure.—In Fig. 105 the pressure on the bottom of the vessel *a* is, of course, equal to the weight of the water it contains. If the area of the bottom of the vessel *b*, and the depth of the liquid contained in it, are the same as in the vessel *a*, the pressure on the bottom of *b* will be the same as on the bottom of *a*. Suppose the bottoms of the vessels *a* and *b* are 6 inches square, and that the part *c d* in the vessel *b* is 2 inches square, and that they are filled with water. Then, the weight of one cubic inch of water is $\frac{62.5}{1,728}$ pound =

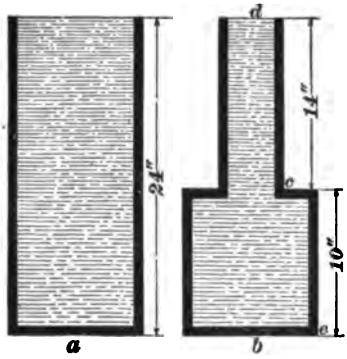


FIG. 105.

.03617 pound. The number of cubic inches in $a = 6 \times 6 \times 24 = 864$ cubic inches; and the weight of the water is $864 \times .03617 = 31.25$ pounds. Hence, the total pressure on the bottom of the vessel a is 31.25 pounds, or 0.868 pound per square inch.

The pressure in b, due to the weight contained in the part $b\ c$ is $6 \times 6 \times 10 \times .03617 = 13.02$ pounds.

The weight of the part contained in $c\ d$ is $2 \times 2 \times 14 \times .03617 = 2.0255$ pounds, and the weight per square inch of area in $c\,d$ is $\frac{2.0255}{4} = .5064$ pound.

566. According to Pascal's law, this weight (pressure) is transmitted equally in all directions, therefore, an extra weight of .5064 pound is imposed on every square inch of the bottom of $b\,c$; the area of this is $6 \times 6 = 36$ square inches, and the pressure on it is, therefore, $36 \times .5064 = 18.23$ pounds, due to water contained in $c\,d$; thus, we have a total pressure on bottom of vessel b of $13.02 + 18.23 = 31.25$ pounds, the same as in vessel a. As a result of the above law, there is also an upward pressure of .5064 pound acting on every square inch of the *top* of the enlarged part $b\,c$.

If an additional pressure of 10 pounds per square inch were applied to the upper surface of both vessels, the total pressure on each bottom would be $31.25 + (6 \times 6 \times 10) = 31.25 + 360 = 391.25$ pounds.

If this pressure were to be obtained by means of a weight placed on each piston (as shown in Figs. 103 and 104), we should have to put a weight of $6 \times 6 \times 10 = 360$ pounds on the piston in vessel a, and one of $2 \times 2 \times 10 = 40$ pounds on the piston in vessel b.

567. The General Law for the Downward Pressure upon the Bottom of any Vessel:

Rule 106.—*The pressure upon the bottom of a vessel containing a fluid is independent of the shape of the vessel, and is equal to the weight of a column of the fluid, the area of whose base is equal to that of the bottom of the vessel, and whose altitude is the distance between the bottom and the upper surface of the fluid plus the pressure per unit of area* *upon the upper surface of the fluid multiplied by the area of the bottom of the vessel.*

568. Suppose that the vessel *b*, in Fig. 105, were inverted, as shown in Fig. 106, the pressure upon the bottom will still be 0.868 pound per square inch, but it will require a weight of 3,490 pounds to be placed upon a piston at the upper surface to make the pressure on the bottom 391.25 pounds, instead of a weight of 40 pounds, as in the other case.

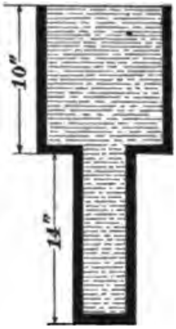


FIG. 106.

EXAMPLE.—A vessel filled with salt water, having a specific gravity of 1.03, has a circular bottom 13 inches in diameter. The top of the vessel is fitted with a piston 3 inches in diameter, on which is laid a weight of 75 pounds; what is the total pressure on the bottom, if the depth of the water is 18 inches?

SOLUTION.—Applying the rule, the weight of 1 cubic inch of the water is $\frac{62.5 \times 1.03}{1,728} = .037254$ lb.

$13 \times 13 \times .7854 \times 18 \times .037254 = 89.01$ pounds = the pressure due to the weight of the water.

$\frac{75}{3 \times 3 \times .7854} = 10.61$ pounds per square inch, due to the weight on the piston.

$$13 \times 13 \times .7854 \times 10.61 = 1,408.29 \text{ pounds.}$$

Total pressure $= 1,408.29 + 89.01 = 1,497.3$ pounds. Ans.

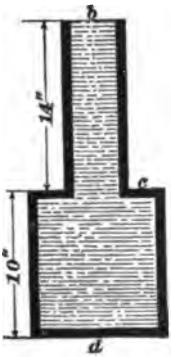


FIG. 107.

569. Upward Pressure.—In Fig. 107 is represented a vessel of exactly the same size as that shown in Fig. 106. There is no upward pressure on the surface *c*, due to the weight of the water in the large part *c d*, but there is an upward pressure on *c*, due to the weight of the water in the small part *b c*. The pressure per square inch, due to the weight of the water in *b c*, was found to be .5064 pound (see Art. **565**), the area of the upper surface *c* of the large part *c d* is $(6 \times 6) - (2 \times 2) = 36 - 4 = 32$ sq. in., and the total upward pressure, due to the weight of the water, is $.5064 \times 32 = 16.2$ pounds.

If an additional pressure of 10 pounds per square inch were applied to a piston fitting in the top of the vessel, the total upward pressure on the surface *c* would be 16.2 + (32 × 10) = 336.2 pounds.

570. General Law for Upward Pressure:

Rule 107.—*The upward pressure on any submerged horizontal surface equals the weight of a column of the liquid whose base has an area equal to the area of the submerged surface, and whose altitude is the distance between the submerged surface and the upper surface of the liquid plus the pressure per unit of area on the upper surface of the fluid multiplied by the area of the submerged surface.*

EXAMPLE.—A horizontal surface 6 inches by 4 inches is submerged in a vessel of water 26 inches below the upper surface; if the pressure on the water is 16 pounds per square inch, what is the total upward pressure on the horizontal surface?

SOLUTION.—Applying the rule, we get 6 × 4 × 26 × .03617 = 22.57 pounds for the upward pressure, due to the weight of the water, and 6 × 4 × 16 = 384 pounds for the upward pressure, due to the outside pressure of 16 pounds per square inch.

Therefore, the total upward pressure = 384 + 22.57 = 406.57 pounds. Ans.

571. Lateral (Sideways) Pressure.—Suppose the top of the vessel shown in Fig. 108 is 10 inches square, and that the projections at *a* and *b* are 1 inch × 1 inch and 10 inches long.

The pressure per square inch on the bottom of the vessel, due to the weight of the liquid, is 1 × 1 × 18 × the weight of a cubic inch of the liquid.

The pressure at a depth equal to the distance of the upper surface *b* is 1 × 1 × 17 × the weight of a cubic inch of the liquid.

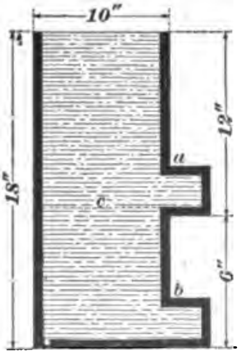


FIG. 108.

Since both of these pressures are transmitted in every direction, they are also transmitted laterally (sideways), and the *pressure per unit of area on the projection b is a mean between the two*, and equals 1 × 1 × 17½ × the weight of a cubic inch of the liquid.

To find the lateral pressure on the projection *a*, imagine that the dotted line *c* is the bottom of the vessel, then the conditions would be the same as in the preceding case, except that the depth is not so great.

The lateral pressure per sq. in. on *a* is thus seen to be 1 × 1 × 11½ × the weight of a cubic inch of the liquid.

572. General Law for Lateral Pressure :

Rule 108.—*The pressure upon any vertical surface, due to the weight of the liquid, is equal to the weight of a column of the liquid whose base has the same area as the vertical surface, and whose altitude is the depth of the center of gravity of the vertical surface below the upper surface of the liquid.*

Any additional pressure is to be added as in the previous cases.

Example.—A well 3 feet in diameter, and 20 feet deep, is filled with water; what is the pressure on a strip of the wall 1 inch wide, the top of which is 1 foot from the bottom of the well? What is the pressure on the bottom? What is the upward pressure per square inch, 2 feet 6 inches from the bottom?

Solution.—Applying the rule, the area of the strip is equal to its length (= circumference of well) multiplied by its height. The length = 36 × 3.1416 = 113.1″; height = 1″; hence, area of strip = 113.1 × 1 = 113.1 sq. in. Depth of center of gravity of strip = (20 − 1) ft. + ½ inch, the half width of strip, = 228½ inches. Consequently, the total pressure on the strip = 113.1 × 228.5 × .03617 = 934.75 lb. Ans.

The pressure on each square inch of the strip would be $\frac{934.75}{113.1} = 8.265$ pounds, nearly.

36 × 36 × .7854 × 20 × 12 × .03617 = 8,836 pounds = the pressure on the bottom. Ans.

20 − 2.5 = 17.5. 1 × 17.5 × 12 × .03617 = 7.596 pounds = the upward pressure per square inch, 2 feet 6 inches from the bottom. Ans.

573. The effects of lateral pressure are illustrated in Fig. 109. In the figure, *e* is a tall vessel having a stop-cock near its base, and arranged to float upon the water, as shown. When this vessel is filled with water, the lateral pressures at any two points of the surface of the vessel opposite to each other are equal. Being equal, and acting in opposite directions, they destroy each other, and no motion can result; but if the stop-cock be opened there will be no resistance to the pressure acting on that part, and the water will flow out; at the same time, the pressure on the corresponding part of the opposite side of the vessel remains, and this, being no longer balanced, causes the vessel to move backwards through the water in a direction opposite to that of the issuing jet.

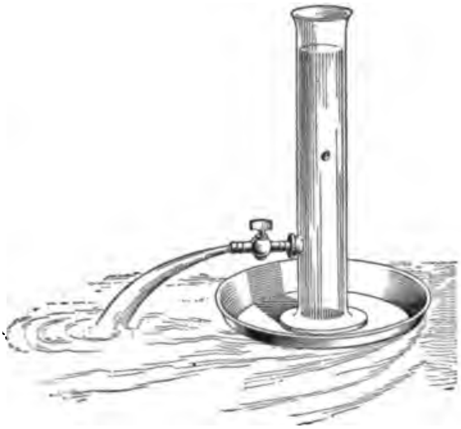


FIG. 109.

574. Since the pressure on the bottom of a vessel, due to the weight of the liquid, is dependent only upon the height of the liquid, and not upon the shape of the vessel, it follows that if a vessel has a number of radiating tubes (see Fig. 110) the water in each tube will be on the same level, no matter

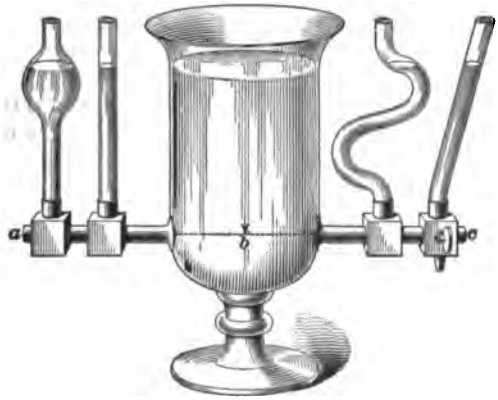


FIG. 110.

what may be the shape of the tubes. For, if the water were higher in one tube than in the others, the downward pressure on the bottom, due to the height of the water in this tube, would be greater than that due to the height of the water in the other tubes. Consequently, the upward pressure would also be greater; the equilibrium would be destroyed, and the water would flow from this tube into the vessel, and rise in the other tubes until it was at the same level in all, when it would be in equilibrium. This principle is expressed in the familiar saying, *water seeks its level.*

575. The above principle explains why city water reservoirs are located on high elevations, and why water on leaving the hose nozzle spouts so high.

If there were no resistance by friction and air, the water would spout to a height equal to the level of the water in the reservoirs. If a long pipe were attached to the nozzle whose length was equal to the vertical distance between the nozzle and the level of the water in the reservoir, the water would just reach the end of the pipe. If the pipe were lowered slightly, the water would trickle out.

Fountains, canal locks, and artesian wells are examples of the application of this principle.

EXAMPLE.—The water level in a city reservoir is 150 feet from the level of the street; what is the pressure of the water per square inch on the hydrant?

SOLUTION.—Apply rule **106,** $1 \times 150 \times 12 \times .03617 = 65.106$ pounds per square inch.

NOTE.—In measuring the height of the water to find the pressure which it produces, the ***vertical*** height, or distance, between the level of the water and the point considered is always taken. This vertical height is called the ***head.***

The weight of a column of water 1 in. square and 1 ft. high is $62.5 \div 144 = .434$ lb., very nearly. Hence, if the depth (head) be given, the pressure per square inch may be found by multiplying the depth in feet by .434. The constant .434 is the one ordinarily employed in practical calculations.

576. In Fig. 111, let the area of the piston *a* be 1 square inch, of *b*, 40 square inches. According to Pascal's law, 1 pound placed upon *a* will balance 40 pounds placed upon *b*.

Suppose that *a* moves downwards 10 inches, then, 10 cubic inches of water will be forced into the tube *b*. This will be distributed over the entire area of the tube *b*, in the form of a cylinder whose cubical contents must be 10 cubic inches, whose base has an area of 40 square inches, and whose altitude must be $\frac{10}{40} = \frac{1}{4}$ of an inch; that is, a movement of 10 inches of the piston *a* will cause a movement of $\frac{1}{4}$ of an inch in the piston *b*.

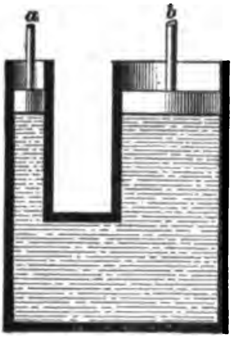


FIG. 111.

Here is the old principle of machines: *The power multiplied by the distance through which it moves equals the weight multiplied by the distance through which it moves.*

Since, if 1 pound on the piston *a* represents the power *P*, the equivalent weight *W* on *b* may be obtained from the equation, $W \times \frac{1}{4} = P \times 10$, whence $W = 40\ P = 40$ lb.

577. Another familiar fact is also recognized, for the velocity ratio of *P* to *W* is $10 : \frac{1}{4}$, or 40; and, since in any machine the weight equals the power multiplied by the velocity ratio, $W = P \times 40$, and when $P = 1$, $W = 40$.

This principle is made use of in the hydraulic press represented in Fig. 112. As the man depresses the lever *O*, he forces down the piston *a* upon the water in the cylinder *A*. The water is forced through the bent tube *d* into the cylinder in which the large ram, or plunger, *C* works, and causes it to rise, thus lifting the platform *K*, and compressing the bales. If the area of *a* be 1 sq. in., and that of *c* be 100 sq. in., the velocity ratio will be 100.

If the length of the lever between the hand and the fulcrum is 10 times the length between the fulcrum and the piston *a*, the velocity ratio of the lever will be 10, and the total velocity ratio of the hand to the piston *C* will be 10 × 100 = 1,000.

Hence, a force of 100 pounds applied by the hand will raise 100 × 1,000 = 100,000 pounds. But, if the average movement of the hand per stroke is 10 inches, it will require $\frac{1,000}{10} = 100$ strokes to raise the platform 1 inch, and it is again seen that what is gained in power is lost in speed.

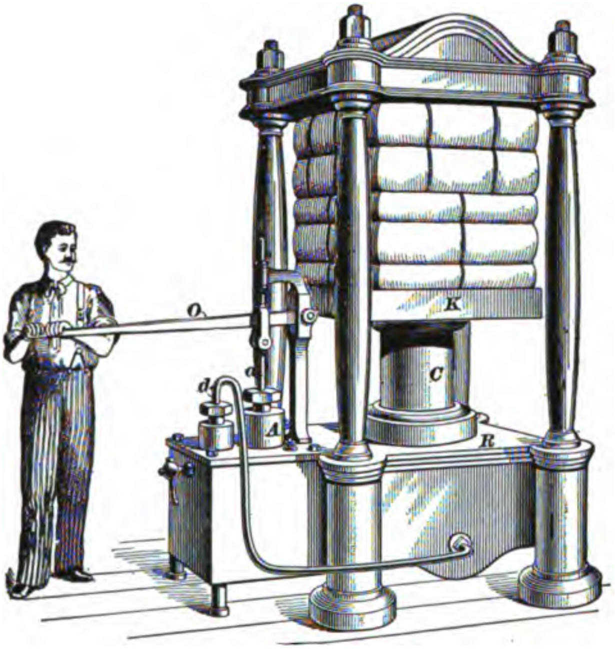


FIG. 112.

578. Applications of this principle are seen in the hydraulic machines used for forcing locomotive drivers on their axles, etc., and for testing the strength of boiler shells.

EXAMPLE.—A suspended vertical cylinder is tested for the tightness of its heads by filling it with water. A pipe whose inside diameter is ¼ of an inch, and whose length is 20 feet, is screwed into a hole in the upper head, and then filled with water. What is the pressure per square inch on each head, if the cylinder is 40 inches in diameter and 60 inches long?

SOLUTION.—Area of heads $= 40^2 \times .7854 = 1,256.64$ sq. in.

Pressure per square inch on the bottom head, due to the weight of the water in the cylinder, $= 1 \times 60 \times .03617 = 2.17$ pounds.

$(\frac{1}{4})^2 \times .7854 = .04909$ sq. in., the area of the pipe.

$.04909 \times 20 \times 12 \times .03617 = .426$ pound = the weight of water in the pipe = the pressure on a surface area of .04909 sq. in.

The pressure per square inch, due to the water in the pipe, is $\frac{1}{.04909} \times .426 = 8.68$ pounds per square inch upon the upper head. Ans.

The total pressure per square inch on the lower head is $8.68 + 2.17 = 10.85$ pounds. Ans.

Example.—In the last example, if the pipe be fitted with a piston weighing ¼ of a pound, and a 5-pound weight be laid upon it, what is the pressure per sq. in. upon the upper head?

Solution.—In addition to the pressure of .426 pound on the area of .04909 sq. in., there is now an additional pressure upon this area of $5 + \frac{1}{4} = 5.25$ pounds, and the total pressure upon this area is $.426 + 5.25 = 5.676$ pounds. The pressure per square inch is $\frac{1}{.04909} \times 5.676 = 115.6$ pounds. Ans.

Example.—In Fig. 113, the plunger *A*, is 10" in diameter, and is forced outwards by means of a small pump *B*, which supplies the press

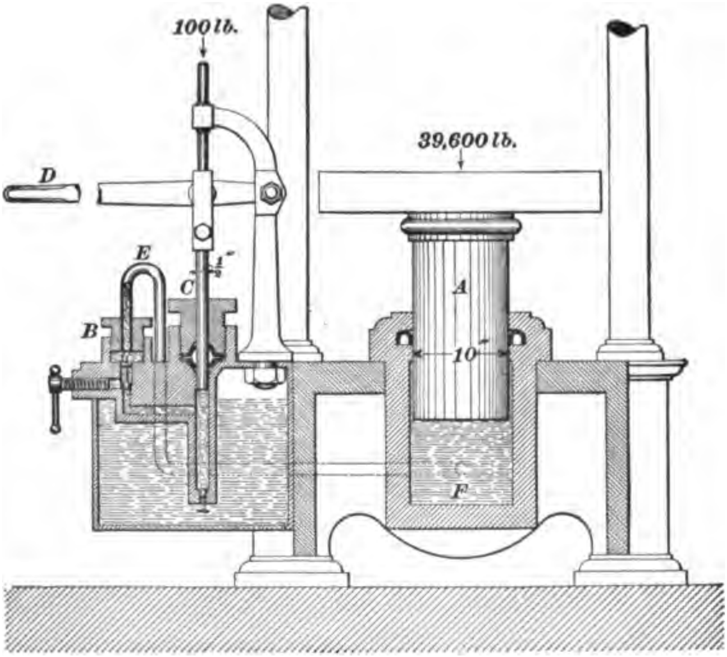


Fig. 113.

cylinder with water. The plunger *C* of the pump *B* is ½" in diameter. If a force of 100 lb. be applied to *C* by means of the lever *D*, how great a weight can the plunger *A* raise, if the plunger itself weighs 400 lb.?

SOLUTION.—First find the pressure per sq. in. which the plunger *C* exerts upon the water. Area of $C = (\frac{1}{2})^2 \times .7854 = .19635$ sq. in. Since *C* exerts a pressure of 100 lb. upon an area of .19635 sq. in., it will exert a pressure of $100 \times \frac{1}{.19635}$ lb. upon an area of 1 sq. in. This pressure is transmitted by the water in the tube *E* to the cylinder *F*, where it forces the plunger *A* upwards as the water is forced into *F*. The pressure per sq. in. on the bottom of *A* is the same as that exerted by *C*. Hence, the total pressure is $10^2 \times .7854 \times \frac{100}{.19635} = 40{,}000$ lb. This result, less the weight of *A*, equals the load lifted. Therefore, $40{,}000 - 400 = 39{,}600$ lb. Ans.

579. In these examples on the hydraulic press, no allowance has been made for the power lost in overcoming the friction between the cup leathers and the plunger; this varies according to the condition of the leathers, and, of course, the smoothness of the plunger; when the leathers are in good condition the loss is about 5% of the total pressure on the ram; when the leathers are old, stiff, and dirty the loss may amount to 15% or more.

BUOYANT EFFECTS OF WATER.

580. In Fig. 114 is shown a 6-inch cube, entirely submerged in water. The lateral pressures are equal and in opposite directions. The upward pressure acting on the lower surface of the cube $= 6 \times 6 \times 21 \times .03617$; the downward pressure acting on the top of the cube $= 6 \times 6 \times 15 \times .03617$, and the difference $= 6 \times 6 \times 6 \times .03617 =$ the volume of the cube in cubic inches × the weight of one cubic inch of water. That is, the upward pressure exceeds the downward pressure by the weight of a volume of water equal to the volume of the body.

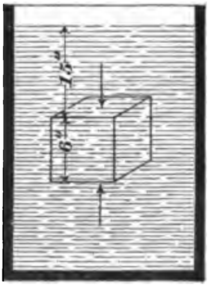


FIG. 114.

This excess of upward pressure over the downward pressure acts against gravity; that is, the water presses the body *up* with a greater force than it presses it *down;* consequently, *if a body is immersed in a fluid, it will lose in weight an amount equal to the weight of the fluid it displaces.* This is called the **principle of Archimedes,** because it was first stated by him.

581. This principle may be experimentally demonstrated with the beam scales, as shown in Fig. 115.

From one scale pan suspend a hollow cylinder of metal *t*, and below that a solid cylinder *a* of the same size as the hollow part of the upper cylinder. Put weights in the other scale pan until they exactly balance the two cylinders. If *a* be immersed in water, the scale pan containing the weights will descend, showing that *a* has lost some of its weight. Now fill *t* with water, and the volume of water that can be poured into *t* will equal that displaced by *a*. The scale pan that contains the weights will gradually rise until *t* is filled, when the scales balance again.

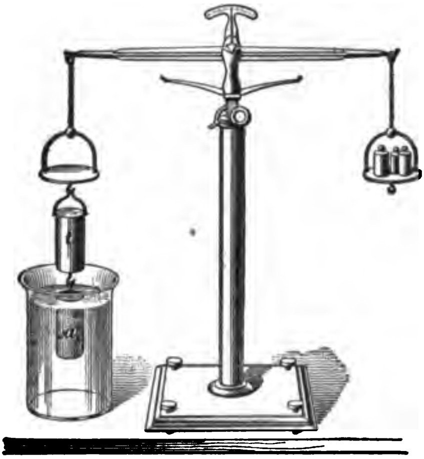


FIG. 115.

If a body be lighter than the liquid in which it is immersed, the upward pressure will cause it to rise, and project partly out of the liquid, until the weight of the body and the weight of the liquid displaced are equal. If the immersed body be heavier than the liquid, the downward pressure plus the weight of the body will be greater than the upward pressure, and the body will fall downwards until it touches bottom or meets an obstruction. If the weights of equal volumes of the liquid and the body are equal, the body will remain stationary, and be in equilibrium in any position or depth beneath the surface of the liquid.

582. An interesting experiment in confirmation of the above facts may be performed as follows: Drop an egg into a glass jar filled with fresh water. The mean density of the egg being a little greater than that of water, it will fall to the bottom of the jar. Now, dissolve salt in the water, stirring it so as to mix the fresh and salt water. The salt water will presently become denser than the egg, and the egg will rise. Now, if fresh water be poured in until the egg and water have the same density, the egg will remain stationary in any position that it may be placed below the surface of the water.

EXAMPLES FOR PRACTICE.

1. Suppose a cylinder to be filled with water and placed in an upright position. If the diameter of the cylinder be 19", and its total length inside be 26", what will be the total pressure on the bottom when a pipe ½" in diameter and 12 ft. long is screwed into the cylinder head and filled with water? The pipe is vertical. Ans. 1,748.2 lb.

2. In the last example, what is the total pressure against the upper head? Ans. 1,476.6 lb.

3. In example 1, a piston is fitted to the upper end of the pipe, and an additional force of 10 lb. is applied to the water in the pipe. (*a*) What is the total pressure on the bottom of the cylinder? (*b*) On the upper head? Ans. $\begin{cases} (a)\ 16{,}184 \text{ lb.} \\ (b)\ 15{,}906 \text{ lb.} \end{cases}$

4. In example 3, what is the pressure per square inch in the pipe 2" from the upper cylinder head? Ans. 56.0656 lb. per sq. in.

5. A water tower 80 feet high is filled with water. A pipe 4" in diameter is so connected to the side of the tower that its center is 8 feet from the bottom. If the pipe is closed by a flat cover, what is the total pressure against the cover? Ans. 420 lb.

6. In the last example, what is the upward pressure per square inch 10 feet from the bottom of the tower? Ans. 30.3828 lb. per sq. in.

7. A cube of wood, one edge of which measures 3 feet, is sunk until the upper surface is 40 feet below the level of the water; what is the total force which tends to move the cube upwards? Ans. 1,687.5 lb.

PNEUMATICS.

583. Pneumatics is that branch of mechanics which treats of the properties of gases.

584. The most striking feature of all gases is their extreme expansibility. *If we inject a portion of gas, however small it may be, into a vessel, it will expand and fill that vessel.* If a bladder or football be partly filled with air, and placed under a glass jar (called a receiver), from which the air has been exhausted, the bladder or football will immediately expand, as shown in Fig. 116. The force which a gas always exerts when confined in a limited space is called **tension.** The word *tension* in this case means pressure, and is only used in this sense in reference to gases.

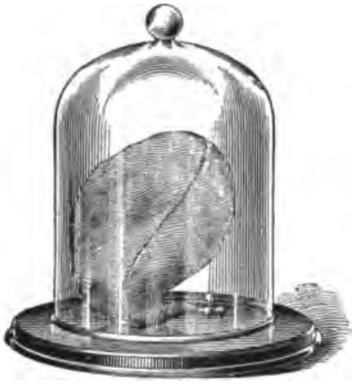


FIG. 116.

585. As *water* is the most common type of fluids so *air* is the most common type of gases. It was supposed by the ancients that air had no weight, and it was not until about the year 1650 that it was proven that air really had weight. A cubic inch of air, under ordinary conditions, weighs .31 grain, nearly. At a temperature of 32° and a pressure of 14.7 pounds per square inch, the ratio of the weight of air to water is about 1 : 774; that is, air is only $\frac{1}{774}$ as heavy as water. In Art. **580,** it was shown that if a body were immersed in water, and weighed less than the volume of water displaced, the body would rise and project partly out of the water. The same is true to a certain extent of air. If a vessel made of light material be filled with a gas lighter than air, so that the total weight of the vessel and gas is less than the air they displace, the vessel will rise. It is on this principle that balloons are made.

586. Since air has weight, it is evident that the enormous quantity of air that constitutes the atmosphere must exert a considerable pressure upon the earth. This is easily proven by taking a long glass tube closed at one end, and filling it with mercury. If the finger be placed over the open end so as to keep the mercury from running out, and the tube inverted and placed in a cup of mercury, as shown in Fig. 117. the mercury will fall, then rise, and after a few oscillations will come to rest at a height above the top of the mercury in the glass equal to about 30 inches. This height will always be the same under the same atmospheric conditions. Now, if the atmosphere has weight, it must press upon the upper surface of the mercury in the glass with equal intensity upon every square unit, except upon that part of the surface occupied by the tube. In order that there may be equilibrium, the weight of the mercury in the tube must be equal to the pressure of the air upon a portion of the surface of the mercury in the glass, equal in area to the inside of the tube. Suppose that the area of the inside of the tube is 1 square inch, then, since mercury is 13.6 times as heavy as water, the weight of the mercury column is $.03617 \times 13.6 \times 30 = 14.7574$ lb. The actual height of the mercury is a little less than 30 inches, and the actual weight of a cubic inch of distilled water is a little less than .03617 lb. When these considerations are taken into account, the average weight of the mercurial

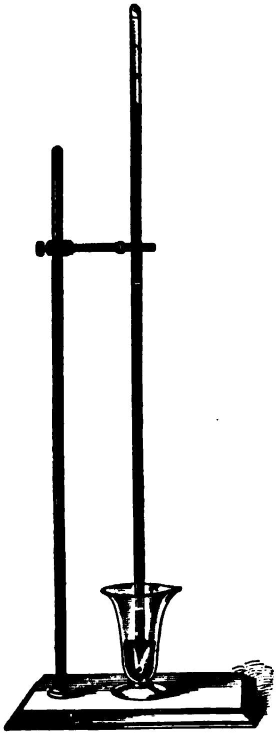


FIG. 117.

column at the level of the sea, when the temperature is 60°, is 14.69 lb., or practically 14.7 lb. Since this weight, when exerted upon 1 square inch of the liquid in the glass, just produces equilibrium, it is plain that the pressure of the outside air is 14.7 lb. upon every square inch of surface.

587. Vacuum.—The space between the upper end of the tube and the upper surface of the mercury is called a **vacuum,** meaning that it is an entirely empty space, and does not contain any substance, solid, liquid, or gaseous. If there were a gas of some kind there, no matter how small the quantity might be, it would expand, filling the space, and its tension would cause the column of mercury to fall and become shorter, according to the amount of gas or air present. The space is then called a **partial vacuum.** If the mercury fell 1 inch, so that the column was only 29 inches high, we should say, in ordinary language, that there were *29 inches of vacuum.* If it fell 8 inches, we should say that there were 22 inches of vacuum; if it fell 16 inches, we should say that there were 14 inches of vacuum, and so on. Hence, when the vacuum gauge of a condensing engine shows 26 inches of vacuum, there is enough air in the condenser to produce a pressure of $\frac{30 - 26}{30} \times 14.7 = \frac{4}{30} \times 14.7 = 1.96$ lb. per sq. in. In all cases where the mercury column is used to measure a vacuum, the height of the column in inches gives the number of inches of vacuum. Were the column only 5″ high, the vacuum would be 5″.

If the tube had been filled with water instead of mercury, the height of the column of water to balance the pressure of the atmosphere would have been $30 \times 13.6 = 408$ inches $= 34$ feet. This means that, if a tube be filled with water, inverted, and placed in a dish of water in a manner similar to the experiment made with the mercury, the height of the column of water will be 34 feet.

588. The **barometer** is an instrument used for measuring the pressure of the atmosphere. There are two kinds in general use—the mercurial barometer and the aneroid barometer. The **mercurial barometer** is shown in Fig. 118. The principle is the same as the inverted tube shown in Fig. 117. In this case the tube and the cup at the bottom are protected by a brass or iron casing. At the top of the tube is a graduated scale which can be read to $\frac{1}{1000}$ of an inch by means of a vernier. Attached to the casing is an accurate thermometer for determining the temperature of the outside air at the time the barometric observation is taken. This is necessary, since mercury expands when the temperature is increased, and contracts when the temperature falls; for this reason a standard temperature is assumed, and all barometer readings are reduced to this temperature. This standard temperature is usually taken at 32° F., at which temperature the height of the mercurial column is 30 inches. Another correction is made for the altitude of the place above sea-level, and a third correction for the effects of capillary attraction.

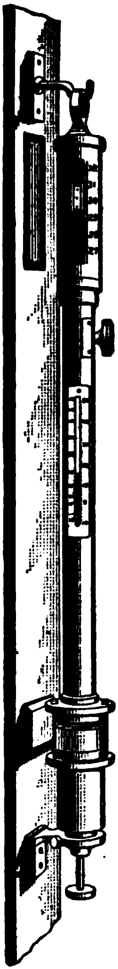


FIG. 118.

589. In Fig. 119 is an illustration of an **aneroid barometer.** These instruments are made in various sizes, from the size of a watch up to 8 or 10 inches in diameter. They consist of a cylindrical box of metal with a top of thin, elastic corrugated metal. The air is exhausted from the box. When the atmospheric pressure increases, the top is pressed inwards, and when it is diminished, the top is pressed outwards by its own elasticity, aided by a spring beneath. These movements of the cover are transmitted and multiplied by a combination of delicate levers, which act upon an index hand, and cause it to move either to the right or left, over a graduated scale. These barometers are self-correcting (compensated) for variations in temperature. They are very portable, occupying but a small space, and are so delicate that they are said to show a difference in the atmospheric pressure when transferred from the table to the floor. The mercurial barometer is the standard.

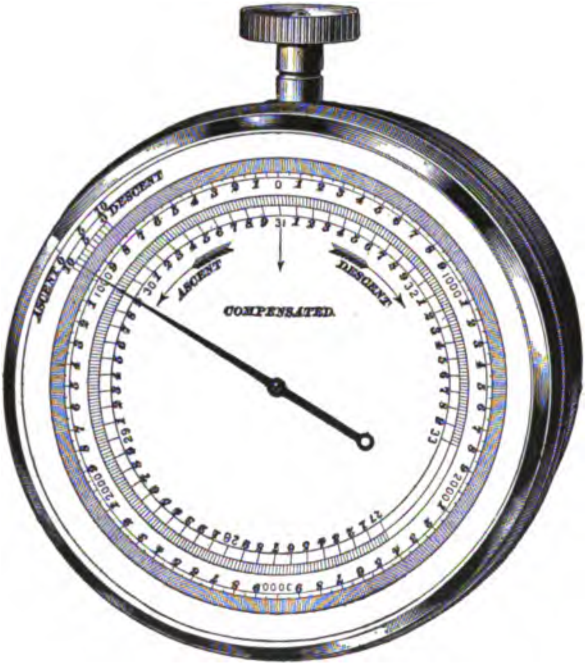


FIG. 119.

590. With air, as with water, the lower we get the greater the pressure, and the higher we get the less the pressure. At the level of the sea, the height of the mercurial column is about 30 inches; at 5,000 feet above the sea, it is 24.7 inches; at 10,000 feet above the sea, it is 20.5 inches; at 15,000 feet, it is 16.9 inches; at 3 miles, it is 16.4 inches, and at 6 miles above the sea-level, it is 8.9 inches.

The density or weight of the atmosphere also varies with the altitude; that is, a cubic foot of air at an elevation of 5,000 feet above the sea-level will not weigh as much as a cubic foot at sea-level. This is proven conclusively by the fact that at a height of 3½ miles the mercurial column measures but 15 inches, indicating that half the weight of the entire atmosphere is below that. It is known that the height of the earth's atmosphere is at least 50 miles; hence, the air just before reaching the limit must be in an exceedingly rarefied state. It is by means of barometers that great heights are measured. The aneroid barometer has the heights marked on the dial, so that they can be read directly. With the mercurial barometer, the heights must be calculated from the reading.

591. The atmospheric pressure is everywhere present, and presses all objects in all directions with equal force. If a book is laid upon the table, the air presses upon it in every direction with an equal average force of 14.7 pounds per square inch. It would seem as though it would take considerable force to raise a book from the table, since, if the size of the book were 8 inches by 5 inches, the pressure upon it would be $8 \times 5 \times 14.7 = 588$ lb.; but there is an equal pressure *beneath* the book which counteracts the pressure on the top. It would now seem as though it would require a great force to open the book, since there are two pressures of 588 pounds each acting in opposite directions and tending to crush the book; so it would, but for the fact that there is a layer of air between each leaf acting upwards and downwards with a pressure of 14.7 pounds per square inch. If two metal plates be made as perfectly smooth and flat as it is possible to get them, and the edge of one be laid upon the edge of the other, so that one may be slid upon the other and thus exclude the air, it will take an immense force, compared to the weights of the plates, to separate them. This is because the full pressure of 14.7 pounds per square inch is then exerted upon each plate, with no counteracting equal pressure between them.

If a piece of flat glass be laid upon a flat surface that has been previously moistened with water, it will require considerable force to separate them; this is because the water helps to fill up the pores in the flat surface and glass, and thus creates a partial vacuum between the glass and the surface, thereby reducing the counter pressure beneath the glass.

592. Tension of Gases.—In Fig. 117, the space above the column of mercury was said to be a vacuum, and it was also said that if any gas or air were present, it would expand, its tension forcing the column of mercury downwards. If sufficient gas be admitted to cause the mercury to stand at 15 inches, the tension of the gas is evidently $\frac{14.7}{2} = 7.35$ lb. per square inch, since the pressure of the outside air, 14.7 lb. per square inch, now balances only 15 instead of 30 inches of mercury; that is, it balances only half as much as it would if there were no gas in the tube; therefore, the pressure (tension) of the gas in the tube is 7.35 pounds. If more gas be forced into the tube until the top of the mercurial column is just level with the mercury in the cup, the gas in the tube will then have a tension equal to the outside pressure of the atmosphere. Suppose that the bottom of the tube is fitted with a piston, and that the total length of the inside of the tube is 36 inches. If the piston be shoved upwards so that the space occupied by the gas is 18 inches long instead of 36 inches, the temperature remaining the same as before, it will be found that the tension of the gas within the tube is 29.4 lb. It will be noticed that the volume occupied by the gas is only half that in the tube before the piston was moved, while the pressure is twice as great, since $14.7 \times 2 = 29.4$ lb. If the piston be shoved up, so that the space occupied by the gas is only 9 inches, instead of 18 inches, the temperature still remaining the same, the pressure will be found to be 58.8 pounds per square inch. The volume has again been reduced one-half, and the pressure increased two-fold, since $29.4 \times 2 = 58.8$ lb. The space now occupied by the gas is 9 inches long, whereas, before the piston was moved, it was 36 inches long; as the tube was assumed to be of uniform diameter throughout its length, the volume is now $\frac{9}{36} = \frac{1}{4}$ of its original volume, and its pressure is $\frac{58.8}{14.7} = 4$ times its original pressure. Moreover, if the temperature of the confined gas remains the same, the pressure and volume will always vary in a similar way.

593. The law which states these effects is called *Mariotte's law*, and is as follows:

Mariotte's Law.—*The temperature remaining the same, the volume of a given quantity of gas varies inversely as the pressure.*

The meaning of the law is this: If the volume of a gas be diminished to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, etc., of its former volume, the tension will be increased 2, 3, 5, etc., times, or if the outside pressure be increased 2, 3, 5, etc., times, the volume of the gas will be diminished to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, etc., of its original volume, the temperature remaining constant. It also means that if a gas is under a certain pressure, and this pressure is diminished to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{10}$, etc., of its original intensity, the volume of the confined gas will be increased 2, 4, 10, etc., times—its tension decreasing at the same rate.

Suppose 3 cubic feet of air to be under a pressure of 60 pounds per square inch in a cylinder fitted with a movable piston, then the product of the volume and pressure is $3 \times 60 = 180$. Let the volume be increased to 6 cubic feet, then the pressure will be 30 pounds per square inch, and $30 \times 6 = 180$, as before. Let the volume be increased to 24 cubic feet; it is then $\frac{24}{3} = 8$ times its original volume, and the pressure is $\frac{1}{8}$ of its original pressure, or $60 \times \frac{1}{8} = 7\frac{1}{2}$ lb., and $24 \times 7\frac{1}{2} = 180$, as in the two preceding cases. It will now be noticed that if a gas be enclosed within a confined space, and allowed to expand without losing any heat, *the product of the pressure and the corresponding volume for any one position of the piston is the same as for any other position.* If the piston were forced inwards so as to compress the air, the same results would be obtained.

594. If the volume of the vessel and the pressure of the gas are known, and it is desired to know the pressure after the first volume has been changed:

Rule 109.—*Divide the product of the first, or original, volume and pressure by the new volume; the result will be the new pressure.*

Or, let p = original pressure;
p_1 = final pressure;
v = volume corresponding to the pressure p;
v_1 = volume corresponding to the pressure p_1.

Then,
$$p_1 = \frac{p\,v}{v_1}.$$

Example.—At the point of cut-off in a steam engine, the amount of steam in the cylinder is 862 cu. in. The pressure at this point is 120 lb. per sq. in. What will be the pressure of the steam when the piston has reached the end of its stroke, and the volume is 1,800 cu. in.?

Solution.—Applying the rule, $\frac{862 \times 120}{1{,}800} = 57.47$ lb. per sq. in. Ans.

595. If it is required to determine the volume after a change in the pressure:

Rule 110.—*Divide the product of the original volume and pressure by the new pressure; the result will be the new volume.*

Or, using the same letters as before,
$$v_1 = \frac{p\,v}{p_1}.$$

Example.—At the commencement of compression, the volume of the steam is 380 cu. in., and the pressure is 18 lb. per sq. in. At the end of compression, the pressure is 112 lb. per sq. in. What is the final volume?

Solution.—Applying the rule, $\frac{380 \times 18}{112} = 61.07$ cu. in. Ans.

Example.—A vessel contains 10 cu. ft. of air at a pressure of 15 lb. per sq. in., and has 25 cu. ft. of air of the same pressure forced into it; what is the resulting pressure?

Solution.—The original volume = 10 + 25 = 35 cu. ft. The original pressure is 15 lb. per sq. in. The final volume is 10 cu. ft. Hence, applying rule **109,** $\frac{35 \times 15}{10} = 52.5$ lb. per sq. in. Ans.

It must be remembered that in the preceding examples the temperature is supposed to remain constant.

EXAMPLES FOR PRACTICE.

1. A vessel contains 25 cubic feet of gas, at a pressure of 18 lb. per sq. in.; if 125 cu. ft. of gas having the same pressure are forced into the vessel, what will be the resulting pressure? Ans. 108 lb. per sq. in.

2. The volume of steam in the cylinder of a steam engine at cut-off is 1.35 cu. ft., and the pressure is 85 lb. per sq. in. The pressure at the end of the stroke is 25 lb. per sq. in. What is the new volume? Ans. 4.59 cu. ft.

3. A receiver contains 180 cu. ft. of gas, at a pressure of 20 lb. per sq. in.; if a vessel holding 12 cu. ft. be filled from the larger vessel until its pressure is 20 lb. per sq. in., what will be the pressure in the larger vessel? Ans. 18⅔ lb. per sq. in.

4. A spherical shell has a part of the air within it removed, forming a partial vacuum; if the outside diameter of the shell is 18", and the pressure of the air within is 5 lb. per sq. in., what is the total pressure tending to crush the shell? Ans. 9,873.42 lb.

PNEUMATIC MACHINES.

596. The Air Pump.—*The air pump is an instrument for removing air from a given space.* A section of the principal parts is shown in Fig. 120, and the complete instrument in Fig. 121. The closed vessel *R* is called the **receiver,** and the space which it encloses is that from which it is desired to remove the air. It is usually made of glass,

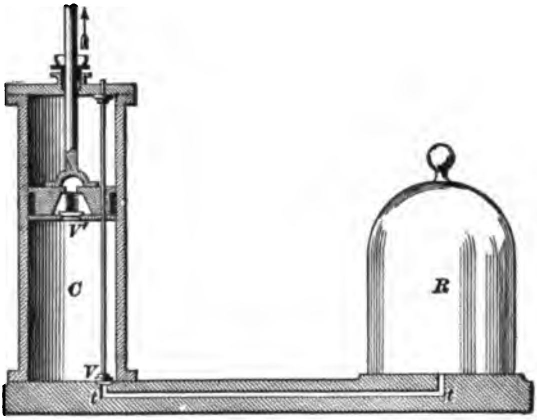


FIG. 120.

and the edges are ground so as to be perfectly air-tight. When made in the form shown, it is called a **bell-jar receiver.** The receiver rests upon a horizontal plate, in the center of which is an opening communicating with the pump cylinder C by means of the passage $t\ t$. The pump piston fits the cylinder accurately, and has a valve V' opening upwards. Where the passage $t\ t$ joins the cylinder, is another valve V, also opening upwards. When the piston is raised, the valve V' closes, and, since no air can get into the cylinder from above, the piston leaves a vacuum behind it. The pressure upon V being now removed, the tension of the air in the receiver R causes V to rise; the air in the receiver and passage $t\ t$ then expands so as to occupy the additional space provided by the upward movement of the piston. The piston is now pushed down, the valve V closes, the valve V' opens, and the air in C escapes. The lower valve V is sometimes supported, as shown in Fig. 120, by a metal rod passing through the piston, and fitting it somewhat tightly. When the piston is raised or lowered, this rod moves with it. A button near the upper end of the rod confines its motion within very narrow limits, the piston sliding upon the rod during the greater part of the journey.

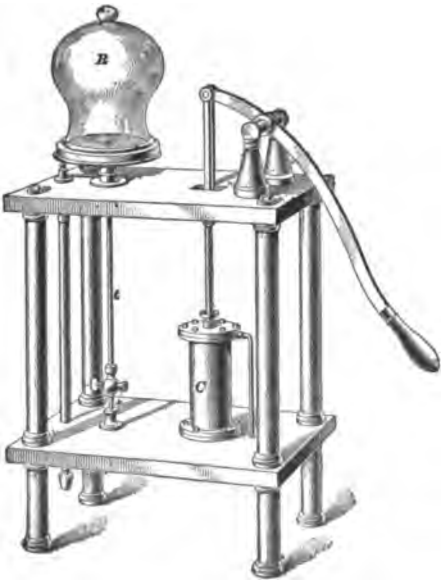


FIG. 121.

In the complete form of the instrument shown in Fig. 121, communication between receiver and pump is made by means of the tube t.

597. Degrees and Limits of Exhaustion.—Suppose that the volume of R and t together is four times that of C, Fig. 121, and that there are say 200 grains of air in R and t, and 50 grains in C when the piston is at the top of the cylinder. At the end of the first stroke, when the piston is again at the top, 50 grains of air in the cylinder C will have been removed, and the 200 grains in R and t will occupy the space R, t, and C. The ratio between the sum of the spaces R and t, and the total space $R + t + C$ is $\frac{4}{5}$; hence, $200 \times \frac{4}{5} = 160$ grains = the weight of air in R and t after the first stroke. After the second stroke, the weight of the air in R and t would be $(200 \times \frac{4}{5}) \times \frac{4}{5} = 200 \times (\frac{4}{5})^2 = 200 \times \frac{16}{25} = 128$ grains. At the end of the third stroke the weight would be $[200 \times (\frac{4}{5})^2] \times \frac{4}{5} = 200 \times (\frac{4}{5})^3 = 200 \times \frac{64}{125} = 102.4$ grains. At the end of n strokes the weight would be $200 \times (\frac{4}{5})^n$. It is evident that *it is impossible to remove all of the air that is contained in R and t by this method.* It requires an exceedingly good air pump to reduce the tension of the air in R to $\frac{1}{50}$ of an inch of mercury. When the air has become so rarefied as this, the valve V' will not lift, and, consequently, no more air can be exhausted.

598. Magdeburg Hemispheres.—By means of the two hemispheres shown in Fig. 122, it can be proven that the atmosphere presses upon a body equally in all directions. They were invented by Otto Von Guericke, of Magdeburg, and are called the *Magdeburg hemispheres.* One of the hemispheres is provided with a stop-cock, by which it can be screwed on to an air pump. The edges fit accurately, and are well greased, so as to be air-tight. As long as the hemispheres contain air, they can be separated with no difficulty; but when the air in the interior is pumped out by means of an

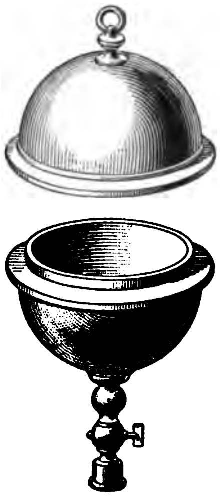


FIG. 122.

air pump, they can be separated only with great difficulty. The force required to separate them will be equal to the area of the largest circle of the hemisphere in square inches multiplied by 14.7 pounds.

This force will be the same in whatever position the hemisphere may be held, thus proving that the pressure of air upon it is the same in all directions.

NOTE.—A theoretically perfect vacuum is sometimes called a ***Torricellian vacuum.***

599. The Weight Lifter.—The pressure of the atmosphere is very clearly shown by means of an apparatus like that illustrated in Fig. 123. Here a cylinder fitted with a piston is held in suspension by a chain. At the top of the cylinder is a plug *A* which can be taken out. This plug is removed, the piston pushed up (the force necessary being equal to the weight of the piston and rod *B*) until it touches the cylinder head. The plug is then screwed in, and the piston will remain at the top until a weight has been hung on the rod equal to the number of pounds obtained by multiplying the number of square inches in the piston by 14.7 pounds, and subtracting therefrom the weight of the piston and rod in pounds.

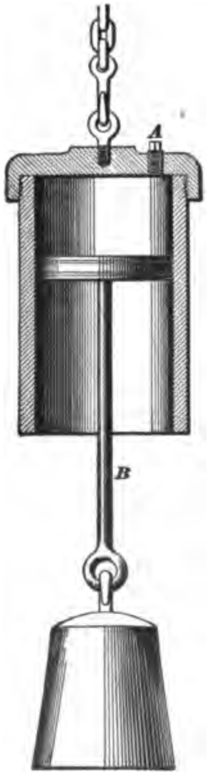


FIG. 123.

If a force sufficiently great were employed to pull the piston downwards, and then any less weight were attached, as shown in Fig. 123, the piston would ascend and carry the weight up with it.

600. Suppose the weight to be removed, and the piston to be supported, say midway of the length of the cylinder. Let the plug be removed, and air admitted above the piston, then screw the plug back into its place; if the piston be shoved upwards, the further up it goes the greater will be the force necessary to push it, on account of the compression of the air. If the piston is of large diameter it will also require a great force to pull it out of the cylinder, as a little consideration will show. For example, let the diameter of the piston be 20 inches, the length of the cylinder 36 inches, and the weight of the piston and rod 100 pounds. If the piston is in the middle of the cylinder, there will be 18 inches of space above it, and 18 inches of space below it. The area of the piston is $20^2 \times .7854 = 314.16$ square inches, and the atmospheric pressure upon it is $314.16 \times 14.7 = 4{,}618$ lb., nearly. In order to shove the piston upwards 9 inches, the pressure upon it must be twice as great, or 9,236 pounds, and to this must be added the weight of the piston and rod, or $9{,}236 + 100 = 9{,}336$ lb. The force necessary to cause the piston to move upwards 9 inches would then be $9{,}336 - 4{,}618 = 4{,}718$ lb. Now, suppose the piston to be moved downwards until it is just on the point of being pulled out of the cylinder. The volume above it will then be twice as great as before, and the pressure one-half as great, or $4{,}618 \div 2 = 2{,}309$ lb. The total upward pressure will be the pressure of the atmosphere less the weight of the piston and rod, or $4{,}618 - 100 = 4{,}518$ lb., and the force necessary to pull it downwards to this point will be $4{,}518 - 2{,}309 = 2{,}209$ lb.

601. Air Compressors.—For many purposes compressed air is preferable to steam or other gas for use as a motive power. In such cases **air compressors** are used to compress the air. These are made in many forms, but the most common one is to place a cylinder, called the **air cylinder,** in front of the cross-head of a steam engine, so that the piston of the air cylinder can be driven by attaching its piston rod to the cross-head, in a manner similar to a steam pump. A cross-section of the air cylinder of a compressor of this kind is shown in Fig. 124, in which *A* is the piston, and *B* is the piston rod, driven by the cross-head of a steam engine, not shown in the figure. Both ends of the lower half of the cylinder are fitted with inlet valves *D* and *D'* which allow the air to enter the cylinder, and both ends of the upper half are fitted with discharge valves *F* and *F'* which allow the air to escape from the cylinder after it has been compressed to the required pressure.

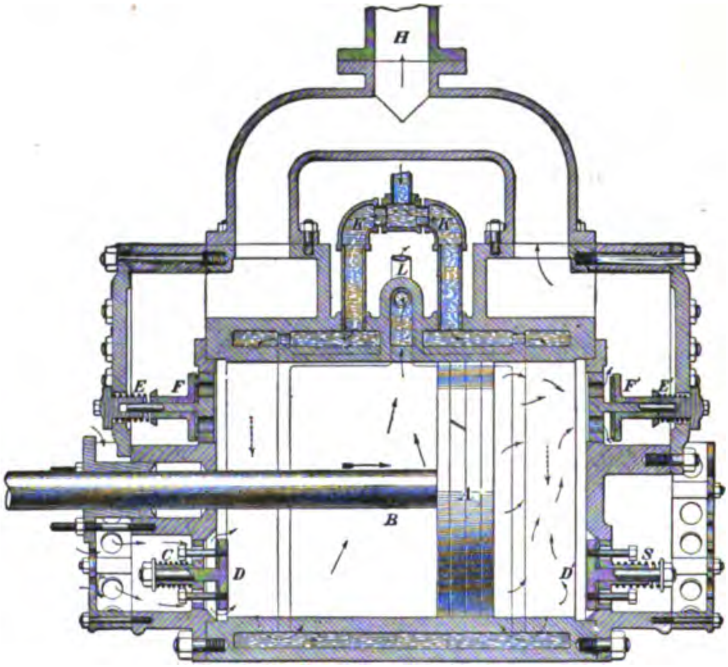


FIG. 124.

602. Suppose the piston *A* to be moving in the direction of the arrow, then the inlet valves *D* in the left-hand end of the cylinder from which the piston is moving will be forced inwards by the pressure of the atmosphere, which overcomes the resistance of a light spring *C*, thus allowing the air to flow in and fill the cylinder. On the other side of the piston, the air is being compressed, and, consequently, it acts with the springs *S* to force the inlet valves *D'* in the right-hand end of the cylinder to their seats. In the right-hand end of the cylinder, the discharge valves *F'* are opened when the pressure of the air in the cylinder is great enough to overcome the resistance of the light springs E' and the tension of the air in the passages leading to the discharge pipe H, and in the left-hand end of the cylinder the discharge valves F are pressed against their seats by the springs E and the tension of the air in the passages. Suppose it is desired to compress the air to 59 pounds per square inch, and we wish to find at what point of the stroke the discharge valves will open. Now, 59 pounds per square inch $= 4$ atmospheres, very nearly; hence, the volume must be $\frac{1}{4}$ of the volume at atmospheric pressure, or the valves will open when the piston has traveled $\frac{3}{4}$ of its stroke.

Considerable heat is developed by the compression of the air; this heat increases the work required to compress the air. The loss of work is reduced by surrounding the cylinder with a **water-jacket,** a hollow space which receives cold water from the pipes K, K to be discharged when heated through the pipe L.

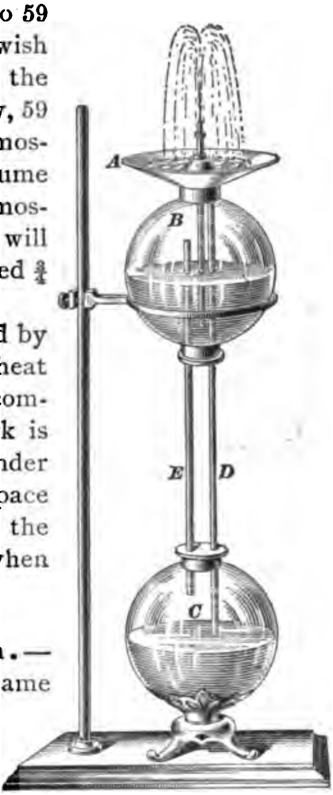


FIG. 125.

603. Hero's Fountain.—Hero's fountain derives its name from its inventor, Hero, who lived at Alexandria 120 B.C.; it is shown in Fig. 125. It depends for its operation upon the elastic properties of air. It consists of a brass dish A and two glass globes B and C. The dish communicates with the lower part of the globe C by means of a long tube D, and another tube E connects the two globes. A third tube passes through the dish A to the lower part of the globe B. This last tube being taken out, the globe B is partially filled with water, the tube is then replaced, and water is poured into the dish. The water flows through the tube *D* into the lower globe, and expels the air, which is forced into the upper globe. The air thus compressed acts upon the water and makes it jet out as represented in the figure. Were it not for the resistance of the atmosphere and friction, the water would rise to a height above the water in the dish equal to the difference of the level of the water in the two globes.

604. The Siphon.—The action of the *siphon* illustrates the effect of atmospheric pressure. It is simply a bent tube of unequal branches, open at both ends, and is used to convey a liquid from a higher point to a lower, over an intermediate point higher than either. In Fig. 126, *A* and *B* are two vessels, *B* being lower than *A*, and *A C B* is the bent tube, or siphon. Suppose this tube to be filled with water and placed in the vessels as shown, with the short branch *A C* in the vessel *A*. The water will flow from the vessel *A* into *B*, as long as the level of the water in *B* is below the level of the water in *A*, and the level of the water in *A* is above the lower end of the tube *A C*. The atmospheric pressure upon the surfaces of *A* and *B* tends to force the water up the tubes *A C* and *B C*. When the siphon is filled with water each of these pressures is counteracted in part by the pressure of the water in that branch of the siphon which is immersed in the water upon which the pressure is exerted. The atmospheric pressure opposed to the weight of the longer column of water will, therefore, be more resisted than that opposed to the weight of the shorter column; consequently, the pressure exerted upon the shorter column

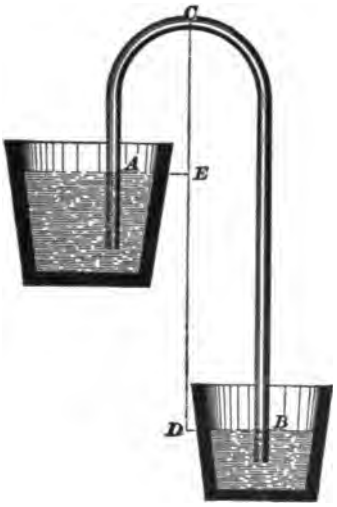


FIG. 126.

will be greater than that upon the longer column, and this excess pressure will produce motion.

605. Let A = the area of the tube;

$h = D\,C$ = the vertical distance between the surface of the water in B, and the highest point of the center line of the tube;

$h_1 = E\,C$ = the distance between the surface of the water in A, and the highest point of the center line of the tube.

The weight of the water in the short column is $.03617 \times A\,h_1$, and the resultant atmospheric pressure tending to force the water up the short column is $14.7 \times A - .03617 \times A\,h_1$. The weight of the water in the long column is $.03617\,A\,h$, and the resultant atmospheric pressure tending to force the water up the long column is $14.7\,A - .03617\,A\,h$. The difference between these two is $(14.7\,A - .03617\,A\,h_1) - (14.7\,A - .03617\,A\,h) = .03617\,A\,(h - h_1)$. But $h - h_1 = E\,D$ = the difference between the levels of the water in the two vessels. In the above, h and h_1 were taken in inches, and A in square inches.

It will be noticed that the short column must not be higher than 34 feet for water, or the siphon will not work, since the pressure of the atmosphere will not support a column of water that is higher than 34 feet.

606. The Injector.—A section of an injector is shown in Fig. 127. There are many different kinds of these instruments, but the principle is the same in all. When they are used for lifting water from a point below the discharge orifice, and forcing it into the boiler of a steam engine or locomotive, they depend for their lifting action upon the creation of a partial vacuum by the action of steam. In the injector, Fig. 127, F is the connection for the steam pipe from the boiler, P is the connection for the pipe from the water supply, N is the connection to which the discharge pipe leading to the boiler is attached, and the waste water and steam are discharged through the **overflow** nozzle O.

607. The method of operation is as follows: The valve *B* is first opened by turning the wheel *W*; the **primer** valve *R* is then opened by the handle *J*, thus permitting steam to flow through the passage *E* and a connection, not shown in the figure, to the nozzle *u*. From *u* the jet of steam rushes out through *O*. A passage connects the chamber surrounding *u* with the space above the valve *L*. The jet of steam from *u* out through *O* carries with it the air in the chamber to which *O* is connected, thus forming a partial vacuum in the space above *L*; the air in the passages *D*, *C*. *G*, *H*, *K*, *T*, and in the water pipe connected at *P* is thus drawn out through the valve *L*, and a partial vacuum is formed which permits the pressure of the atmosphere to force water through *P* until it finally fills the passages and flows out through *L* and the overflow nozzle *O*. As soon as water appears at *O*, the valve *R* is closed and the main steam valve *A* is opened by the wheel *S*, thus admitting steam to the passages *C*, *H*, *K*. This steam draws water from *G* through the opening surrounding *H* and discharges it through *K* with such a high velocity that it rushes past the opening *T* into the nozzle *M* and thence into the boiler.

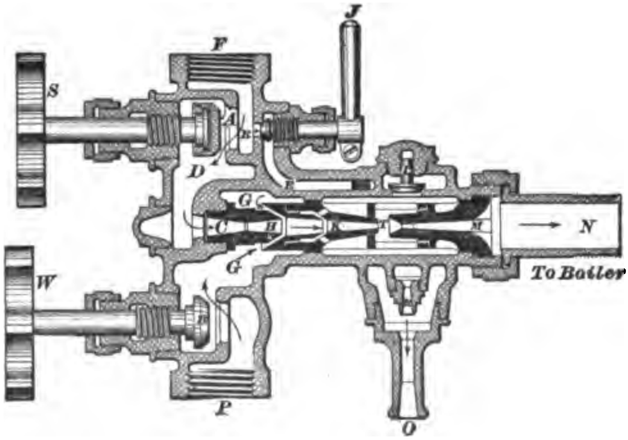


FIG. 127.

PUMPS.

608. The Suction Pump.—A section of an ordinary suction pump is shown in Fig. 128. Suppose the piston to be at the bottom of the cylinder, and to be just on the point of moving upwards in the direction of the arrow. As the piston rises it leaves a vacuum behind it, and the atmospheric pressure upon the surface of the water in the well causes it to rise in the pipe *P* for the same reason that the mercury rises in the barometer tube. The water rushes up the pipe and lifts the valve *V*, filling the empty space in the cylinder *B* displaced by the piston. When the piston has reached the end of its stroke the water entirely fills the space between the bottom of the piston and the bottom of the cylinder, and also the pipe *P*. The instant that the piston begins its down stroke, the water in the chamber *B* tends to fall back into the well, and its weight forces the valve *V* to its seat, thus preventing any downward flow of the water. As the piston descends the water must give way to it, and, since the valve *V* is closed, the valves *u*, *u* must open, and thus allow the water to pass through the piston, as shown in the right-hand figure. When the piston has reached the end of its downward stroke, the weight of the water above closes the valves *u*, *u*. All the water resting on the top of the piston is then lifted with the piston on its upward stroke, and discharged through the spout *A*, the valve *V* again opening, and the water filling the space below the piston as before.

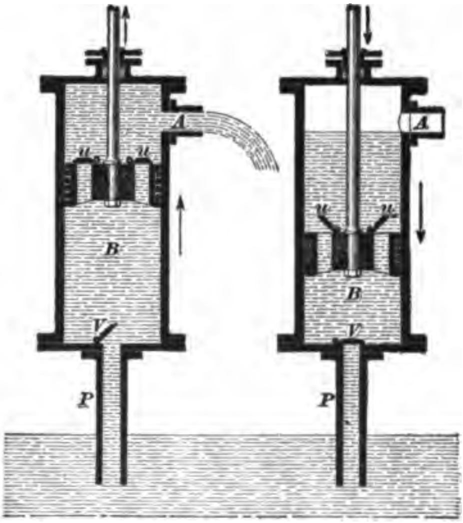


FIG. 128.

609. It is evident that the distance between the piston when at top of its stroke and the surface of the water in the well must not exceed 34 feet, the highest column of water which the pressure of the atmosphere will sustain, since, otherwise, the water in the pipe would not rise up and fill the cylinder as the piston ascended. In practice, this distance should not exceed 28 feet. This is due to the fact that there is a little air left between the bottom of the piston and the bottom of the cylinder, a little air leaks through the valves, which are not perfectly air-tight, and a pressure is needed to raise the valve against its weight, which, of course, acts downwards. There are many varieties of the suction pump, differing principally in the valves and piston, but the principle is the same in all.

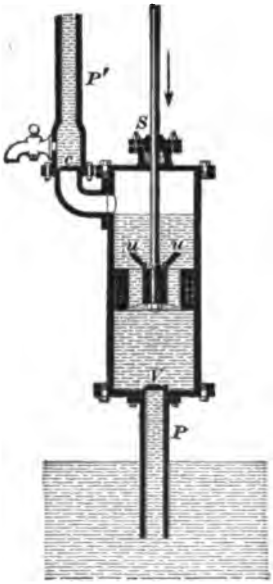


FIG. 129.

610. The Lifting Pump.—A section of a lifting pump is shown in Fig. 129 These pumps are used when water is to be raised to greater heights than can be done with the ordinary suction pump. As will be perceived, it is essentially the same as the pump previously described, except that the spout is fitted with a cock and has a pipe attached to it, leading to the point of discharge. If it is desired to discharge the water at the spout, the cock may be opened; otherwise, the cock is closed, and the water is lifted by the piston up through the pipe *P'* to the point of discharge, the valve *C* preventing it from falling back into the pump, and the valve *V* preventing the water in the pump from falling back into the well.

611. Force Pumps.—The force pump differs from the lifting pump in several important particulars, but chiefly in the fact that the piston is solid; that is, it has no valves. A section of a suction and force pump is shown in Fig. 130. The water is drawn up the suction pipe as before, when the piston rises; but when the piston reverses, the pressure on the water caused by the descent of the piston opens the valve V' and *forces* the water up the delivery pipe P'. When the piston again begins its upward movement, the valve V' is closed by the pressure of the water above it, and the valve V is opened by the pressure of the atmosphere on the water below it, as in the previous cases. For an arrangement of this kind, it is not necessary to have a stuffing-box. The water may be forced to almost any desired height. The force pump differs again from the lifting pump in respect to its piston rod, which should not be longer than is absolutely necessary in order to prevent it from *buckling*, while in the lifting pump the length of the piston rod is a matter of indifference.

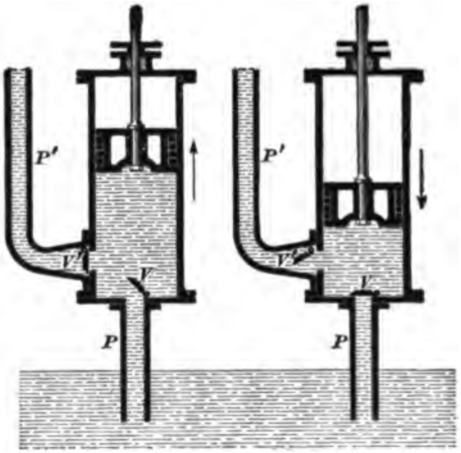


Fig. 130.

612. Plunger Pumps.—When force pumps are used to convey water to great heights, the pressure of the water in the cylinder becomes so great that it becomes extremely difficult to keep the water from leaking past the piston, and the constant repairing of the piston packing becomes a nuisance. To obviate this difficulty the piston is made very long, as shown in Fig. 131, and is then called the **plunger.** The suction valve in this case consists of two clack valves inclined to each other and resting upon a square pin A; they are prevented from flying back too far during the up stroke of the plunger by the two uprights I, I. During the down stroke of the plunger, the valves at *A* are closed and the valve *B* in the delivery pipe is open A little air is always carried into the cylinder of a pump with the entering water. In force pumps this fact becomes a serious consideration, since after repeated strokes the air accumulates, and during the down stroke of the plunger it is compressed. After a time it would become sufficient to entirely prevent the water from entering through the suction valve, the pressure on the top of the valve being greater than that of the atmosphere below. In the pump shown in the figure, the plunger is a trifle smaller than the cylinder, and the air collects around the plunger below the stuffing-box. To remove this air a narrow passage *C* (shown by the dotted lines), that can be closed at its upper end by the cock *D*, connects the interior of the pump with the atmosphere when the cock is open. It is evident that this cock must not be opened except during the down stroke of the plunger, for, if it were open during the up stroke, the pressure below the plunger being less than the pressure of the atmosphere above, the air would rush in instead of being expelled.

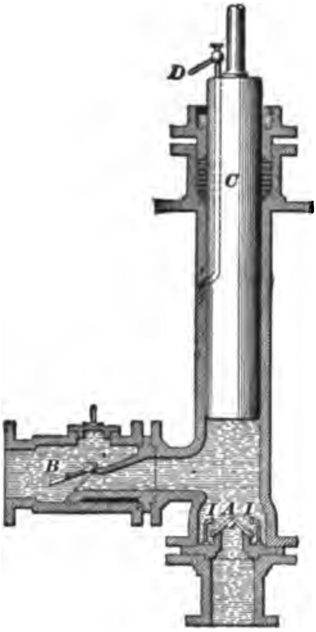


FIG. 131.

AIR CHAMBERS.

613. In order to obtain a continuous flow of water in the delivery pipe, with as nearly a uniform velocity as possible, an **air chamber** is usually placed on the delivery pipe of force pumps as near to the pump cylinder as the construction of the machine will allow. The air chambers are usually pear-shaped, with the small end connected to the pipe. They are filled with air, which the water compresses during the discharge. During the suction, the air thus compressed expands and acts as an accelerating force upon the moving column of water, a force which diminishes with the expansion of the air and helps to keep the velocity of the moving column more nearly uniform. An air chamber is sometimes placed upon the suction pipe. These air chambers not only tend to promote a uniform discharge, but also to equalize the stresses upon the pump and prevent shocks due to the incompressibility of water. They subserve the same purpose on pumps that a fly-wheel does on the steam engine. Unless the pump moves very slowly, it is absolutely necessary to have an air chamber on the delivery pipe.

STEAM PUMPS.

614. Steam pumps are force pumps operated by steam acting upon the piston of a steam engine directly connected to the pump, and in many cases cast with the pump. A section of a double-acting steam pump, showing the steam and water cylinders, with other details, is illustrated in Fig. 132. Here *G* is the steam piston, and *R* the piston rod,

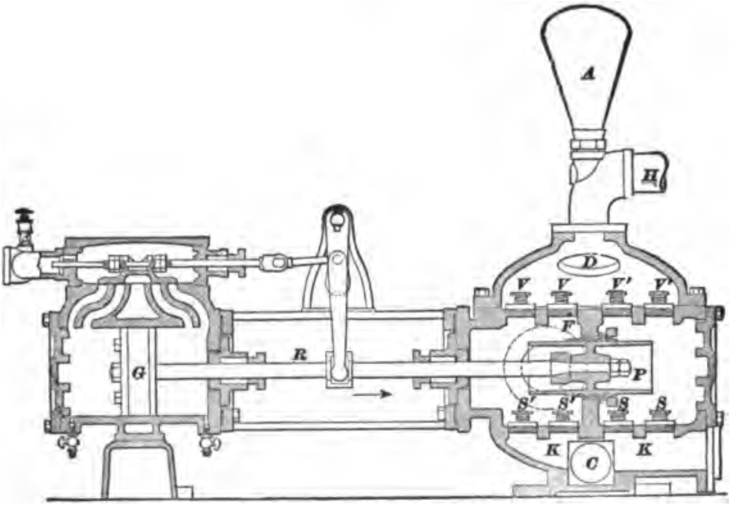


FIG. 132.

which is secured at its other end to the pump plunger *P*. *F* is a partition cast with the cylinder, which prevents the water in the left-hand half from communicating with that in the right-hand half of the cylinder. Suppose the piston to be moving in the direction of the arrow. When the piston has arrived at the end of its stroke, the water space in the left-hand half of the pump cylinder will have been increased by an amount equal to the area of the cross-section of the plunger, multiplied by the length of the stroke, and the volume of the right-hand half of the cylinder will have been diminished by a like amount. In consequence of this, a volume of water in the right-hand half of the cylinder equal to the volume displaced by the plunger in its forward movement will be forced through the valves *V*,' *V*', through the orifice *D*, into the air chamber *A*, and then discharged through the delivery pipe *H*. By reason of the partial vacuum in the left-hand half of the pump cylinder, owing to this movement of the plunger, the water will be drawn from the reservoir through the suction pipe *C* into the chamber *K K*, lifting the valves *S*', *S*' and filling the space displaced by the plunger. During the return stroke the water will be drawn through the valves *S*, *S* into the right-hand half of the pump cylinder, and at the same time water will be discharged from the left-hand half through the valves *V*, *V*, out through the pipe *H*, as before. Each one of the four suction and four discharge valves is kept to its seat, when not working, by light springs, as shown.

615. There are many varieties and makes of steam pumps, the majority of which are double-acting. In many cases two steam pumps are placed side by side, having a common delivery pipe. This arrangement is called a **duplex pump.** It is usual to so set the steam pistons of duplex pumps that when one is completing the stroke, the other is in the middle of its stroke. A double-acting duplex pump made to run in this manner, and having an air chamber of sufficient size, will deliver water with nearly a uniform velocity.

In mine pumps for forcing water to great heights, the plungers are made solid, and in most cases extend through the pump cylinder. In many steam pumps, pistons are used instead of plungers, but when very heavy duty is required, plungers are preferred.

STRENGTH OF MATERIALS.

616. When a force is applied to a body, it changes either its form or its volume. A force, when considered with reference to the internal changes it tends to produce in any solid, is called a **stress.**

Thus, if we suspend a weight of 2 tons by a rod, the stress in the rod is 2 tons. This stress is accompanied by a lengthening of the rod, which increases until the internal stress or resistance is in equilibrium with the external weight.

617. Stresses may be classified as follows:

- Tensile, or pulling stress.
- Compressive, or pushing stress.
- Transverse, or bending stress.
- Shearing, or cutting stress.
- Torsional, or twisting stress.

618. A **unit stress** is the amount of stress on a unit of area, and may be expressed either in pounds per square inch, or in tons per square foot; or, it is the load per square inch or square foot on any body.

Thus, if 10 tons are suspended by a wrought-iron bar which has an area of 5 square inches, the unit stress is 2 tons per square inch, because $\frac{10}{5} = 2$ tons.

619. **Strain** is the deformation or change of shape of a body resulting from stress.

For example, if a rod 100 feet long is pulled in the direction of its length, and if it is lengthened 1 foot, it is strained $\frac{1}{100}$th of its length, or 1 per cent.

620. **Elasticity** is the power which a body has of returning to its original form after the external force on it is withdrawn, providing the stress has not exceeded the elastic limit.

Consequently, we see from this that all material is lengthened or shortened when subjected to either tensile or compressive stress, and the change of the length is directly proportional to the stress, within the elastic limit.

For stresses within the elastic limits, materials are perfectly elastic, and return to their original length on removal of the stresses; but, when their elastic limits are exceeded, the changes of their lengths are no longer regular, and a permanent **set** takes place; the destruction of the material has then begun.

621. The **measure of elasticity** of any material is the change of length under stress within the elastic limit.

622. The **elastic limit** is that unit stress under which the permanent set becomes visible.

The elasticity of wrought iron is practically the same as that of steel; that is, each material will change an equal amount of length under the same stress within the elastic limits.

The elastic limit of steel is higher than that of wrought iron; consequently, the former will lengthen or shorten more than the latter before its elasticity is injured.

TENSILE STRENGTH OF MATERIALS.

623. The tensile strength of any material is the resistance offered by its fibers to being pulled apart.

The tensile strength of any material is proportional to the area of its cross-section.

Consequently, when it is required to find the safe tensile strength of any material, we have only to find the area at the minimum cross-section of the body, and multiply it by its strength per square inch, as given in the following table under the heading "Working Stress."

NOTE.—The minimum cross-section referred to in the above paragraph is that section of the material which is pierced with holes; such as bolt or rivet holes in iron, or knots in wood, if there are any.

624. In the following table is given the average breaking and working tensile stress of different materials:

TABLE 17.

Material.	Breaking Stress in Pounds per Square Inch.	Working Stress in Pounds per Square Inch.
Timber...........	10,000	600 to 1,200
Cast Iron.........	16,000	1,500 to 3,500
Wrought Iron.....	50,000	5,000 to 12,000
Steel..............	70,000	6,000 to 13,000

The above table shows that the tensile breaking strength of cast iron is 16,000 pounds per square inch of cross-section, and that the working strength is from 1,500 to 3,500 pounds per square inch of cross-section.

625. In machinery, such as steam engines, where the parts are subjected to shocks, or are alternately compressed and extended, it is not safe to subject cast iron to a stress of more than 1,500 pounds per square inch of section, wrought iron to more than 5,000 pounds per square inch of section, or steel to more than 6,000 pounds per square inch of section.

But in structures in which the strains are constantly in one direction, as is the case with steam boilers, wrought iron may be strained with from 6,000 to 8,000 pounds per square inch of section, or steel with from 8,000 to 10,000 pounds per square inch of section.

Consequently, strict attention must be given to the *nature of the load* the given structure has to bear, and fix the working stress accordingly.

NOTE.—For structures on which the load is applied suddenly, use the smaller working stresses given in the table, and for those on which the load is applied gradually, use the larger working stresses.

RULES AND FORMULAS FOR TENSILE STRENGTH.

626. Let $W =$ safe load in pounds;

$A =$ area of minimum cross-section;

$S =$ working stress in pounds per square inch, as given in the foregoing table.

Rule 111.—*The working load in pounds for any bar subjected to a tensile stress is equal to the minimum sectional area of the bar, multiplied by the working stress in pounds per square inch, as given in the table.*

That is, $W = A\,S.$

Example.—A bar of good wrought iron which is 3" square is to be subjected to a steady tensile stress; what is the maximum load that it should carry?

Solution.—From what has been said above in regard to the materials and to the nature of the load, it will be safe in this case to use a working stress of 12.000 pounds per square inch.

Applying the rule, we have

$$W = 3 \times 3 \times 12{,}000 = 108{,}000 \text{ pounds. Ans.}$$

Rule 112.—*The minimum sectional area of any bar subjected to a tensile stress should be equal to the load in pounds, divided by the working stress in pounds per square inch, as given in the table.*

That is, $A = \frac{W}{S}.$

Example.—What should be the area of a wrought-iron bar to carry a steady load of 108,000 pounds, if it is to resist a tensile stress of 12,000 pounds per square inch?

Solution.—Applying the rule,

$$A = \frac{108{,}000}{12{,}000} = 9 \text{ sq. in. Ans.}$$

Rule 113.—*The working stress in pounds per square inch is equal to the load in pounds divided by the minimum sectional area of the bar.*

That is, $S = \frac{W}{A}.$

Example.—A bar of wrought iron 3" square, subjected to tensile stress, carries a load of 108,000 pounds; what is the stress per square inch?

Solution.—Applying the rule,

$$S = \frac{108,000}{3 \times 3} = 12,000 \text{ lb. per sq. in.} \quad \text{Ans.}$$

CHAINS.

627. Chains made of the same size iron vary in strength, owing to the different kinds of links from which they are made.

It is a good practice to anneal old chains which have become brittle by overstraining. This renders them less liable to snap from sudden jerks. It reduces their tensile strength, but increases their toughness and ductility, which are sometimes more important qualities.

When annealing, care should be taken that a sufficient heat be applied, otherwise no benefit will be gained; the chains ought to be heated to a cherry red, say 1300° F. at the least.

628. Rules and Formulas for the Strength of Chains:

Let W = safe load in pounds;
D = diameter of the iron, in inches, from which the links are made.

Rule 114.—*The safe load in pounds of a stud-link wrought-iron chain is equal to 18,000, multiplied by the square of the diameter of the iron from which the links are made.*

$$\text{That is, } W = 18,000\, D^2.$$

Example.—What is the maximum load that should be carried by a stud-link wrought-iron chain, if its links are made from ¾-inch round iron?

Solution.—Applying the rule, $W = 18,000\, D^2$.

Substituting the value of D^2, we have $W = 18,000 \times \frac{3}{4} \times \frac{3}{4} = 10,125$ pounds. Ans.

Rule 115.—*The safe load in pounds of a close-link wrought-iron chain is equal to 12,000, multiplied by the square of the diameter of the iron from which the links are made.*

That is, $W = 12{,}000\ D^2$.

Example.—What is the maximum load that should be carried by a close-link wrought-iron chain, if its links are made from ¾-inch round iron?

Solution.—Applying the rule, $W = 12{,}000\ D^2$.

Substituting the value of D^2, we have $W = 12{,}000 \times \frac{3}{4} \times \frac{3}{4} = 6{,}750$ pounds. Ans.

HEMP ROPES.

629. The strength of hemp ropes does not depend so much upon the quality of the material and the cross-section of the rope, as upon the method of manufacture and the amount of twisting.

The ropes in common use are three-strand, shroud-laid rope, and hawser or cable-laid rope.

The strongest ropes are three-strand shroud-laid, made without tar. Ropes made with tar are less flexible, and are reduced in strength about 25 per cent., but have better wearing qualities.

630. Rules and Formulas for the Strength of Hemp Ropes:

Let W = maximum working load in pounds;
C = circumference of rope in inches.

Rule 116.—*The maximum working load in pounds that should be allowed on any hemp rope is equal to the square of the circumference of the rope, multiplied by 100.*

That is, $W = 100\ C^2$.

Example.—What is the maximum load in pounds that should be carried by a hemp rope which has a circumference of 8 inches?

Solution.—Substituting the value of C in the formula, $W = 100 \times 8^2 = 6{,}400$ lb. Ans.

Rule 117.—*The circumference of any hemp rope is equal to the square root of the maximum working load in pounds which it is capable of carrying, multiplied by .1.*

That is, $C = .1 \sqrt{W}$.

Example.—A maximum working load of 1,000 pounds is to be carried by a hemp rope; what should be the circumference of the rope?

Solution.—Applying the rule, $C = .1 \sqrt{1,000} = 3.16''$. Ans.

When measuring ropes, the circumference is sought instead of the diameter, because the ropes are not round and the circumference therefore is not 3.1416 times the diameter. For three strands the circumference is about 2.86 d; for seven strands, 3 d.

631. The above formulas are very convenient for use, and easily remembered, but it is well to remark that the values thus given apply to ropes of a *good average* quality.

WIRE ROPES.

632. Wire rope is made of iron and steel wire. It is stronger than hemp rope, and, to carry the same load, is of smaller diameter.

In substituting steel for iron rope, the object in view should be to gain an increase of wear from the rope, rather than to reduce the size.

A steel rope to be serviceable should be of the best obtainable quality, because ropes made from low grades of steel are inferior to good iron ropes.

633. Formulas for the Strength of Wire Ropes:

Let W = maximum of working load in pounds;
C = circumference of rope in inches.

Rule 118.—*The maximum working load in pounds that should be allowed on any iron wire rope is equal to the square of the circumference of the rope in inches, multiplied by 600.*

That is, $W = 600\ C^2$.

Example.—What is the maximum load in pounds that should be carried by an iron wire rope whose circumference is $4\frac{1}{2}$ inches?

Solution.—Applying the formula,

$$W = 600 \times 4.5^2 = 12,150 \text{ lb. Ans.}$$

Rule 119.—*The circumference of any iron wire rope in inches is equal to the square root of the maximum working load in pounds, multiplied by .0408.*

That is, $C = .0408\sqrt{W}$.

Example.—A maximum working load of 12,150 pounds is to be carried by an iron wire rope; what should be the minimum circumference of the rope?

Solution.—Applying the formula,

$$C = .0408\sqrt{12,150} = 4\tfrac{1}{2} \text{ inches. Ans.}$$

Rule 120.—*The above rules and formulas are also made applicable when computing the safe strength of steel wire rope, by substituting the constant 1,000 for the constant 600, and .0316 for .0408.*

Example.—What is the maximum load in pounds that should be carried by a steel wire rope, the circumference of which is 4½ inches?

Solution.—Applying the rule, $W = 1,000 \times 4.5^2 = 20,250$ lb. Ans.

Example.—A maximum working load of 10,485 pounds is to be carried by a steel wire rope; what should be the minimum circumference of the rope?

Solution.—Applying the rule, $C = .0316\sqrt{10,485} = 3.24$ inches. Ans.

EXAMPLES FOR PRACTICE.

1. What should be the diameter of a steel piston rod of a steam engine to resist tension, if the piston is 19" in diameter and the pressure is 85 lb. per sq. in.? Ans. 2¼", nearly.

2. What safe load will a cast-iron bar of rectangular cross-section 7½" by 3½" support if subjected to shocks? The bar is in tension. Ans. 39,375 lb.

3. What is the stress per sq. in. on a piece of timber 8" square, which is subjected to a steady pull of 60,000 pounds? Ans. 937.5 lb. per sq. in.

4. What should be the safe load for a close-link wrought-iron chain whose links are made from ⅞" iron? Ans. 9,187.5 lb.

5. What safe load may a hemp rope carry whose circumference is 4"? Ans. 1,600 lb.

6. What should be the allowable working load for a steel wire rope whose circumference is 3¾"? Ans. 14,062.5 lb.

7. What should be the circumference of an iron wire rope to support a load of 20,000 lb.? Ans. 5¾", nearly.

CRUSHING STRENGTH OF MATERIALS.

634. The crushing strength of any material is the resistance offered by its fibers to being pushed together.

To obtain only compression, the length of a rod should not be more than 5 times greater than its least diameter, or its least thickness when it is a rectangular rod.

If a bar is long compared with its cross dimensions, the load if sufficiently great will cause it to bend sideways under the compressive force, and we have, then, not only compression, but compression compounded with bending.

Experimental tests on pillars have shown that their strengths are approximately inversely proportional to the squares of their lengths. That is, if there are two pillars of the same material, having the same cross-section, but one is twice as long as the other, the long one will sustain only about one-quarter the load of the short one.

635. Attention should be given to the ends of pillars, as their shape has great influence upon their strength. In Fig. 133 are shown three pillars with differently shaped ends

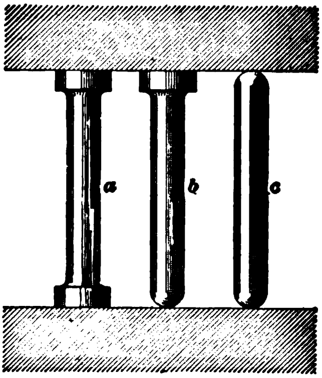


FIG. 133.

It has been proven by the aid of higher mathematics that, theoretically, a pillar having flat or fixed ends, as shown at *a*, is four times as strong as one that has round or movable ends, as shown at *c*, and one and seven-ninths times as strong as one having one flat and one round end, as shown at *b*; *b* is thus two and one-fourth times as strong as *c*. It has also been found that if three pillars, *a*, *b*, *c*, which have the same cross-section, are to carry the same load and be of equal strength, their lengths must be as the numbers 2, 1½, and 1, respectively.

In practice, however, the ends of the pillars *b* and *c* are not generally made as shown by the figure, but have holes at their ends into which pins are fitted which are fastened to some other piece; as, for example, the connecting-rod of an engine. In such cases, it has been found that *a* is two times as strong as *c*, and that *b* is one and one-half times as strong as *c*. That is, in actual practice, a column fixed as at *c* is really $\frac{1}{2}$ as strong as one fixed as at *a*, instead of being only $\frac{1}{4}$ as strong, as given above.

Green or wet timber has only one-half the strength of dry and seasoned timber; consequently, its crushing strength is only one-half of that given in the table below.

636. In the following table is given the mean crushing strength of some short specimens of materials in tons (of 2,000 pounds) per square inch:

TABLE 18.

Materials.	Crushing Strength in Tons per Square Inch.
Cast Iron	40
Wrought Iron	18
Mild Steel	26
Cast Copper	5
Cast Brass	4.5
Timber (dry) endwise	3.5
Brick	1
Stone	3

637. Formula for the Strength of Pillars:

The following formula is applicable to pillars which are commonly used in practice, the lengths of which are about from 10 to 40 times their least diameter, or, if rectangular, their least thickness as indicated by d.

Let C = crushing strength in tons per square inch of a short specimen of the material, as given in the above table;

S = sectional area in square inches;

L = length in inches;

d = least thickness of rectangular pillar, or diameter of round pillar in inches;
W = breaking load in tons;
A = the area of the two flanges;
B = the area of the web;
a = constant, given in one of the following three tables:

TABLE 19.

CONSTANTS FOR WROUGHT-IRON PILLARS.

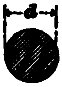
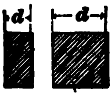
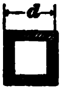
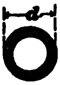
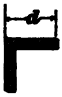
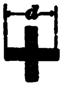
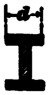
Cross-section of Pillar.	When Both Ends of the Pillar are Flat or Fixed.	When One End of the Pillar is Flat or Fixed, and the Other Round or Movable.	When Both Ends of the Pillar are Round or Movable.
 Round.	2,250	1,500	1,125
 Square or Rectangle.	3,000	2,000	1,500
 Thin Square Tube.	6,000	4,000	3,000
 Thin Round Tube.	4,500	3,000	2,250
 Angle with Equal Sides.	1,500	1,000	750
 Cross with Equal Arms	1,500	1,000	750
 I Beam.	$3{,}000 \times \frac{A}{A+B}$	$2{,}000 \times \frac{A}{A+B}$	$1{,}500 \times \frac{A}{A+B}$

TABLE 20.

CONSTANTS FOR CAST-IRON PILLARS.

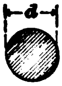
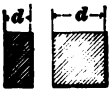
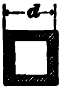
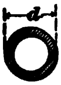
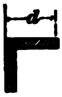
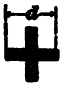
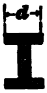
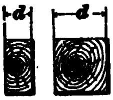
Cross-section of Pillar.	When Both Ends of the Pillar are Flat or Fixed.	When One End of the Pillar is Flat or Fixed, and the Other Round or Movable.	When Both Ends of the Pillar are Round or Movable.
 Round.	281.25	187.5	140.625
 Square or Rectangle.	375.	250.	187.5
 Thin Square Tube.	750.	500.	375.
 Thin Round Tube.	562.5	375.	281.25
 Angle with Equal Sides.	187.5	125.	93.75
 Cross with Equal Arms.	187.5	125.	93.75
 I Beam.	$375 \times \frac{A}{A+B}$	$250 \times \frac{A}{A+B}$	$125 \times \frac{A}{A+B}$

TABLE 21.

CONSTANTS FOR WOODEN PILLARS.

Cross-section of Pillar.	When Both Ends of the Pillar are Flat or Fixed.	When One End of the Pillar is Flat or Fixed, and the Other Round or Movable.	When Both Ends of the Pillar are Round or Movable.
Round.	187.5	125.	93.75
 Square or Rectangle.	250.	166.66	125.
Hollow Square Made of Boards.	500.	333.33	250.

638. Rule 121.—*The breaking load of a pillar in tons is equal to the crushing strength of a short specimen of the material as given in Table 18, Art.* **636**, *multiplied by the sectional area of the pillar in square inches, and the product divided by 1 plus the quotient obtained by dividing the square of the length of the pillar in inches by the square of the diameter (or least thickness, if rectangular) multiplied by the value of a.*

That is,
$$W = \frac{CS}{1 + \frac{L^2}{a d^2}}.$$

The result obtained by the formula must be divided by 6 to get the safe working load.

Note.—If the length of the pillar is given in feet, be sure to reduce it to inches before substituting in the formula.

Example.—A wooden pillar, 6 inches square and 144 inches long, is fixed at both ends; what load will it sustain with safety?

Solution.—By rule **121**, $W = \frac{CS}{1 + \frac{L^2}{a d^2}}$.

Substituting the values of C, S, L^2, a, and d^2, we have

$$W = \frac{3.5 \times 6 \times 6}{1 + \dfrac{144 \times 144}{250 \times 6 \times 6}} = 38.14 \text{ tons, nearly.}$$

Which, divided by 6, gives $\frac{38.14}{6} = 6.357$ tons, or the load it is capable of sustaining with safety. Ans.

EXAMPLE.—A wrought-iron pillar, 4 inches in diameter and 60 inches long, is fixed at one end and movable at the other; what load will it sustain with safety?

SOLUTION.—By the rule,

$$W = \frac{18 \times 4 \times 4 \times .7854}{1 + \dfrac{60 \times 60}{1{,}500 \times 4 \times 4}} = 196.69 \text{ tons.}$$

Which, divided by 6, gives $\frac{196.69}{6} = 32.78$ tons, nearly, or the load it is capable of sustaining with safety. Ans.

EXAMPLE.—What load will a cast-iron pillar sustain with safety, if it is 20 feet long, and if its cross-section is a cross with equal arms, two of which are equal to 10 inches in length (as *d*, see table), and whose arms are 1 inch thick; both ends of the pillar movable.

SOLUTION.—Area of cross-section $= (10 \times 1) + 2(4.5 \times 1) = 19$ square inches; 20 feet = 240 inches.

$$W = \frac{40 \times 19}{1 + \dfrac{240 \times 240}{93.75 \times 10 \times 10}} = 106.38 \text{ tons.}$$

Which, divided by 6, gives $\frac{106.38}{6} = 17.73$ tons, the load it is capable of sustaining with safety. Ans.

639. When using this formula, first obtain the value of C from Table 18, Art. **636.** Next, calculate the area of the cross-section of the pillar. Then, find the value of a from one of the last four tables. Finally, be sure that the length of the pillar has been reduced to inches before substituting in the formula.

To find the proper value of a in any example, first turn to the table dealing with the material in question, and find the figure corresponding to the given cross-section; in the horizontal line containing this are three numbers corresponding to the different conditions of the ends of the column. From these numbers select the one corresponding to the given conditions of the column to be calculated, and this will be the required value of a.

EXAMPLES FOR PRACTICE.

1. What load may be safely carried by a hollow cylindrical cast-iron pillar, 20 ft. long, inside diameter 8", and outside diameter 10"? Both ends of the pillar are fixed. Ans. 93.13 tons.

2. A rectangular wooden column is 14 ft. long, and has one end rounded; if the cross-section is 12" × 8", what load will be required to break it? Ans. 92.15 tons.

3. A solid wrought-iron column, which has both ends movable, is 3" in diameter and 8 ft. long; what load will it safely support? Ans. 11.1 tons.

TRANSVERSE STRENGTH OF MATERIALS.

640. The transverse strength of any material is the resistance offered by its fibers to being broken by bending. As, for example, when a beam, bar, rod, etc., which is supported at its ends, is broken by a force applied between its supports.

The transverse strength of any beam, bar, rod, etc., is proportional to the product of the square of its depth multiplied by its width; consequently, it is more economical to increase the depth than the width.

TABLE 22.

CONSTANTS FOR TRANSVERSE STRENGTH.

Material.	Constant in Pounds.	Material.	Constant in Pounds.
Metals:		Woods:	
Cast Iron	100	Birch	35
Wrought Iron	150	Elm	25
Structural Steel	160	Ash	45
Copper	50	Beech	30
Brass	55	Hickory	50
		Maple	60
		Oak (American)	45
		Pine (Pitch)	40
		Pine (White)	30

641. A **cantilever** is a beam, bar, rod, etc., fixed at one end and subjected to a transverse stress, as shown in

Fig. 134. It has a tendency to overthrow the wall or structure to which it is attached.

The strength of a cantilever varies inversely as the distance of the load from the point of fixing; and the stress upon any section varies directly as the distance of the load from that section.

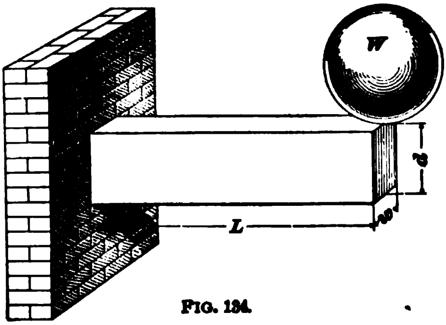


FIG. 134.

The strength of a beam, bar, rod, etc., which has both its ends supported, but not fixed, and which carries a load midway between its supports, is four times that of a beam of the same length, fixed into a wall at one end, and carrying a load at the other, as in Fig. 134.

A cantilever uniformly loaded will sustain twice as great a load as one in which the load is applied at the free end; and a beam resting on two supports and uniformly loaded will sustain twice as great a load as it would if the load were all applied at the middle.

In Table 22 is given the safe transverse strength of bars of different kinds of material, one inch square and one foot long, with the load suspended from one end, the other being fixed, as shown in Fig. 134.

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642. Rules and Formulas for the Transverse Strength of Beams:

Let $d =$ the depth of beam in inches;
$w =$ the width of the beam in inches;
$L =$ the length of the beam between its supports, in feet, or, for cantilever, the distance between load and fixed end;
$S =$ the safe transverse strength, as given in Table 22;
$W =$ the safe load in pounds.

For a rectangular or square cantilever to which the load is applied at one end, as shown in Fig. 134:

Rule 122.—*The maximum safe load in pounds that should be allowed at the end of any rectangular or square cantilever is equal to the square of the depth in inches multiplied by its width in inches multiplied by the constant given in the table, and the product divided by its length in feet.*

That is, $W = \frac{d^2 w S}{L}$.

Example.—What is the maximum safe load that can be placed at one end of a cast-iron bar which projects 4 feet, the depth being 6 inches, and the width 3 inches?

Solution.—By the rule, $W = \frac{d^2 w S}{L}$.

Substituting the values of d, w, S, and L, we have

$$W = \frac{6 \times 6 \times 3 \times 100}{4} = 2{,}700 \text{ pounds. Ans.}$$

643. For a cylindrical cantilever to which the load is applied at one end, as shown in Fig. 135:

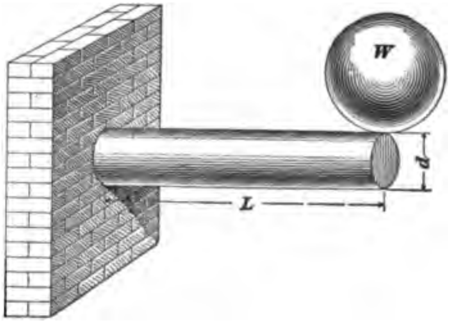

FIG. 135.

Rule 123.—*The maximum safe load in pounds that should be allowed at the end of any cylindrical cantilever is equal to the cube of its diameter in inches multiplied by .6 of the constant given in the table, and the product divided by its length in feet.*

That is, $W = \frac{d^3 \times .6S}{L}$.

Example.—What is the maximum load that can be placed with safety at one end of a cast-iron bar 4 inches in diameter that projects 3 feet?

SOLUTION.—By the rule, $W = \frac{d^3 \times .6S}{L}$.

Substituting the values of d, S, and L, we have

$$W = \frac{4 \times 4 \times 4 \times .6 \times 100}{8} = 1{,}280 \text{ pounds. Ans.}$$

644. When the load is uniformly distributed on a cantilever of any cross-section, as shown in Fig. 136, it will sustain a load twice as great as when the load is applied at one end. For example, if the cantilevers in the two examples above were to carry a uniformly distributed load, they would sustain 2,700 × 2 = 5,400 pounds, and 1,280 × 2 = 2,560 pounds respectively.

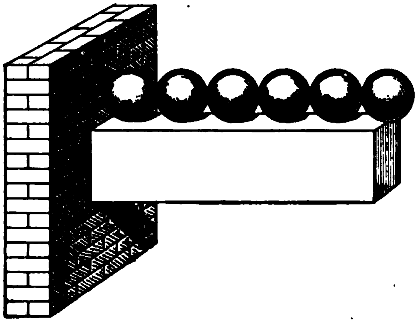

FIG. 136.

For a rectangular or square beam the ends of which merely rest upon supports, and loaded in the middle, as shown in Fig. 137:

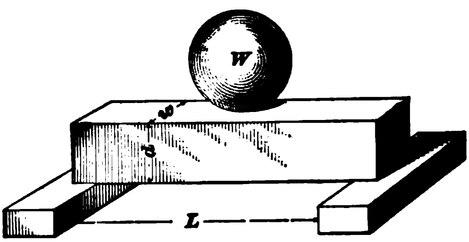

FIG. 137.

Rule 124.—*The maximum safe load in pounds that any rectangular or square beam is capable of sustaining at the middle, when its ends merely rest upon supports, is equal to four times the square of its depth in inches multiplied by its width in inches multiplied by the constant given in the table, and the product divided by the distance between its supports in feet.*

That is, $$W = \frac{4\,d^2\,w\,S}{L}.$$

EXAMPLE.—What maximum safe load is a bar of cast iron capable of sustaining in the middle between the supports on which its ends merely rest, if its depth is 6 inches, its width 3 inches, and the distance between the supports is 4 feet?

SOLUTION.—By the rule, $W = \frac{4 \times 6^2 \times 3 \times 100}{4} = 10,800$ lb. Ans.

645. For a cylindrical beam supported at its ends and loaded in the middle, as shown in Fig. 138:

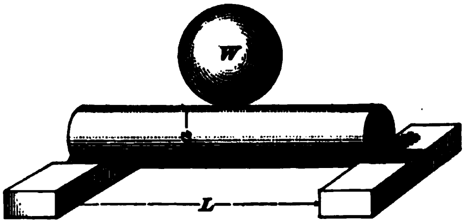

FIG. 138.

Rule 125.—*The maximum safe load in pounds that any cylindrical beam is capable of sustaining at the middle, when its ends merely rest upon supports, is equal to four times the cube of its diameter multiplied by .6 of the constant given in the table, and the product divided by the distance between its supports in feet.*

That is, $$W = \frac{4\,d^3 \times .6\,S}{L}.$$

EXAMPLE.—What maximum safe load is a bar of cast iron capable of sustaining in the middle between the supports on which its ends merely rest, if it is 4 inches in diameter, and if the distance between its supports is 3 feet?

SOLUTION.—By the rule, $W = \frac{4 \times 4^3 \times .6 \times 100}{3} = 5,120$ lb. Ans.

646. When the load is uniformly distributed on a beam of any cross-section, as shown in Fig. 139, it will sustain a load twice as great as when the load is applied in the middle between the supports.

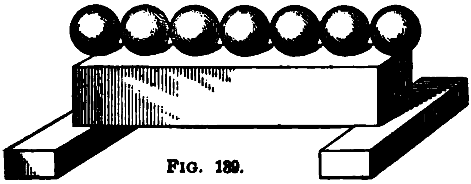

FIG. 139.

For example, if the beams in the last two examples were to carry a uniformly distributed load, they would sustain 10,800 × 2 = 21,600 pounds, and 5,120 × 2 = 10,240 pounds, respectively.

SHEARING, OR CUTTING, STRENGTH OF MATERIALS.

647. The shearing strength of any material is the resistance offered by its fibers to being cut in two.

Thus, the pressure of the cutting edges of an ordinary shearing machine, Fig. 140, causes a shearing stress in the plane *a b*. The unit shearing force may be found by dividing the force *P* by the area of the plane *a b*.

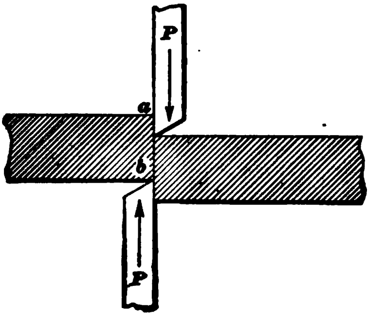

FIG. 140.

648. Fig. 141 shows a piece in **double shear**; here the central piece *c d* is forced out while the ends remain on their supports *M* and *N*.

The shearing strength of any body is directly proportional to its area.

In the following table are given the greatest and safe shearing strengths per square inch of different kinds of materials:

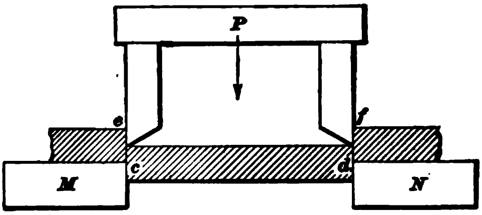

FIG. 141.

TABLE 23.

Material.	Greatest Shearing Stress in Pounds per Square Inch.	Safe Shearing Stress in Pounds per Square Inch.
Cast Iron.........	18,000	1,500 to 3,000
Wrought Iron.....	40,000	4,000 to 10,000
Steel	60,000	5,000 to 12,000

649. Formula for the Shearing Strength of Materials:

Let a = area of cross-section in inches;
S = safe shearing stress as given in the table;
W = safe load in pounds.

Rule 126.—*The safe load that any body which is subjected to a shearing stress is capable of sustaining is equal to the area of its cross-section in inches multiplied by its safe shearing stress, as given in the table.*

That is, $W = a\,S$.

Example.—If the beam in Fig. 141 is made of wrought iron, 4 inches in depth and 2 inches in width, what *steady* shearing stress is it capable of sustaining with safety?

Solution.—Applying the rule, $W = 4 \times 2 \times 10{,}000 = 80{,}000$ lb. This result must be multiplied by 2, since the beam is sheared in two places, along the lines ec and fd. Hence, the stress which the beam will safely sustain is $80{,}000 \times 2 = 160{,}000$ lb.

Example.—What force is required to punch a hole ⅝" in diameter through a steel plate ½" thick?

Solution.—It is evident that punching is but shearing in a circle instead of a straight line. The area punched (sheared) is equal to the thickness of the plate multiplied by the circumference of a circle having the same diameter as the punched hole. For, if the plate were cut through one of the diameters of the punched hole, and the two semicircles were straightened out, the punched surface would be a rectangle, which would have a length equal to the circumference of a circle whose diameter was equal to that of the hole, and a breadth equal to the thickness of the plate. In this case, the area $= \frac{5}{8} \times 3.1416 \times \frac{1}{2} = .98175$ sq. in. Table 23 gives the ultimate shearing strength of steel as 60,000 lb. per sq. in. Hence, the total force required is $.98175 \times 60{,}000 = 58{,}905$ lb. Ans.

EXAMPLES FOR PRACTICE.

1. What is the greatest load that can be safely carried by a steel rectangular cantilever at its extreme end, if the bar is 2" wide, 3" deep, and 2 ft. 6" long? Ans. 1,152 lb.

2. What is the greatest uniform load that can be safely carried by a white-pine girder, 6" wide, 8" deep, 16 ft. long, and supported at its ends? Ans. 5,760 lb.

3. A cast-iron bar, 1¾" in diameter and 5 ft. 8" long, is supported at its ends; what load will it safely sustain in the middle? Ans. 245 lb.

4. What force is required to punch a 1¼" hole through a wrought-iron plate $\frac{7}{16}$" thick? Ans. 68,723 lb.

5. What force is required to cut off the end of a cast-iron bar whose diameter is 2¼"? Ans. 88,357 lb.

LINE SHAFTINGS.

650. A line of shafting is one continuous run, or length, composed of lengths of shafts joined together by couplings.

The **main line** of shafting is that which receives the power from the engine or motor, and distributes it to the other lines of shafting or to the various machines to be driven.

Line shafting is supported by hangers, which are brackets provided with bearings, bolted either to the walls, posts, ceilings, or floors of the building. Short lengths of shafting, called **countershafts,** are provided to effect changes of speed, and to enable the machinery to be stopped or started.

Shafting is usually made cylindrically true, either by a special rolling process, when it is known as **cold-rolled shafting,** or else it is turned up in a machine called a lathe. In the latter case it is called **bright shafting.** What is known as **black shafting** is simply bar iron rolled by the ordinary process, and turned where it receives the couplings, pulleys, bearings, etc.

Bright turned shafting varies in diameter by ¼ inch up to about 3½ inches in diameter; above this diameter the shafting varies by ½ inch. The actual diameter of a bright shaft is $\frac{1}{16}$ of an inch less than the commercial diameter, it being designated from the diameter of the ordinary round bar iron from which it is turned. Thus, a length of what is called 3-inch bright shafting is only 2$\frac{15}{16}$ inches in diameter.

Cold-rolled shafting is designated by its commercial diameter; thus, a length of what is called 3-inch shafting is 3 inches in diameter.

651. In the following table is given the maximum distance between the bearings of some continuous shafts which are used for the transmission of power:

TABLE 24.

Diameter of Shaft in Inches.	Distance Between Bearings in Feet.	
	Wrought-Iron Shaft.	Steel Shaft.
2	11	11.5
3	13	13.75
4	15	15.75
5	17	18.25
6	19	20.
7	21	22.25
8	23	24.
9	25	26.

Pulleys from which considerable power is to be taken should always be placed as close to a bearing as possible.

652. The diameters of the different lengths of shafts composing a line of shafting may be proportional to the quantity of power delivered by each respective length. In this connection, the positions of the various pulleys depend upon the distance between the pulley and the bearing, and upon the amount of power given off by the pulleys. Suppose, for example, that a piece of shafting delivers a certain amount of power, then, it is obvious that the shaft will deflect or bend less if the pulley transmitting that power be placed close to a hanger or bearing, than if it be placed midway between the two hangers or bearings. It is impossible to give any rule for the proper distance of bearings which could be used universally, as in some cases the requirements demand that the bearings be nearer together than in others.

If the work done by a line of shafting is distributed quite equally along its entire length, and the power can be applied near the middle, the strength of the shaft need be only half as great as would be required if the power were applied at one end.

653. To compute the horsepower that can be transmitted by a shaft of any given diameter:

Let $D =$ diameter of shaft;
$R =$ revolutions per minute;
$H =$ horsepower transmitted;
$C =$ constant given in Table 25.

TABLE 25.

CONSTANTS FOR LINE SHAFTING.

Material of Shaft.	No Pulleys Between Bearings.	Pulleys Between Bearings.
Steel or Cold-Rolled Iron	65	85
Wrought Iron..........	70	95
Cast Iron..............	90	120

In the above table the bearings are supposed to be spaced so as to relieve the shaft of excessive bending; also, in the third vertical column, an average number and weight of pulleys, and power given off, is assumed.

In determining the above constants, allowance has been made to insure the stiffness as well as strength of the shaft. Cold-rolled iron is considerably stronger than ordinary turned wrought iron; the increased strength being due to the process of rolling, which seems to compress the metal and so make it denser, not merely skin deep, but practically throughout the whole diameter. We have, then, the following:

Rule 127.—*The horsepower that a shaft will transmit equals the product of the cube of the diameter and the number of revolutions, divided by the value of C for the given material.*

That is,
$$H = \frac{D^3 \times R}{C}.$$

Example.—What horsepower will a 3-inch wrought-iron shaft transmit which makes 100 revolutions per minute, there being no pulleys between bearings?

Solution.—$H = \frac{D^3 \times R}{C}$.

Substituting, we have

$$H = \frac{3 \times 3 \times 3 \times 100}{70} = 38.57 \text{ horsepower. Ans.}$$

If there were the usual amount of power taken off, as mentioned above, we should take $C = 95$. Then, $H = \frac{27 \times 100}{95} = 28.42$ horsepower. Ans.

654. To compute the number of revolutions a shaft must make to transmit a given horsepower:

Rule 128.—*The number of revolutions necessary for a given horsepower equals the product of the value of C for the given material and the number of horsepower, divided by the cube of the diameter.*

That is,

$$R = \frac{C \times H}{D^3}.$$

Example.—How many revolutions must a 3-inch wrought-iron shaft make per minute to transmit 28.42 horsepower, power being taken off at intervals between the bearings?

Solution.—$R = \frac{C \times H}{D^3}$.

Substituting, we have $R = \frac{95 \times 28.42}{3 \times 3 \times 3} = 100$ revolutions. Ans.

655. To compute the diameter of a shaft that will transmit a given horsepower, the number of revolutions the shaft makes per minute being given:

Rule 129.—*The diameter of a shaft equals the cube root of the quotient obtained by dividing the product of the value of C for the given material and the number of horsepower by the number of revolutions.*

That is,

$$D = \sqrt[3]{\frac{C \times H}{R}}.$$

Example.—What must be the diameter of a wrought-iron shaft to transmit 38.57 horsepower, the shaft to make 100 revolutions per minute, no power being taken off between bearings?

Solution.—$D = \sqrt[3]{\frac{C \times H}{R}}$.

Substituting, we have

$$D = \sqrt[3]{\frac{70 \times 38.57}{100}} = \sqrt[3]{27} = 3 \text{ in. Ans.}$$

As the speed of shafting is used as a multiplier in the calculations of the horsepower of shafts, a shaft having a given diameter will transmit more power in proportion as its speed is increased. Thus, a shaft which is capable of transmitting 10 horsepower when making 100 revolutions per minute, will transmit 20 horsepower when making 200 revolutions per minute. We may, therefore, say *the horsepowers transmitted by two shafts are directly proportional to the number of revolutions.*

EXAMPLES FOR PRACTICE.

1. What horsepower will a 2½" wrought-iron shaft transmit when running at 110 revolutions per minute, it being used for transmission only? Ans. 24.55 horsepower.

2. A 6" cast-iron shaft transmits 150 horsepower; how many revolutions per minute must it make, no power being taken off between bearings? Ans. 62½ R. P. M.

3. What should be the diameter of a wrought-iron shaft to transmit 100 horsepower at 150 revolutions per minute, power being taken off between bearings? Ans 4 in., nearly.

4. The diameter of a steam-engine shaft is 8"; what horsepower will it transmit, if made of steel, when making 150 revolutions per minute? Ans. 1,181.54 horsepower.

5. The machines driven by a certain line of wrought-iron shafting take their power from various points between the bearings; and, if all were working together at their full capacity, they would require 65 horsepower to drive them. What diameter should the shaft be if it runs at 150 revolutions per minute? Ans. 3½ in.

STEAM AND STEAM BOILERS.

HEAT.

656. Heat *is a form of energy.* It is, in fact, a motion of the molecules composing matter. It was stated in Art. **434** that all matter is composed of molecules; now, these molecules are not in a state of rest, but are moving, or vibrating back and forth, with a greater or less velocity, and it is this movement of the molecules that causes the sensations of warmth or cold. If the motion is slow, the body appears cold to the touch; when the vibrations are rapid, the body becomes warm or hot.

It was shown in Art. **545,** rule **100,** that a body in motion has kinetic energy, the amount of which is measured in foot-pounds, and is found by multiplying the weight of the body by the square of its velocity and dividing by 64.32. Since the molecules composing matter are in motion, they must possess kinetic energy, and we are justified, therefore, in saying that *heat, this motion of the molecules, is a form of energy.*

657. Temperature is a term used to indicate how hot or cold a body is; i. e., to indicate the rate of vibration of the molecules of a body. A hot body has a high temperature; a cold body, a low temperature. When a body, as, for example, an iron bar, receives heat from any source, its temperature rises; on the other hand, when a body loses heat, its temperature falls.

The temperature is *not* a measure of the *quantity* of heat a body possesses. *Temperature* may be considered to be a measure of the velocity of the molecules of a body as they vibrate to and fro, while the *quantity of heat* may be considered to be the kinetic energy of the molecules composing the body. A small iron rod may be heated to whiteness and yet possess a very small quantity of heat.

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Its temperature is very high, which simply indicates that the molecules of the rod are vibrating with an extremely high velocity.

Temperature is measured by an instrument called the thermometer, which is so familiar as to scarcely need description. It consists of a thin glass tube, at one end of which is a bulb filled with mercury. Upon being heated the mercury expands in proportion to its temperature. Thermometers are graduated in different ways. In the Fahrenheit thermometer, which is generally used in this country, the point where the mercury stands when the instrument is placed in melting ice is marked 32°. The point indicated by the mercury when the thermometer is placed in water boiling in the open air at the level of the sea is marked 212°. The tube between these two points is divided into 180 equal parts called degrees.

658. Effects of Heat.—Suppose we take a vessel filled with water. Let the vessel be a cylinder fitted with a piston, as shown in Fig. 142. The water is say at the freezing point, and the millions of molecules composing the water are moving to and fro with a comparatively small velocity. Place the vessel in a fire or furnace. Heat is communicated to the molecules of water, and they begin to move faster and faster and faster. That is, their kinetic energy increases, and, if a thermometer were inserted in the vessel, it would be found that the temperature of the water rises. Consequently, one effect of heat is to raise the temperature of the body to which it is applied. But, after reaching a certain temperature, the molecules of the water not only move faster, but they move further from each other and their paths are longer. It is plain that if the molecules are further apart than they were originally, the whole body of them must take up more space. In other

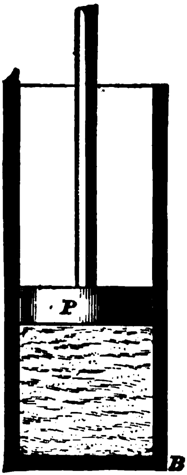

FIG. 142.

words, after reaching a certain temperature, the water expands as heat is added. Hence, another effect of heat is to expand bodies to which it is applied. Common examples of the expansion of bodies by heat are seen in the setting of tires, the expansion of the rails of a railway in summer, etc.

659. The heat supplied to the vessel of water has so far done three things: 1. Raised the temperature of the water and thus increased the kinetic energy of the molecules. (Let the amount of heat expended for this purpose be denoted by S.) 2. A certain quantity of heat has been used in expanding the water; that is, in pushing its molecules further apart against the force of cohesion. (Denote the amount of heat so expended by I.) 3. Since the water expands, it must raise the piston P against the pressure of the atmosphere, and, consequently, more heat must be used to expand the water than would be required if there were no pressure on the upper side of the piston. (Call this extra quantity of heat, W.)

If we denote by Q the total heat given up to the vessel of water, we have

$$Q = S + I + W.$$

660. Ordinarily, the greater part of the heat given to a body is spent in raising its temperature, and but little is used in expanding the body. That is, the quantity S is nearly equal to the quantity Q, while the quantities I and W are extremely small.

Suppose that the piston is removed from the cylinder of Fig. 142 and a thermometer inserted. As the vessel becomes more and more heated, the temperature indicated by the thermometer will rise until it reaches 212°. So far most of the heat has been used to raise the temperature of the water. But now, no matter how much heat is added to the water, the thermometer stands at 212° and can not be made to rise higher. This is the reason: When the temperature reaches 212° the molecules of water have been set into such rapid motion that the force of cohesion is no longer able to hold them and they tend to separate. In other words, the water is changing to a gas (steam), and all of the heat is being used to effect this change. The temperature of the steam will remain at 212° until all the water is changed to steam; then, if more heat is applied, the temperature of the steam will begin to rise.

Suppose we take a block of ice at a temperature of say 14° and heat it. If a thermometer is placed in contact with the ice, its temperature will rise until it reaches 32° and will then remain stationary. As soon as this temperature is reached the ice begins to melt or change to water, and the heat, instead of raising the temperature further, is all used to effect this change of state. Here, then, is another effect produced by heat. It will change a solid to a liquid or a liquid to a gas.

661. Latent Heat.—The heat which is expended in changing a body from the solid to the liquid state or from the liquid to the gaseous state is called **latent heat.** The portion of the heat applied which raises the temperature, and which, therefore, affects the thermometer, is sometimes called **sensible heat.**

662. Measurement of Heat.—Since heat is not a substance, it can not be measured directly in pounds or quarts; but, like force, it may be measured by the effects it produces. Suppose a certain quantity of heat raises the temperature of a pound of water from 52° to 53°. It will take the same quantity of heat to raise the temperature of a pound from 53° to 54°, and it will take double the quantity to raise the temperature of the pound of water from 52° to 54° that it took to raise the temperature from 52° to 53°. The unit quantity of heat is the quantity required to raise the temperature of a pound of water from 62° to 63°. This unit is called the **British thermal unit,** or B. T. U.

663. Relation Between Heat and Work.—Suppose that in the experiment shown in Fig. 142, the piston had been allowed to remain in the cylinder while the water was being changed to steam. Steam at 212° occupies nearly 1,700 times the space that the water originally occupied.

Hence, the piston would be lifted in the cylinder to make room for the steam which was being formed. But to raise the piston, work must be done. Here, then, is an example of *work being performed by heat.* On the other hand, work will produce heat. If two blocks of wood are rubbed briskly together, they will become warm and may even ignite. The work of friction causes the journals and bearings of fast-running machines to heat. A small iron rod may be raised to a very considerable heat by pounding it on an anvil.

664. Since work may be changed into heat, and heat into work, it seems probable that there is some fixed ratio between the unit of heat (B. T. U.) and the unit of work, the foot-pound. By a careful series of experiments Dr. Joule, of England, discovered this ratio.

He found that one B. T. U. was equal to 772 foot-pounds; later and more careful experiments show that 778 foot-pounds is more nearly correct. This number, 778 foot-pounds, is called the **mechanical equivalent** of one B. T. U.

665. We have, then, the following important law: *Heat may be changed to work, or work to heat; 778 foot-pounds of work are required to produce one B. T. U., and, conversely, the expenditure of one B. T. U. produces 778 foot-pounds of work.*

EXAMPLE 1.—The burning of a pound of coal gives out sufficient heat to raise 14,000 pounds of water from 62° to 63°. If all this heat is utilized, how high will it lift a weight of 700 pounds?

SOLUTION.—Since one B. T. U. raises a pound of water from 62° to 63°, it requires 14,000 B. T. U. to raise 14,000 lb. of water from 62° to 63°. Hence, the burning of the pound of coal gives out 14,000 B. T. U. One B. T. U. is equivalent to 778 foot-pounds; hence, 14,000 B. T. U. are equivalent to $14,000 \times 778 = 10,892,000$ foot-pounds. Then, the height to which the weight can be raised is $10,892,000 \div 700 = 15,560$ feet. Ans.

EXAMPLE 2.—A cannon ball weighing 60 pounds moves with a velocity of 1,800 ft. per sec. Suppose the ball were suddenly stopped and all its kinetic energy changed into heat. How many B. T. U. would be developed? If all this heat were applied to 100 lb. of water at a temperature of 60°, to what temperature would the water be raised?

SOLUTION.—By rule **100,** Art. **545,** the kinetic energy of the cannon ball is $\frac{W v^2}{64.32} = \frac{60 \times 1,300^2}{64.32} = 1,576,492$ foot-pounds. But 778 foot-pounds = 1 B. T. U. Therefore, the number of B. T. U. developed is $1,576,492 \div 778 = 2,026.3$ B. T. U. Since 1 B. T. U. raises the temperature of a pound of water 1 degree, it will take 100 B. T. U. to raise 100 pounds of water 1 degree. Hence, 2,026.3 B. T. U. will raise 100 pounds of water $2,026.3 \div 100 = 20.263°$, and the final temperature of the water will be $60° + 20.263° = 80.263°$. Ans.

666. Specific Heat.—One B. T. U. raises the temperature of a pound of water one degree; will it have the same effect on a pound of mercury? Heat two one-pound iron balls to the temperature of boiling water, 212°; having now the same weights and temperatures, each ball has the same quantity of heat. Place one of these balls in a vessel, into which slowly pour enough water at a temperature of 60° that the iron will be cooled to 70° while the water is heated to the same temperature. Now, place the other hot ball in another vessel, into which pour mercury at a temperature of 60° until the iron and mercury reach a common temperature of 70°. In each case the ball was cooled from 212° to 70°; each, therefore, gave up the same quantity of heat. When, however, we consider its effects, we find that it raised less than $\frac{1}{2}$ pound of water through a range of 10°, while $14\frac{1}{2}$ pounds of mercury, nearly 30 times as much, was raised through the same range. It is plain, therefore, that to raise a pound of mercury from 62° to 63° requires $\frac{1}{30}$ the heat necessary to raise a pound of water from 62° to 63°. Hence, we say the **specific heat** of the mercury is $\frac{1}{30}$, or .0333.

667. *The* **specific heat** *of a body is the ratio between the quantity of heat required to warm that body one degree and the quantity of heat required to warm an equal weight of water one degree.*

EXAMPLE 1.—It is found that to raise the temperature of 20 lb. of iron from 62° to 63° requires 2.276 B. T. U. What is the specific heat of iron?

SOLUTION.—To raise 20 lb. of water from 62° to 63° requires 20 B. T. U. The specific heat of the iron is, according to the above definition, the ratio between the quantities of heat required to warm the iron and the water, respectively, through one degree, that is, it is the ratio 2.276 : 20 = 2.276 ÷ 20 = .1138. Ans.

Example 2.—The specific heat of silver is .057. How many B. T. U. are required to raise the temperature of 22 lb. of silver from 50° to 60°?

Solution.—To raise the temperature of a pound of water one degree requires one B. T. U. Since the specific heat of silver is .057, only .057 B. T. U. is required to raise one pound of silver one degree. Hence, to raise 22 pounds of silver 10 degrees must require $.057 \times 22 \times 10 = 12.54$ B. T. U. Ans.

Rule 130.—*To find the number of B. T. U. required to raise the temperature of a body a given number of degrees, multiply the specific heat of the body by its weight in pounds and by the number of degrees.*

Denote the number of B. T. U. required by U; the specific heat by c; the weight by W, and let t and t_1 be the temperatures before and after the heat is applied, respectively.

Then, $$U = c\,W\,(t_1 - t).$$

The specific heat of some of the more common substances is given in the following table:

TABLE 26.

Substance.	Specific Heat.	Substance.	Specific Heat.
Water	1.0000	Ice	.5040
Sulphur	.2026	Steam (superheated)	.4805
Iron	.1138	Air	.2375
Copper	.0951	Oxygen	.2175
Silver	.0570	Hydrogen	3.4090
Tin	.0562	Nitrogen	.2438
Mercury	.0333	Carbon Monoxide	.2479
Lead	.0314	Carbon Dioxide	.2170

668. Latent Heat of Fusion.—This term is applied to the quantity of heat required to change a pound of a given substance from the solid to the liquid state. The only case of interest to the engineer is the heat required to change a pound of ice to water. Careful experiments show that about 144 B. T. U. are required to change a pound of ice at 32° to water at 32°. Hence, the latent heat of water is 144 B. T. U.

669. The **latent heat of steam** is the quantity of heat required to change a pound of water at 212° into steam at 212°. Experiment shows that this quantity of heat is about 966 B. T. U. This shows that the heat required to change a pound of water at 212° to steam is 966 times as great as the quantity required to raise the temperature of a pound of water from 62° to 63°. The latent heat of steam is different for different temperatures.

EXAMPLE.—How many B. T. U. are required to change 5 pounds of ice at 15° into steam at 212°?

SOLUTION.—To raise the temperature of the ice from 15° to 32° (the melting temperature) requires, according to rule **130**,

$$c\,W(t_1 - t) = .504 \times 5 \times (32 - 15) = 42.84 \text{ B. T. U.}$$

To change the ice to water requires 144 B. T. U. for each pound, or $144 \times 5 = 720$ B. T. U. To raise the water from 32° to 212° requires, according to the same rule,

$$c\,W(t_1 - t) = 1 \times 5 \times (212 - 32) = 5 \times 180 = 900 \text{ B. T. U.}$$

Finally, to change the water to steam requires 966 B. T. U. per pound, or $966 \times 5 = 4{,}830$ B. T. U. Therefore, in all, $42.84 + 720 + 900 + 4{,}830 = 6{,}492.84$ B. T. U. are required. Ans.

Expressed in foot-pounds, the work required to effect the above change would be $6{,}492.84 \times 778 = 5{,}051{,}429.5$ foot-pounds, or work enough to lift a weight of 1,000 pounds nearly a mile.

Since a pound of ice requires 144 B. T. U. to change it to water, it follows that when a pound of water at 32° changes to ice (freezes), 144 B. T. U. are given out in the process. Similarly, the condensation of a pound of steam into water at 212° liberates 966 B. T. U. This principle is applied in heating buildings by steam. The steam passes through the radiators and condenses. The latent heat thus set free warms the building.

670. Temperature of Mixtures.—It is often desirable to calculate the final temperature of a mixture of different substances at different temperatures. The following law is to be observed in such cases: *The quantity of heat in a mixture is the same as the quantity of heat contained in the substances before being combined.* If two substances of different temperatures are placed together, they both finally attain the same temperature; the heat lost by the one in coming from a higher to a lower temperature is gained by the other in passing from a lower to a higher temperature.

Rule 131. — *To find the temperature of a mixture of several substances, multiply together the weight, specific heat, and temperature of each substance separately, and add the products. Next multiply together the weight and specific heat of each of the substances separately, and add these products. Divide the former sum by the latter. The result will be the temperature of the mixture.*

EXAMPLE.—15 pounds of water at 42° and 30 pounds of mercury at 70° are placed in the same vessel, and a ball of lead weighing 19 pounds, and having a temperature of 110°, is immersed in the mixture. What will be the final temperature of the contents?

SOLUTION.—Applying the rule just given, the product of the weight, specific heat, and temperature of the water is $15 \times 1 \times 42 = 630$; of the mercury, $30 \times .0333 \times 70 = 69.93$; of the lead, $19 \times .0314 \times 110 = 65.626$, and the sum is $630 + 69.93 + 65.626 = 765.556$.

The product of the weight and specific heat of the water is $15 \times 1 = 15$; of the mercury, $30 \times .0333 = .999$; of the lead, $19 \times .0314 = .5966$, and the sum is $15 + .999 + .5966 = 16.5956$. Then, the temperature of the mixture is $765.556 \div 16.5956 = 46.13°$. Ans.

671. A particularly important case is the mixture of steam and water. Let W and t_1 represent the weight and temperature of the steam, and let w and t represent the weight and temperature of the water. Let T represent the final temperature of the mixture, and L the latent heat of the steam at the given temperature. Then, the temperature of a mixture of steam and water may be found by means of the following rule:

Rule 132. — *When steam and water are mixed, the steam condenses. To find the final temperature of the mixture, add together the latent heat and the temperature of the steam,* *and multiply the sum by the weight of the steam. To this product add the product of the weight and temperature of the water, and divide the sum so obtained by the sum of the weights of the steam and water. The quotient will be the temperature of the mixture.*

That is, $$T = \frac{W(L + t_1) + wt}{W + w}.$$

EXAMPLE.—If 8 pounds of steam at 212° are run into a barrel containing 300 lb. of water at 43°, what will be the final temperature?

SOLUTION.—The weight of the steam $W = 8$ lb.; the latent heat $L = 966$ B. T. U.; the temperature t_1 is 212°. The weight of the water is $w = 300$ lb.; the temperature is 43°. Hence, by the last rule, the final temperature T is

$$T = \frac{W(L + t_1) + wt}{W + w} = \frac{8 \times (966 + 212) + 300 \times 43}{8 + 300} = 72\tfrac{1}{2}°. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

1. A body weighing 143 pounds falls 62 feet. If the energy of the body at the end of the fall be changed into heat, how many B. T. U. would be developed? Ans. 11.39 B. T. U.

2. An expenditure of 210 B. T. U. per min. will develop how many horsepower? Ans. 4.95 H. P.

3. Supposing 1/7 of the total heat of the coal to be used in doing work, how many pounds of coal must be burned per hour to run a 40-horsepower engine? Each pound of the coal gives out 13,500 B. T. U. Ans. 52.8 lb.

4. From what height must a block of ice fall, in order that the heat developed by its collision with the earth may be just enough to melt it, supposing that all of the energy gained during the fall is converted into heat? Ans. 112,032 feet.

5. A bar of iron weighing 20 pounds, and having a temperature of 350°, is plunged into a tank containing 130 pounds of water at 55°. To what temperature will the water be raised? Ans. 60°.

6. How many pounds of ice at 32° can be melted by 3 pounds of steam at 212°? Ans. 23.875 lb.

SUGGESTION.—Each pound of ice requires 144 B. T. U. to melt it; each pound of steam in changing to water at 32° gives up $966 + (212 - 32) = 1{,}146$ B. T. U.

7. How many B. T. U. are required to raise the temperature of 26 pounds of copper from 57° to 93°? Ans. 89.1 B. T. U.

8. Four pounds of a certain substance at a temperature of 212° are mixed with one pound of water at 50°; the specific heat of the substance being .03125, what is the resulting temperature of the mixture? Ans. 68°.

9. How many B. T. U. are required to change 13 pounds of water at 59° into steam at 212°? Ans. 14,547 B. T. U.

10. Twenty pounds of steam at 212° are run into a tank containing 340 lb. of water at 42°. What will be the final temperature of the mixture? Ans. 105.11°.

STEAM.

672. **Steam** is *water vapor;* that is, it is *water* changed into a *gaseous state* by the application of *heat.*

The process of changing water (or other liquid) into vapor by means of heat is called **evaporation** or **vaporization.**

673. When an open vessel containing water is placed in contact with fire, the air contained in the water is first driven off, and escapes from the surface. The water in contact with the part of the vessel nearest the fire receives heat first and expands. Its specific gravity is reduced; that is, it becomes lighter than the cooler water above it, and it rises to the surface, cooler water taking its place. In this manner the water keeps up a circulation until it reaches a temperature of 212°. At this stage the molecules nearest the fire attain such a velocity of vibration that they rise through the water above them, overcome the pressure of the air, and escape in the form of gas. When this occurs, the water boils.

It is plain that if the pressure on the surface of the water is increased, it will require more work to force the molecules to the surface against this increased pressure. That is, more heat must be expended upon the water to make it boil, and, therefore, the boiling point will be raised. We have seen that when water boils in the open air, exposed, therefore, to the atmospheric pressure of 14.7 lb. per sq. in., the boiling begins when a temperature of 212° is reached. If the pressure on the surface is increased to say 32 lb. per sq. in., the water will not boil until it reaches a temperature of 254°. On the other hand, if the pressure is lowered to 6 lb. per sq. in., the water will boil when it reaches 170°. Hence, we have the following law:

An increase of pressure on the surface of a liquid raises the temperature at which it boils; a decrease of pressure lowers the temperature at which it boils.

674. Steam in contact with water is called *saturated steam.* This is the condition of steam in a boiler. According to the law just given, the temperature of saturated steam depends upon the pressure only. When the steam in a boiler shows a gauge pressure of 60 pounds, its temperature *must be* 307°. A thermometer placed in a boiler could be used to tell the pressure of the steam. It would be as accurate, though not as convenient, as a steam gauge.

The reader is cautioned against the idea that saturated steam necessarily implies "wet" steam. It may be perfectly free from water particles, but it is saturated if in any way in contact with water, so that the pressure and temperature are mutually dependent. In the steam boiler, for example, the space above the water, if viewed through a glass-covered opening, appears perfectly transparent as though filled with air, provided the boiler is not working. This shows that the steam which fills this space is perfectly "dry." When, however, the boiler is steaming rapidly, the violent ebullition may throw up a certain amount of water in the form of spray, which mingles with the steam, giving it the misty appearance shown in the exhaust from an engine. Steam in this condition is **"wet" saturated steam.**

675. Steam separated from water may be heated like air or any other gas until its temperature is higher than the boiling point. To illustrate, put a little water in a cylinder as at (*a*), Fig. 143. Suppose that the area of the cylinder is 100 sq. in.; then the pressure of the atmosphere upon the piston is $14.69 \times 100 = 1,469$ lb. (The number 14.69 is a little more exact than 14.7.)

When a part of the water is changed to steam, as at (*b*), Fig. 143, the steam is in a saturated state, and at this pressure its temperature can not be higher than 212°. When, however, the water is all changed to steam, as at (*c*), any further addition of heat raises its temperature, while the pressure remains at 14.69 lb. per sq. in. Steam in this latter condition is **superheated steam.** The specific heat of superheated steam at constant pressure is .4805, or say .48 for ordinary purposes; that is, .48 of one B. T. U. will raise the temperature of a pound of superheated steam one degree. The temperature of saturated steam can not be raised if the pressure remains constant. All the heat is expended in changing water to steam, and until all the water is vaporized, the temperature remains constant.

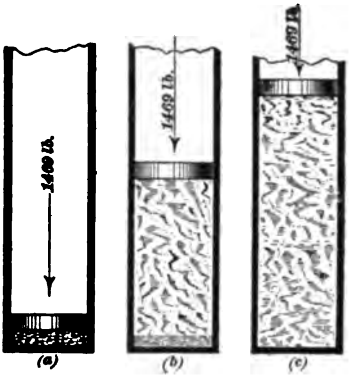

FIG. 143.

STEAM TABLES.

676. Whenever the pressure of saturated steam is changed, there are other properties which also change with it. These properties are the following:

1. The temperature of the steam, or, what is the same thing, the boiling point.

2. The number of B. T. U. required to raise a pound of water from 32° to the boiling point corresponding to the given pressure. This is called the **heat of the liquid.**

3. The number of B. T. U. required to change the water at the boiling temperature into steam at the same temperature. This is called the **latent heat of vaporization,** or, simply, **latent heat.**

4. The number of heat units required to change a pound of water at 32° to steam of the required temperature and pressure. This is called the **total heat of vaporization,** or, simply, **total heat.**

It is plain that the *total heat* is the sum of the *heat of the liquid* and the *latent heat.* That is, total heat = heat of liquid + latent heat.

5. The **specific volume** of the steam at the given pressure; that is, the number of cubic feet occupied by a pound of steam of the given pressure.

6. The **density** of the steam; that is, the weight of one cubic foot of the steam at the given pressure.

677. All the above properties vary with the pressure of the steam. For example, if steam is at atmospheric pressure, the temperature is at 212°; the heat of the liquid is 180.531 B. T. U.; the latent heat, 966.069 B. T. U.; the total heat, 1,146.6 B. T. U. A pound of steam at this pressure occupies 26.37 cu. ft., and a cubic foot of the steam weighs about .037928 lb. When the pressure is 70 lb. per sq. in. above vacuum, the temperature is 302.774°; the heat of the liquid is 272.657 B. T. U.; the latent heat is 901.629 B. T. U.; the total heat, 1,174.286 B. T. U. A pound of the steam occupies 6.076 cu. ft., and a cubic foot of the steam weighs .164584 lb.

These properties have been determined by direct experiment for all ordinary steam pressures. They are given in the Table of the Properties of Saturated Steam.

678. Explanation of the Table.—Column 1 gives the pressures from 1 to 300 pounds. These pressures are above vacuum. The steam gauges fitted on steam boilers register the pressure above the atmosphere. That is, if the steam is at atmospheric pressure, 14.7 lb. per sq. in., the gauge registers 0. Consequently, the atmospheric pressure must be added to the reading of the gauge to obtain the pressure above vacuum. In using the table, care must be taken *not* to use the gauge pressures without first adding 14.7 lb. per sq. in.

Pressures registered above vacuum are called **absolute pressures.** The pressures given in column 1 are *absolute.* Absolute pressure per sq. in. = gauge pressure per sq. in. + 14.7.

Column 2 gives the temperature of the steam when at the pressure shown in column 1.

Column 3 gives the **heat of the liquid.** It will be noticed that the values in column 3 may be obtained approximately by subtracting 32° from the temperature in column 2. If the specific heat of water were exactly 1.00 at all temperatures, it would, of course, take exactly $212 - 32 = 180$ B. T. U. to raise a pound of water from 32° to 212°. But, actually, the specific heat of water is slightly greater than 1.00 when the temperature of the water is over 62°, and it, therefore, takes 180.531 B. T. U. to raise the temperature of a pound from 32° to 212°.

Column 4 gives the **latent heat of vaporization,** which is seen to decrease slightly as the pressure increases.

Column 5 gives the **total heat of vaporization.** The values in column 5 may be obtained by adding together the corresponding values in columns 3 and 4.

Column 6 gives the weight of a cubic foot of steam in pounds. As would naturally be expected, as the pressure rises, the steam becomes denser and weighs more per cubic foot.

Column 7 gives the number of cubic feet occupied by one pound of steam at the given pressure. It will be noticed that the corresponding values of columns 6 and 7 multiplied together always produce 1. That is, for 31.3 pounds gauge pressure (corresponding to 46 lb. in Col. 1) we have $.11088 \times 9.018 = 1.000$, very nearly.

Column 8 gives the ratio of the volume of a pound of steam at the given pressure to the volume of a pound of water at 39.1°. The values in column 8 may be obtained by dividing 62.425, the weight of a cubic foot of water at 39.1°, by the numbers in column 6.

EXAMPLES ON THE USE OF THE STEAM TABLE.

EXAMPLE 1.—Calculate the heat required to change 5 pounds of water at 32° into steam at 92 pounds pressure above vacuum.

SOLUTION.—From column 5, the total heat of one pound at 92 lb. pressure is 1,180.045 B. T. U.

$$1{,}180.045 \times 5 = 5{,}900.225 \text{ B. T. U. Ans.}$$

Example 2.—How many heat units are required to raise 8½ pounds of water from 32° to 250° ?

Solution.—Looking in column 3, the heat of the liquid of one pound at 250.293° is 219.261 B. T. U. 219.261 − .293 = 218.968 B. T. U. = heat of liquid for 250°. Then, for 8½ pounds it is 218.968 × 8½ = 1,861.228 B. T. U. Ans

Example 3.—How many foot-pounds of work will it require to change 60 pounds of boiling water at 80 lb. pressure, absolute, into steam of the same pressure ?

Solution.—Looking under column 4, the latent heat of vaporization is 895.108; that is, it takes 895.108 B. T. U. to change 1 pound of water at 80 lb. pressure into steam of the same pressure. Therefore, it takes 895.108 × 60 = 53,706.48 B. T. U. to perform the same operation upon 60 pounds of water.

$$53{,}706.48 \times 778 = 41{,}783{,}641.44 \text{ foot-pounds. Ans.}$$

Example 4.—Find the volume occupied by 14 pounds of steam at 30 pounds, *gauge* pressure.

Solution.—30 pounds, gauge pressure, = 30 + 14.7 = 44.7 pounds, absolute pressure. The nearest pressure in the table is 44 pounds, and the volume of a pound of steam at that pressure is 9.403 cubic feet. The volume of a pound at 46 pounds pressure is 9.018 cu. ft. 9.403 − 9.018 = .385 cu. ft., the difference in volume for a difference in pressure of 2 lb. $\frac{.385}{2} = .1925$ cu. ft., the difference in volume for a difference in pressure of 1 pound. .1925 × .7 = .135 cu. ft., the difference in volume for a difference in pressure of .7 pound. Therefore, 9.403 − .135 = 9.268 cu. ft. is the volume of one pound of steam at 44.7 pounds pressure. The .135 cu. ft. is subtracted from 9.403 cu. ft., since the volume is less for a pressure of 44.7 lb. than for 44 lb.

$$9.268 \times 14 = 129.752 \text{ cu. ft. Ans.}$$

Example 5.—Find the weight of 40 cu. ft. of steam at a temperature of 254°.

Solution.—The weight of one cubic foot of steam at 254.002° is, from the table, .078839 pound. Neglecting the .002°, the weight of 40 cu. ft. is, therefore,

$$.078839 \times 40 = 3.15356 \text{ pounds. Ans.}$$

Example 6.—How many pounds of steam at 64 pounds pressure, absolute, are required to raise the temperature of 300 pounds of water from 40° to 130°, the water and steam being mixed together ?

Solution.—The number of heat units required to raise one pound of water from 40° to 130° is 130 − 40 = 90 B. T. U. (Actually, a little more than 90 would be required, but the above is near enough for all practical purposes.) Then, to raise 300 pounds from 40° to 130° requires $90 \times 300 = 27,000$ B. T. U. This quantity of heat must necessarily come from the steam. The total heat at 64 pounds pressure, absolute, is (see steam table, column 5) 1,172.466 B. T. U. above 32°. $130° - 32° = 98°$. Then, total heat above 130° is $1,172.466 - 98 = 1,074.466$ B. T. U. = number of heat-units given up by one pound of steam and used to heat the water.

It will, therefore, take $\frac{27,000}{1,074.466} = 25.13$ pounds of steam to accomplish the desired result. Ans.

EXAMPLES FOR PRACTICE.

1. How many foot-pounds of work are required to change 42 pounds of water at the temperature corresponding to a pressure of 88 lb., absolute, into steam at a temperature corresponding to a pressure of 105 lb., absolute? Ans. 29,208,194.15 ft.-lb.

2. How many B. T. U. are required to convert 25 pounds of water at 32° into 109,6 cubic feet of steam? Ans. 29,541.1 B. T. U.

3. Find the number of heat units required to change 11 pounds of water at 32° into steam at 100 pounds, absolute pressure. Ans. 13,000.526 B. T. U.

4. Find the weight of 712 cubic feet of steam at a pressure of 83 pounds, gauge. Ans. 81.689 lb.

5. How many pounds of steam at 47.3 pounds pressure, gauge, are required to raise 120 pounds of water from 55° to 160° at atmospheric pressure? Ans. 12.091 lb.

6. Find the volume of 19 pounds of steam at a pressure of 62 lb. per sq. in., gauge. Ans. 105.952 cu. ft.

TYPES OF MARINE BOILERS.

679. A **steam boiler** is a vessel used to generate steam for different purposes, by the application of heat to water. The heat is generated by the combustion of some fuel, such as coal or wood, in the *furnace*. To carry away the products of combustion and to create a draught, that is, supply the burning fuel with air, the furnace is connected with the *smokestack*, sometimes called the *funnel*. The water used for the generation of steam is supplied to the boiler by the *feed apparatus*, and enters the boiler through the *feed-pipe*.

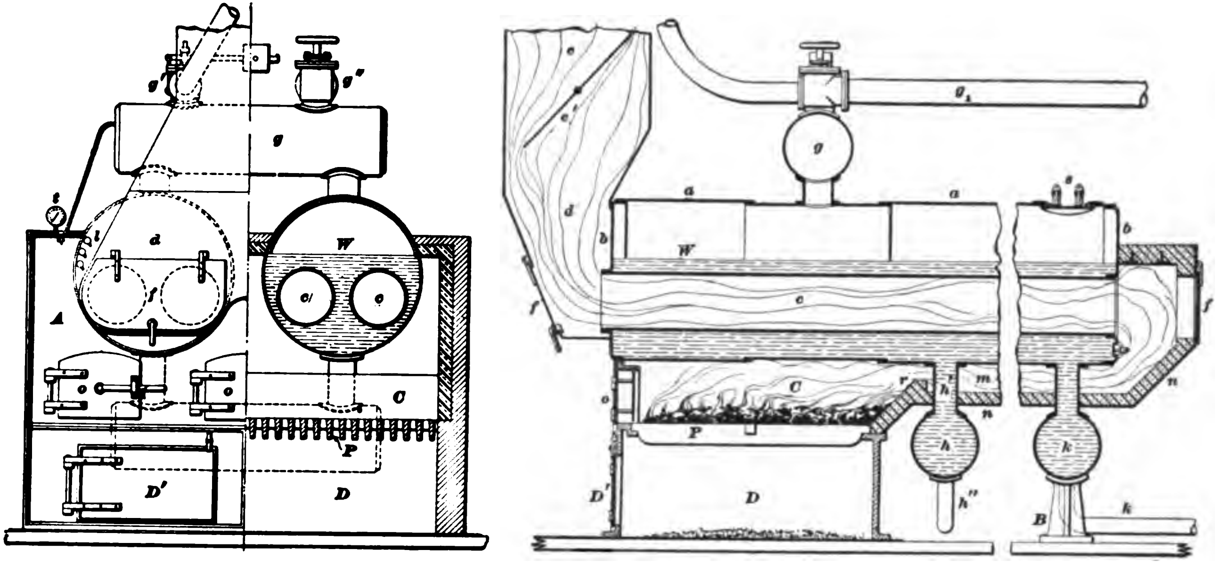

FIG. 144.

The steam generated in the boiler is conveyed to its destination by the *steam pipe*.

680. Where any portion of a boiler, such as a plate or tube, is in contact with the fire or hot gases on one side and water on the other, the surface in contact with the fire is called the **heating surface.** The sum of all such surfaces is called **the total heating surface.**

Boilers may be divided into three classes, **stationary, locomotive,** and **marine boilers.**

Marine boilers only will be treated of here.

THE FLUE BOILER.

681. The simplest form of marine boiler, as used to-day, is the **flue boiler,** shown in Fig. 144. This type of boiler is still in extensive use on Western river steamboats. It consists essentially of a long cylinder *a*, called the **shell,** made of iron or steel plates riveted together. The ends of the boiler are closed by flat or hemispherical plates *b b*, called the **heads** of the boiler. Two or more **flues** *c*, *c*, in some instances as many as six, are fixed to the front and rear heads. To the front of the boiler a sheet-iron casing *d*, known as the **front connection,** is secured; the upper part of this leads to the **smokestack** *e*. To give access to the flues, doors *f* are provided. When two boilers are placed in one setting, as shown in the figure, they are usually connected to a **steam drum,** shown at *g*, the object of this drum being to furnish dry steam. Fitted to the steam drum is the stop valve g'', by means of which communication between the boilers and the engines may be shut off. Connected to the stop valve is the steam pipe g_1 which conveys the steam to the engines. Attached to the top of the steam drum is the **safety valve,** shown at g', which prevents the steam pressure from exceeding the safe working pressure of the boiler. To indicate the steam pressure, a **steam gauge,** shown at *t*, is attached to each set of boilers. To indicate the water level within the boiler, **gauge-cocks,** shown at *l*, are fitted to the front heads. A **manhole** *s*, which is simply a hole cut in the shell and closed by a suitable cover, gives access to the inside of the boiler. The steam drum, safety valve, steam gauge, gauge-cocks, and manholes, will be considered in detail further on. To provide a quiet place for the settlement of the foreign matter held in suspension in the water used for feeding the boiler, a **mud drum** *h*, connected to the shell by the nozzle *h'*, is provided. Attached to the mud drum is a blow-off pipe *h''* provided with a stop-cock, not shown in the figure, by means of which the sediment may be drawn off, or the boiler emptied. A similar drum *k* is attached to the rear of the boilers to receive the feed-water, which passes thence into the boilers. The pipe *k'* leads to the feed-pump.

As usually set, the front ends of the boilers are supported in a cast-iron front *A* resting on the deck. The rear ends are supported by cast-iron brackets *B* placed underneath the feed drum and secured to the deck. Brickwork, lined with firebrick, forms the sides and top of the boiler setting. The bottom of the setting which forms the lower smoke flue *m* is made of wrought-iron plates *n*, *n*, lined with firebrick. The furnace *C* is placed under the front end of the boiler shell. The fuel is thrown in through the **furnace doors** *o*, *o* and burns upon the **grate** *P*, the ashes falling through the grate into the **ash pit** *D*, which is provided with doors *D'*. Behind the furnace is built the firebrick **bridge** *r*. It serves to keep the hot gases in close contact with the under side of the shell. The gases arising from the combustion of the fuel pass from the furnace *C* over the bridge *r* into the smoke flue *m*, thence into the cylindrical boiler flues *c*, *c*, whence they pass into the front connection *d* and up the smokestack, which is provided with a **damper,** shown at *e'*.

682. It will be seen, by referring to Fig. 144, that the brickwork of the setting covers the upper portion of the boiler shell in such a manner as to prevent the hot gases from coming in contact with the shell above the waterline *W*. It is a general rule in boiler construction that *under no circumstances should the fire line be carried above the water-line.* The part of the boiler shell above the brickwork, the steam drum, and the steam pipe are covered with some non-conducting material to prevent the radiation of heat.

In the flue boiler, the heating surface, according to the definition, consists of the part of the shell which is over the furnace, the rear head (both to be measured up to the underside of the top of the setting), the inside of the flues and the front head. The front head is usually omitted in calculating the heating surface.

683. A somewhat similar boiler, known as **a plain cylindrical** boiler, was formerly used. This was a boiler without any flues, set in a similar manner to the flue boiler just described. Owing to the small amount of heating surface presented, a greater number of boilers were required; hence, it was abandoned in favor of the flue boiler.

By extending the principle of the flue boiler, that is, reducing the size and increasing the number of flues, the tubular boiler was evolved. This boiler, which gives a greater heating surface than the flue boiler, has been constructed in a great variety of forms, but most of them have been abandoned, and at the present day the great majority of ocean steamers are fitted with the latest development of the tubular boiler, namely, the *Scotch*, or *drum*, boiler.

Within the last few years, however, there has been an increasing tendency to use water-tube boilers, especially for steam yachts, steam launches, and naval vessels. A number of merchant steamers have also been thus fitted, and it is quite likely that in the near future the Scotch boiler will be replaced to a large extent by various forms of water-tube boilers.

THE SCOTCH BOILER.

684. The **Scotch,** or **drum, boiler** is shown in Fig. 145. It has a cylindrical shell with flat heads. The diameter of these boilers may vary from 10 to 15 or even 20 feet; the length may vary from 7 to 11 feet. The boiler is

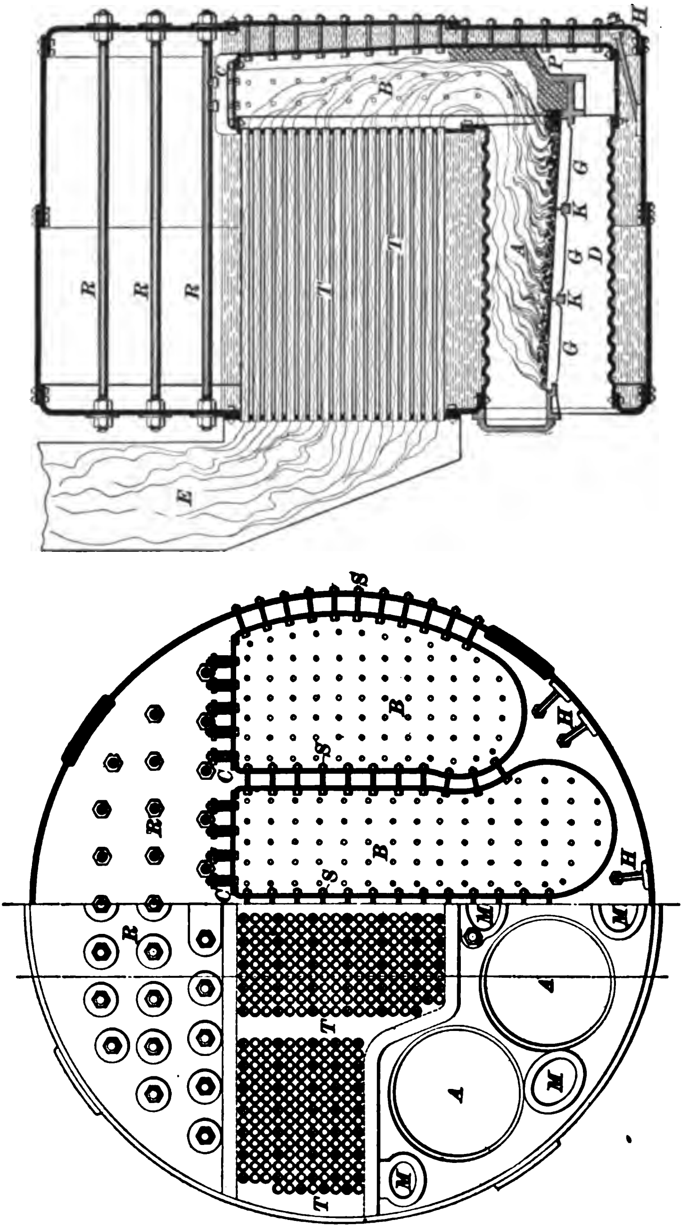

FIG. 145.

provided with two, three, or four large corrugated furnace flues *A*, *A*. (The one shown in the figure has four.) The rear end of each furnace flue opens into a **combustion chamber** *B*. Usually each flue has its own combustion chamber, but in some cases two or more flues open into a common chamber. Nests of tubes *T*, *T* extend from the front plate of the combustion chamber to the front head of the shell. These tubes place the combustion chambers *B*, *B* in communication with the large smoke chamber *E*, commonly known as the front connection. The upper part of this chamber forms the uptake, which in turn leads directly to the smokestack.

The flat heads of the shell are kept from bulging by the heavy stayrods *R*, *R*, and further by the diagonal braces, or **palm stays,** *H*, *H*. About one-third of the tubes (those marked with a cross in the figure) are threaded and provided with nuts, and thus act as stayrods for the flat surfaces occupied by the tubes. The flat sides of the combustion chambers are stayed to each other and to the rear head by the staybolts *S*, *S*. The flat tops of the combustion chambers, called the **crown sheets,** are strengthened by the **girder stays,** or **dogs** *C*, *C*. The manholes *M*, *M* give access to the various parts of the boiler. The various fittings are not shown in the figure, but are attached in convenient places.

The furnaces are placed within the corrugated flues *A*, *A*. As shown, the grate *G* is made in three sections, supported by the cross-bars *K*, *K*. Below the grate is the ash pit *D*. At the rear end of the grate is placed a firebrick plate *P* for the purpose of preventing cold air from sweeping through the ash pit into the combustion chamber without first passing through the grate.

The gases arising from the combustion of the coal pass into the combustion chamber *B*, where they undergo further combustion in contact with the air which passes through the grate. The hot products of combustion then pass through the tubes to the front connection *E*, and out through the stack.

It is seen from the figure that the flues and tubes are completely surrounded by water; likewise, the combustion chambers.

685. In Art. **680** it was stated that the heating surface of a boiler is that surface which is in contact with the fire or hot gases of combustion; the *other* surface of the plate, etc., being in contact with the water. In Fig. 145 the water is shown covering the tubes T, T to a depth of several inches. Hence, from the foregoing definition, the heating surface of a Scotch boiler consists of the following:

1. The part of the furnace flues *above* the grate.
2. The back, sides, and top of the combustion chamber.
3. The front plate of the combustion chamber, known as the **back tube-sheet.**
4. The inner surface of the tubes.
5. The part of the front head pierced by the tubes, known as the **front tube-sheet.**

As the front tube-sheet does not form a very efficient heating surface, it is customary not to consider it as such in computing the heating surface of a boiler.

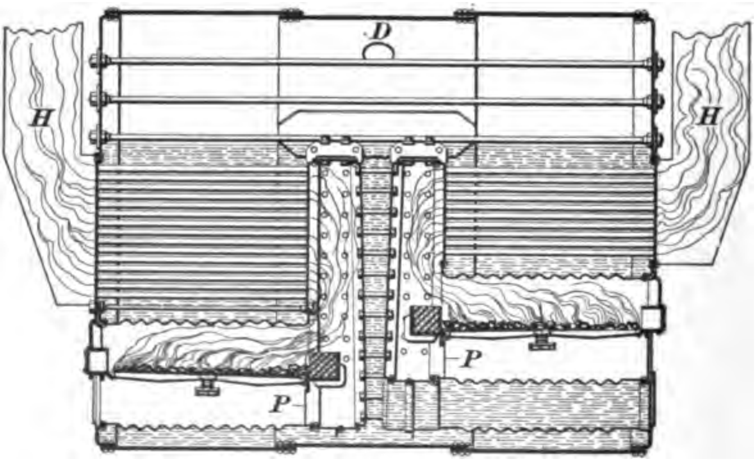

FIG. 146.

The boiler just described is known as a **single-ended**

Scotch boiler, meaning that it is provided with furnaces only at one end.

686. In Fig. 146 is shown a section of a **double-ended Scotch marine boiler.** At the left end of the figure, the section is taken through one of the lower flues, and at the right end through one of the upper flues. The furnace flues are placed in each end of the boiler, the combustion chambers being near the center. Each chamber is connected with the nearest head by a nest of tubes leading to the uptakes *H*, *H*.

The flat surfaces are stayed and braced as just described in the case of the single-ended boiler. (Fig. 145.)

The process of combustion and the path of the gases are the same as for the boiler just mentioned. The plates *P*, *P* prevent air from passing from the ash pit to the combustion chamber. The steam dome, or drum, is attached at *D*.

687. Another form of double-ended boiler is shown in Fig. 147. The furnace flues are placed in each end of the shell, but each opposite pair opens into a common combustion chamber *C*. Each of these combustion chambers has two nests of

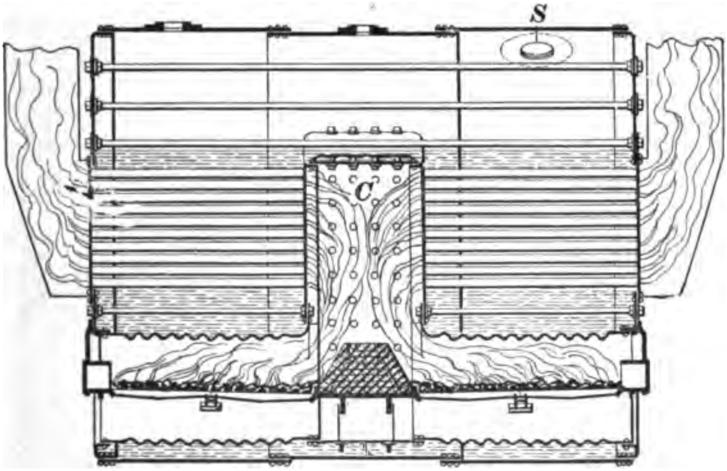

FIG. 147.

tubes, one nest connecting it with one head, the other nest with the other head. The gases from two opposite furnaces mix together in the common combustion chamber and then pass through the two nests of tubes, one-half to one smoke flue, the other half to the other. In other respects, the construction of the boiler is similar to that shown in Fig. 146. The steam dome is attached at *S*.

688. The usual method of setting a "Scotch" boiler is illustrated in Fig. 148. Two **saddles**, or **cradles**, composed of wrought-iron plates *A*, *A*, and angle irons ***a***, ***a***, bent to conform to the shape of the shell, are firmly secured to the framing of the vessel. The boiler rests in these saddles and is secured to them, if the boiler is small, by a strap passing around the boiler. As the weight of the boiler and the water it contains would throw a heavy bending strain on the saddles when the vessel is rolling or pitching, some further means of securing the boiler is necessary. Usually diagonal braces *C*, *C*, *B*, *B*, secured to the top and bottom of the boiler, a pair on each side, are employed for this purpose. One end of the brace is flattened and riveted to the shell, the other end being securely attached

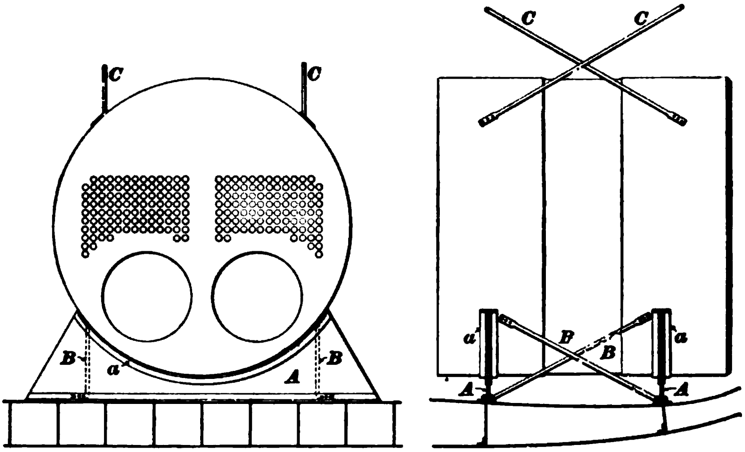

FIG. 148.

to some convenient part of the framing of the vessel. Sometimes the end of the brace is threaded and provided with nuts, and by this means connected to a bracket riveted on the boiler shell; the tension of the brace can thus be adjusted. The arrangement is shown in Fig. 149.

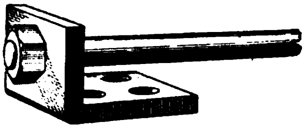

FIG. 149.

THE FIRE-BOX BOILER.

689. A **fire-box marine boiler** is illustrated in Fig. 150. The shell of the boiler is composed of two differently shaped parts riveted together. The rear part of the boiler is cylindrical; the front part is of a rectangular cross-section with vertical sides and a semicircular top. There are one or more furnaces *A* (two in this case), with vertical sides and a round top. A space is left between the two furnaces as well as between the furnaces and the sides of the boiler; these spaces are filled with water, as shown at *a*, *a*, and are known as the **water legs.** From the furnaces the large flues *c*, *c*, *c* lead to the combustion chamber, or **back connection** *B*, common to both furnaces. Two nests of tubes *T* connect the combustion chamber with the front connection, or uptake *C*. It will be noticed that the uptake is inside of the boiler and surrounded by water at the lower end and by steam at the upper end. Such an uptake is called a **wet uptake** to distinguish it from the form of an uptake used with a Scotch boiler, and known as a **dry uptake.** Access to the front and back connections is provided by means of the doors *D*, *D'*. The upper part of the uptake opens directly into the smokestack *S*, which is provided with a damper *F*. At *E* the steam drum is shown connected to the shell by the nozzles *e*, *e*. The manhole is shown at *G*. Handholes *h*, *h*, *h* are provided to facilitate the cleaning out of the water legs. The boiler is supported at the rear by the cast-iron saddle *H*, to which it is firmly bolted, the saddle in turn being securely attached to the timbers or framing of the

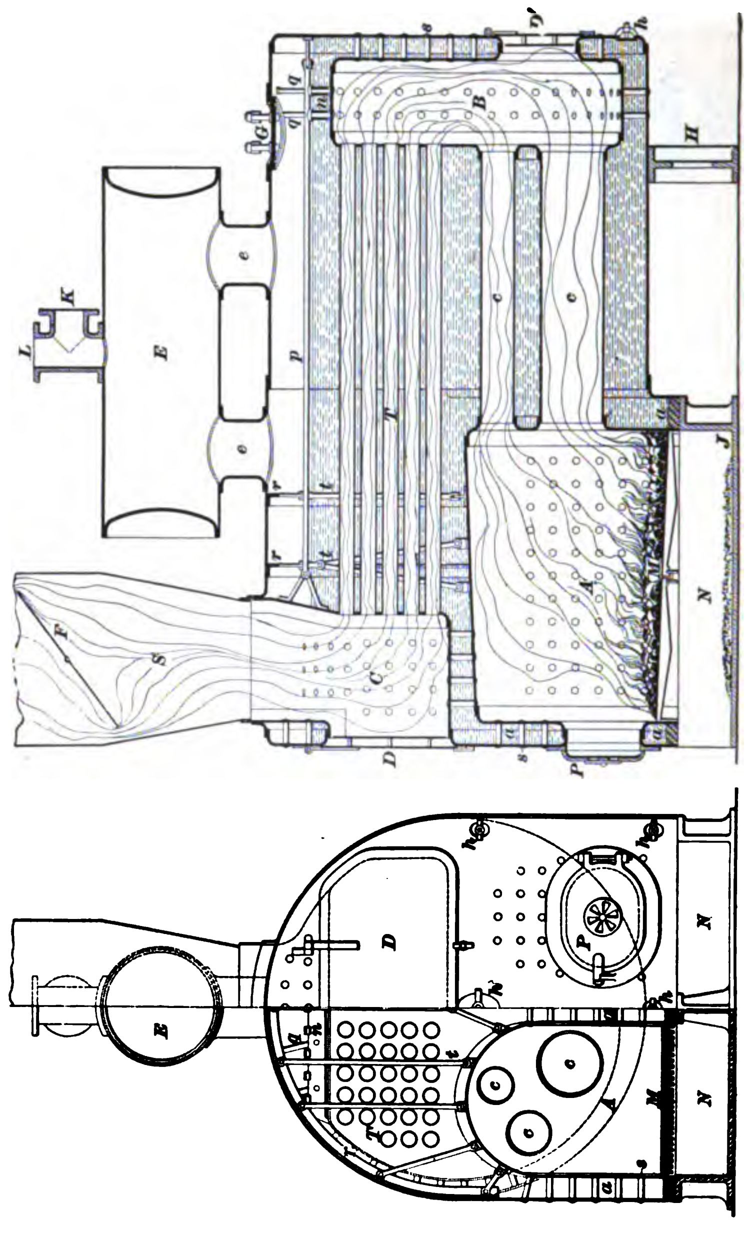

FIG. 150.

bottom of the vessel. The front part of the boiler is fastened to a cast-iron frame, shown at *J*. This frame is bolted to the framing, and also forms the ash pit *N*. Since the flat sides of the furnaces and shells would bulge on account of the pressure, they must be braced or stayed. This is accomplished by the screw stays *s*, *s*. Similar screw stays are also employed to connect the combustion chamber with the rear head, strengthening them both against bulging; this kind of stay is also used between the combustion chamber and outer shell except at the top, which is strengthened by the girder stays *n*, supported by the sling stays *q*, *q*. The top of the furnace is stayed by the toggle braces *t*, *t*, attached to rings, shown at *r*, *r*, made of angle iron and riveted to the shell. Toggle braces *p* are used to stay the flat surfaces of the rear head and of the uptake. At *K* the steam pipe is attached; at *L*, the safety valve; the furnace door is shown at *P*; the grate at *M*.

690. The gases of combustion pass from the furnaces *A* through the flues *c* to the combustion chamber *B*, whence they pass through the tubes *T* to the front connection *C* and up the smokestack *S*. As shown in this figure, the water legs only extend down a little below the grate, the ash pit being formed by the frame *J*. Sometimes there is a water space below the ash pit; that is, the furnace and ash pit are entirely surrounded by water; a boiler constructed in this manner is known as a **wet-bottomed boiler.** The type illustrated in Fig. 150 is called a **dry-bottomed boiler.** Vertical boilers are found but very seldom, and have, in this country, gone almost entirely out of use for marine purposes.

In the fire-box boiler shown, the heating surface is formed by the inside of the furnaces above the grate, the inside of the flues *c*, the sides, top and bottom of the combustion chamber, deducting, of course, the spaces taken up by the flues *c*, *c* and the door *D'*, the inside of the tubes, and the sides and bottom of the uptake. Only in case of a wet uptake is its surface to be taken as heating surface, and then only up to the water-line.

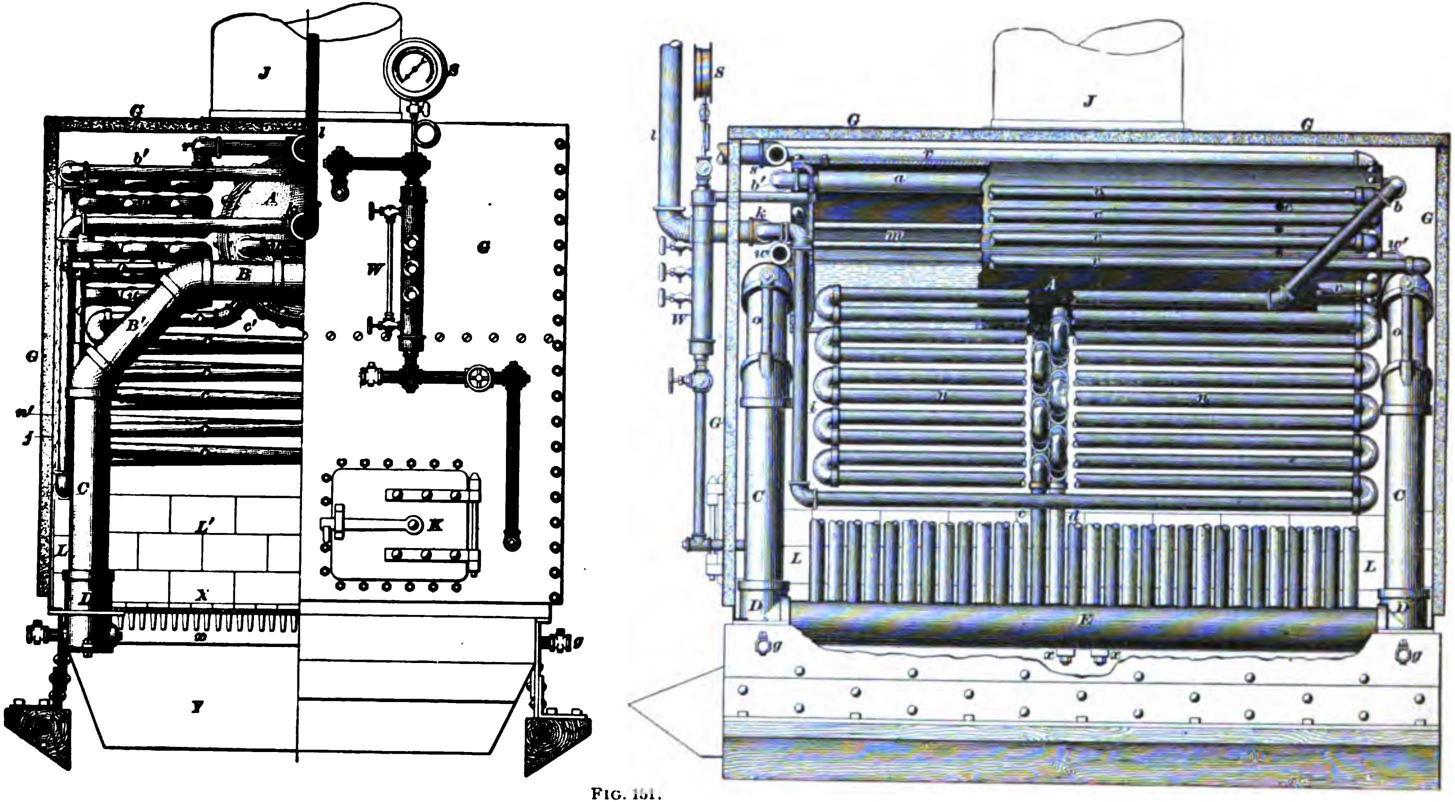

FIG. 151.

WATER-TUBE BOILERS.

691. The boilers previously described are known as **fire-tube boilers.** The gases of combustion pass through the tubes, the water being in contact with the outside of the tubes. Since all fire-tube boilers are of great weight (which weight is increased by the large volume of water contained by them), and, since they also occupy considerable space, their use is objectionable in small vessels. To overcome the above disadvantages, a distinct class of boilers, known as **water-tube boilers,** has been designed. These boilers have come into extensive use during the past few years, and their use is gradually extending to larger vessels. In a water-tube boiler the water to be evaporated is contained *within* the tubes, the gases of combustion being in contact with the outside of the tubes.

THE ROBERTS WATER-TUBE BOILER.

692. **A Roberts water-tube boiler** is shown in Fig. 151. The construction of the boiler is as follows:

A cylindrical steam drum A forms a receptacle for a small body of water, the space above the water forming the steam space; it is made of a steel or iron plate, and closed by flanged heads riveted to the shell, having the heads stayed by the stayrods shown at m. Two side pipes E (only one is shown in the figure) are connected to the front and back of the steam drum by means of the down-flow pipes C, C, the angular down-flow pipes B' and the cross pipes B. It will be noticed that on each of the T's, D D is a flange, to which are bolted the angle irons carrying the jacket G, G. A plate, on which the ends of the grate bars X, as well as the firebrick lining L', are supported, extends across the furnace in the front and rear, the plates being bolted to the flanges of the bottom T's. The grate is composed of two lengths of grate bars, which are held up at the center of the furnace by the bearing bars x, x, which in turn are supported by studs screwed into the bottom of the side pipes. The firebrick lining L rests on the side pipes. Connected to the top of the side pipes are the up-flow coils *c* and *d*; only two are shown in full, the rest being shown broken off; they start alternately from the right and left side pipes. These coils are composed of pipes and return bends, with the return bends tapped "on a spread;" that is, so as to give all the pipes an upward inclination. This is clearly shown in the front elevation, which also shows at *c'* the point at which the up-flow coil *c* enters the steam drum. The coil *d* enters the steam drum at a similar point at the opposite side. Two feed coils *u* and *v* are placed one on each side of the drum, and are supported on the small pipes *o*, *o*. Superheating coils *n* and *n'* are placed one on each side of the furnace, outside of the up-flow coils. The boiler is enclosed at the four sides and the top by a sheet-iron jacket *G G*, lined with some non-heat-conducting material. An opening is provided in the top of the jacket, through which the smokestack *J* connects with the space inside of the jacket, forming the combustion chamber, the space directly over the grate *X* being the furnace. At *W* the water column, mounted with a glass water-gauge and three gauge-cocks, is shown. The steam-gauge is shown at *S*; the furnace door at *K*, and the ash pit at *Y*.

The feed-water enters at *s* and is divided into two streams by a partition cast with the **T**'s. One stream passes through the pipe *r* into the feed coil *u*; the second stream passes through a similar pipe into the feed coil *v*. The feed-water is heated to a high temperature before leaving the coils. On leaving them, the water passing through the coil *u*, enters the steam drum at *w'*, and the water passing through *v* enters at *w*. In operation, the entering feed-water is discharged above the water-line to allow the steam which may have formed in the feed coils to rise to the top of the drum, and the water to fall to the water level. The horizontal layers of the feed coils are separated by cross pipes, shown at *c*, placed there to prevent sagging of the free ends of each layer of pipe composing the coils. In order to show the coil *u* more clearly, these pipes have been omitted in the figure. There is no water in these cross pipes.

693. The boiler being filled with water until the steam drum is about one-quarter full vertically, and the fire started, the water expands and becomes lighter much faster in the up-flow coils *c* and *d* than in the down-flow pipes *C* and *C*; this is due both to the fact of the small pipes absorbing a greater amount of heat, owing to their being directly over the fire, and also to their being of a smaller diameter, because then there is more heating surface for a given volume of water in the pipes. The result is that the water at once commences to rise in the small up-flow coils and to fall in the large down-flow pipes. As soon as steam bubbles commence to form, this movement of the water becomes more rapid, as water holding these bubbles is much lighter than solid water, even if their temperatures are the same. To assist the upward flow of the water, the pipes *c* and *c* are given the upward inclination previously mentioned.

The up-flow coils now begin to throw currents of water mixed with steam bubbles into the drum. The steam bubbles break, the steam rises to the top of the drum, and the solid water flows out of each end into the cross pipes leading to the down-flow pipes. The lower part of the bottom **T**, *D*, of each down-flow pipe forms a **mud,** or **sediment, pocket;** that is, it provides a quiet place for the deposit of foreign matter held in suspension in the feedwater. This deposit may be drawn off by means of the cocks *g*, *g*. Just above the mud pockets the downward currents make a turn at right angles into the side pipes, part of the water flowing into the up-flow coils as it passes along underneath them. The two currents, one from each end of each side pipe, meet in the center of the side pipes and there form an eddy, the only non-circulating water in the boiler excepting that in the mud pockets. The steam at the top of the drum passes into a **spray pipe,** or **dry pipe,** *a*, running from one head to the other, and drilled full of small holes on top for about half its length in the center. By locating the holes near the center, no water can enter the dry pipe when the vessel is pitching. The superheating

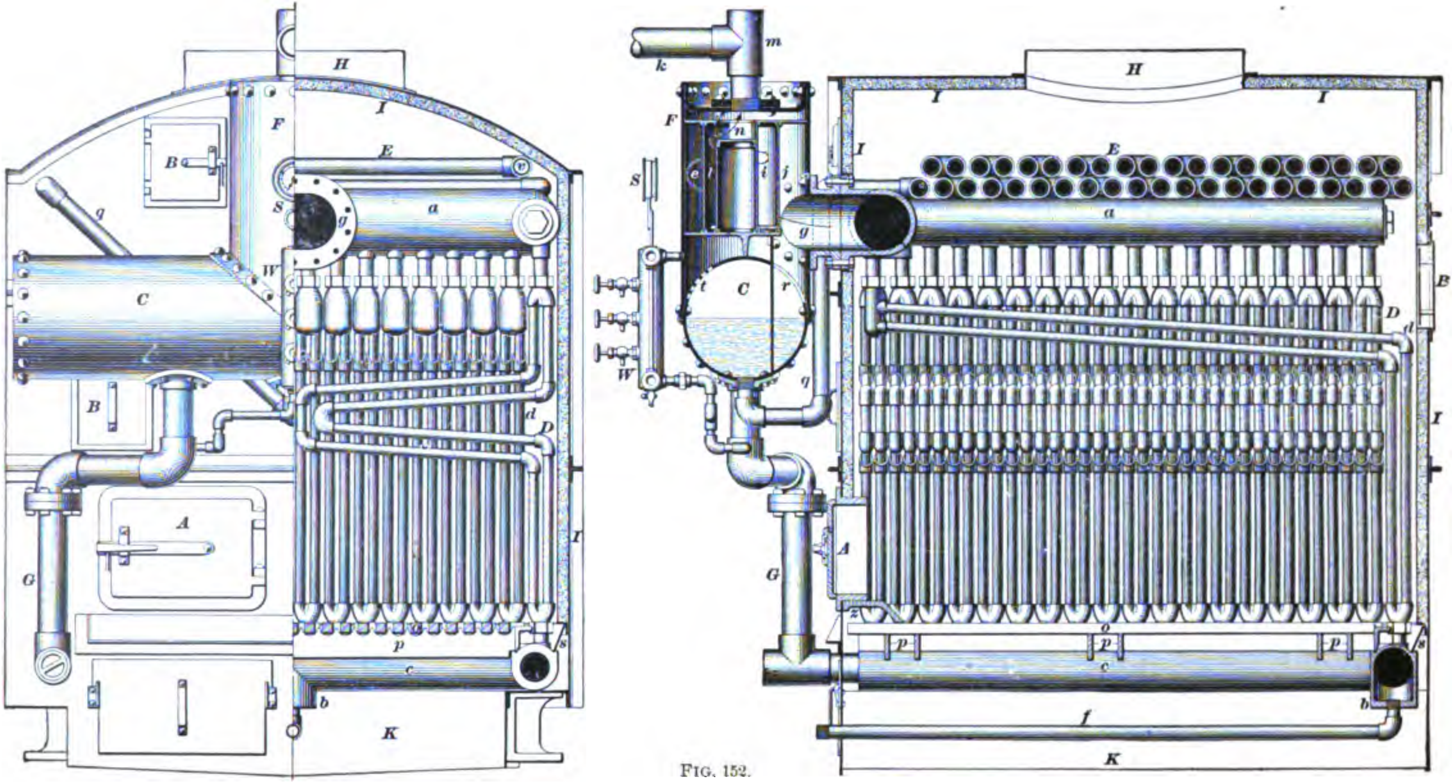

FIG. 152.

coil *n'* is connected to one end of the dry pipe by the pipe *b'*, and the superheating coil *n* to the other end by the pipe *b*. The steam flows into the top of the coils and passes downwards to the bottom, rising upwards again within the pipes *i* and *j* which unite at *k*, whence the steam passes into the pipe *l*. To the upper end of this pipe a **T** is attached to one outlet of which the safety valve is fitted, while to the other outlet the steam pipe leading the steam to the engine is connected. These connections, for want of room, are not shown. The superheating coils *n* and *n'* answer the double purpose of drying the steam and of protecting the jacket from the action of the fire. The object of the angular down-flow pipes *B'* is to prevent any part of the down-flow system (the pipes *B*, *B'*, *C*) from being thrown above the water-line by the list (i.e., inclination) of the vessel, which would greatly interfere with the circulation.

694. The Roberts boiler is an example of one of the two general types into which water-tube boilers are divided; viz., **drowned-tube** boilers, or boilers in which the upper end of the tubes is submerged in water, and **dry-tube** boilers, or boilers in which the upper end of the tubes is filled with steam only, or such water as is carried up by the violent ebullition of the water within the lower part of the tubes. From the construction of the Roberts boiler, it follows that it belongs to the drowned-tube type.

THE ALMY WATER-TUBE BOILER.

695. An example of the dry-tube type is the **Almy water-tube boiler,** shown in Fig. 152. The construction of the boiler is as follows:

A continuous manifold *c* which forms the base of the boiler, extends along the sides and across the back of the boiler. At the top is a similar manifold *a* which extends along the sides and across the front of the boiler. To form the heating surface, a series of up-flow coils *D*, *d*, made up of pipes connected together by elbows, return bends, and four-way **Y** fittings, are connected to the top and bottom manifolds and with each other by means of unions. The coils *D* at the sides of the boiler rise from the manifold to a proper height to form the top of the furnace; they then extend half way across the furnace and return back to the sides; thence up, connecting to the side pipes of the top manifold *a*. The coils *d* which form the back of the furnace rise from the back manifold to a height sufficient to cross over above and at a right angle to those forming the top of the furnace; they then pass to the front and connect to the manifold extending across the front at the top. A feed coil *E*, serving to heat the feed-water, and consisting of two layers of pipe connected together by return bends, thus forming a continuous pipe, rests on the top manifold. The feed-water enters this coil at *v*, and after passing through leaves it by means of the pipe *q* connected to the bottom of the horizontal water reservoir *C*, which extends across the front of the boiler. The top manifold *a* is attached by means of the nozzle *g* to the vertical separator *F*, placed on top of the horizontal water reservoir. A jacket *I I*, lined on the inside with some non-heat-conducting material, encloses the boiler on the four sides and at the top. To the ring *H*, on top of the jacket, the smokestack is attached. The grate is composed of square iron bars *o*, *o* which are supported by the bearing bars *p*, *p*. To prevent any entrance of air to the furnace, otherwise than through the grate, baffle plates *s* extend around the sides and back of the furnace. At the front the dead plate *z* serves the same purpose. The boiler is provided with a water column *W*, mounted with a glass water-gauge and three gauge-cocks, and which also carries the steam-gauge *S*. At *A*, the furnace door is shown. *B*, *B* are doors in the jacket to allow inspection, etc., of the inside, and *K* is the ash pit. In the center of the bottom back manifold, a mud or sediment pocket *b* is located, whence the sediment may be drawn off or the boiler emptied, through the pipe *f*. Both the separator and the water reservoir, as well as the down-flow pipes *G*, *G*, which are connected to the water reservoir and extend down to the manifold at each side of the furnace, are outside the jacket, and hence, in a cool place.

696. The operation of the boiler is as follows: The boiler being filled with water until the reservoir *C* is about half full vertically, the fire is started. The water in the coils *D* and *d* expanding, and, hence, becoming lighter, rises to the top manifold, carrying the steam bubbles with it; cooler water, coming from the reservoir *C* through the down-flow pipes *G*, constantly takes its place, and, in turn, becomes heated and rises. On reaching the top manifold the steam bubbles burst, the steam and entrained water rising to the top; the water thrown into the top manifold by the violent ebullition within the tubes, after flowing along the bottom, falls through the opening *r* in the horizontal water reservoir to the water level. The steam and entrained water pass into the separator *F*, where, by curved partitions, the steam is constrained to move in a spiral path; that is, it passes from *g* into *j*; thence into *e*; thence, into the passage *i*, whence it passes into *l* and out of the separator at *n*, entering there the bottom of the steam pipe *k*. By giving the steam a whirling motion, the particles of entrained water are, by the action of centrifugal force, thrown against the curved partitions; the water drips down on these partitions and flows back into the reservoir *C* through an opening in the base of the separator (not shown in the figure) and through the holes shown at *t*. To the top of the **T** shown at *m*, the safety valve is attached.

From the construction of this boiler, it follows that it belongs to the dry-tube type.

697. In all water-tube boilers the whole or greater part of the heating surface is formed by the outer surface of the tubes of which the boiler is composed. The lower half of the steam drum, measured up to the water-line, is also considered as heating surface.

698. There are various points in favor of water-tube boilers for marine service. It was mentioned before that they occupy less space and weigh less for the same power than fire-tube boilers. In addition to these points, a water-tube boiler is less likely to a disastrous explosion, both on account of the small quantity of water it contains, and also the greater comparative strength of the tubes, owing to their small diameter. Instead of a large body of highly heated water becoming suddenly liberated and converted into steam, as would be the case if a sheet of a fire-tube boiler ruptures, the most serious trouble experienced in this direction with a water-tube boiler is the rupturing of one or more tubes, allowing the water to escape gradually. This undoubtedly insures greater safety to the attendants and to the vessel. An objection often raised against water-tube boilers is that the splitting of one tube will throw the boiler out of service, while a split tube in a fire-tube boiler may be repaired temporarily by plugging it. Another objection often raised is the apparent difficulty of maintaining the water level constant. It is plain that as the water-tube boiler contains but a limited quantity of water, it requires constant feeding to make up for the water evaporated into steam. But, as most engineers having charge of this class of boilers seem to experience no trouble in this direction, the trouble is probably chiefly imaginary. It is claimed that water-tube marine boilers are more efficient, that is, that they utilize more of the heat generated by the combustion of the fuel, than the other type.

699. There is an endless variety of water-tube boilers, but as the principles of operation are practically the same in all of them, no others will be illustrated here. After carefully studying what has been said in regard to water-tube boilers, the student will not experience any difficulty in understanding the operation of any water-tube boiler of which he may be placed in charge.

DONKEY BOILERS.

700. Every sea-going steamer carrying passengers must be provided with an auxiliary, or **donkey, boiler** of sufficient capacity for working the fire pumps. Such boilers must not be placed below the lower decks, except on single-deck vessels. All the rules and regulations governing the construction, inspection, and fittings of the main boilers apply also to donkey boilers. There are no special types of donkey boilers; either Scotch, fire-box, or water-tube boilers may be used. The smokestack of a donkey boiler is usually carried up inside the stack of the main boilers, and held in position by suitable bracing. It is provided with a separate damper, and should not enter the stack below the main damper.

THE CONSTRUCTION OF BOILERS.

BOILER MATERIALS.

701. The **materials** used in boiler construction are *wrought iron*, *steel*, *cast iron*, *semi-steel*, and to some extent *copper* and its alloys, chiefly compositions. The qualities required of boiler material are: (1) Ductility, in order that it may undergo, without injury to its tensile strength, the various processes to which it must be subjected in being made up into boiler parts. (2) Tensile strength to resist the stresses due to the steam pressure. (3) Toughness and elasticity.

702. **Wrought iron** was, until a few years ago, the only material of which boiler shells, fire-boxes, and tubes were made. The wrought iron used for marine boilers is usually of the grade known commercially as C H No. 1 flange iron. This has a tensile strength of from 50,000 to 65,000 pounds, is quite homogeneous, that is, it has an even texture, and is not very fibrous; it does not blister nor crack much in the fire, stands repeated reheating, and flanges well. The iron of high tensile strength is often found to lack homogeneousness as well as ductility. As a measure of ductility, wrought iron used in the construction of marine boilers must show a reduction in area of 15% of the original area for iron of 45,000 pounds tensile strength, and an additional 1% for each additional 1,000 pounds up to 55,000 pounds tensile strength. Wrought iron of 55,000 pounds tensile strength and over, showing a reduction in area of 25%, is deemed to have the lawful ductility.

Iron used for rivets should be of the very best quality; it should be soft and tough; its tensile strength should not exceed 50,000 lb. per sq. in., and a good rivet should bend double while cold without fracture.

703. Steel is now rapidly supplanting iron as a boiler material. The steel used in boiler construction belongs to the class known as soft or mild steel. It exceeds iron in tensile strength, thus permitting the use of thinner sheets; it is as ductile as and more homogeneous than iron, and by the modern processes it can be manufactured more cheaply than iron. Each sheet of steel plate to be used in the construction of marine boilers coming under the jurisdiction of the Board of Supervising Inspectors of Steam Vessels, United States Treasury Department, must be tested by one of the boiler inspectors. A test piece is cut from the plate and pulled asunder in a testing machine. The sample must show an elongation of at least 25 per cent. in a length of 2 inches for thicknesses of plate up to and including one-quarter of an inch; it must show an elongation of at least 25 per cent. in a length of 4 inches for thicknesses over one-quarter of an inch and up to and including seven-sixteenths of an inch. For thicknesses above seven-sixteenths and under one and three-quarter inches, the elongation must be at least 25 per cent. in a length of 6 inches. Steel plates must not contain more than .06 per cent. of phosphorus and .04 per cent. of sulphur, to be determined by a verified analysis. Steel made by the Bessemer process is allowed to be used only for tubes, water pipes, and steam pipes. When used for this purpose, Bessemer, acid, or basic open-hearth steel must show the following physical properties: Elastic limit per square inch, 30,000 to 35,000 lb.; tensile strength, 50,000 to 60,000 lb.; elongation in an 8-inch specimen, 20 to 25 per cent.; reduction of area, 45 to 50 per cent.

All steel plates of one-half inch thickness and under must show a reduction of area of not less than 50%. Steel plate over one-half inch in thickness up to three-quarter inch must show a reduction of area of not less than 45%. All steel plate exceeding a thickness of three-quarter inch is considered to have the lawful ductility if it shows a reduction of area of not less that 32.5%.

The tensile strength should not be much above 65,000 lb., for though higher strengths may be easily obtained, the plates will be harder and less ductile.

The sample taken from each plate must also be capable of being bent to a curve, the inner radius of which is not greater than one and one-half times the thickness of the plate. Before bending, each sample must be heated to a low cherry red, and quenched in water having a temperature of 82° Fahrenheit.

Steel used for rivets is of the grade known commercially as "Extra Soft Steel," having a tensile strength of from 45,000 to 55,000 lb., with an elongation of 28 per cent., and an elastic limit of not less than one-half the ultimate tensile strength. It should not contain more than .04 per cent. of phosphorus and .04 per cent. of sulphur.

703*a*. Iron or steel of high tensile strength is used for the parts of a boiler not subjected to sudden changes of temperature, such as the shell and other parts remote from the intense heat of the fire.

For furnace plates neither iron nor steel exceeding 50,000 pounds tensile strength is used, experience having shown that material of a higher tensile strength is too hard and brittle for this purpose. No iron nor steel plates of a tensile strength less than 45,000 pounds are allowed to be used in the construction of any of the parts of a marine boiler.

704. **Cast iron** is used only in the construction of some of the boiler fittings. The rules and regulations of the Board of Supervising Inspectors provide that no cast-iron nozzles, branch pipes, or elbows shall be used in connecting steam drums, superheaters, branch pipes, or steam pipes to boilers, and in no other part of steam pipes, nor for flanges, unless the cast iron has been officially tested and has shown a tensile strength of not less than 30,000 lb.

704*a*. Semi-steel is the name given to cast iron which has been partially decarburized. It may be used for nozzles, stop-valves, branch pipes, elbows, slip joints, flanges to boilers, tee pipes, and water and gauge cock pipes, or columns, when it has been officially tested and then shown a tensile strength of not less than 24,000 lb.

705. Copper is used chiefly in the construction of the piping connected to the boiler, as it withstands the corrosive action of sea-water better than any other material.

706. Composition metal, an alloy of copper and zinc or copper and tin, is used in the construction of fittings, being somewhat cheaper than pure copper, easier to cast, and withstanding the action of sea-water almost as well.

707. The boilermakers receive the flat plates from the rolling mill. The first operation is to cut or shear the plates to the desired size; after being sheared they are placed in a machine which planes the rough edges. Next, the rivet holes are drilled in the edge of the plate. The plate is then passed through large rolls and bent to a cylindrical form and in such a manner that the corresponding rivet holes in the two edges come opposite one another. It is very much better practice, however, to drill the plates after they are rolled to shape. In this case, one or two holes, to serve as bolt holes for clamping purposes, are put in previous to bending, to receive after bending bolts that hold the plates together while the rest of the rivet holes are drilled; the plates are next taken apart and the burrs removed. They are finally put together again and are then ready for riveting.

RIVETS AND RIVETED JOINTS.

708. Common forms of rivets are shown in Figs. 153 to 157. In Figs. 153 and 154 are shown examples of hand riveting; in the first case, the head is hammered down to a conical shape, while in Fig. 154 the rivet has a **cup,** or **snap,** head. This form of head is produced by first hammering the rivet down roughly and then finishing the head by a cup-shaped die called the **button set.** The dotted lines in the figures show the shape of the rivet shank before being upset by the hammer.

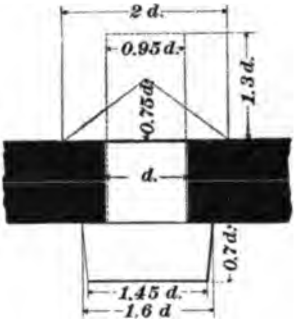

FIG. 153.

The rivets shown in Figs. 155 and 156 are examples of machine riveting. The rivet is placed between two dies which are forced together by heavy steam, hydraulic, or pneumatic pressure. When carefully done, machine riveting is much to be preferred to hand riveting.

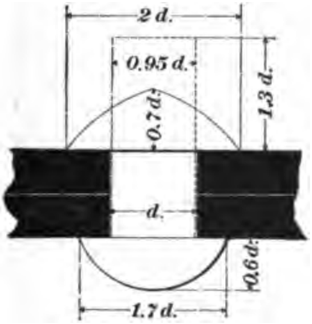

FIG. 154.

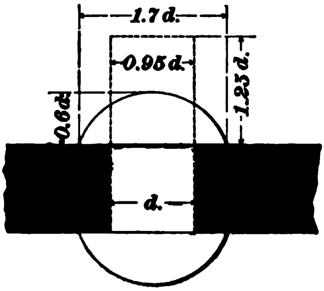

FIG. 155.

A rivet with a **countersunk** head is shown in Fig. 157. Such riveting is sometimes necessary where a smooth surface is needed to attach boiler mountings. As shown in the figures, the rivets, before being headed, are slightly smaller than the hole, so that they may be easily inserted. The upsetting action of heading the rivets causes them to fill the holes when finished.

In Figs. 156 and 157 it will be noticed that there is a slight countersink in the plate just underneath the rivet head. This is considered good practice, since, on account of it, the heads are not so liable to fly off under strain.

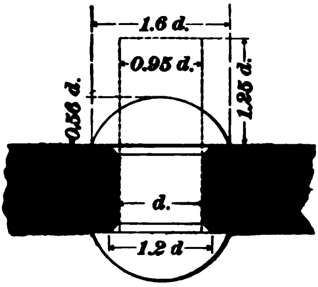

FIG. 156.

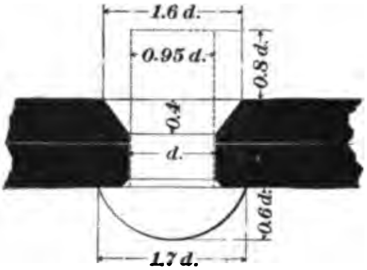

FIG. 157.

709. Riveted joints of different forms are shown in Figs. 158 to 161. When one plate overlaps the other, and the two are joined with one or more lines of rivets, as shown in Figs. 158 and 159, the joint is said to be **lap-riveted.** When, however, the plates are placed edge to edge, as in Figs. 160 and 161, and the joint is covered with one or two plates, usually called *butt straps*, the joint is called a **butt joint.**

710. Fig. 158 represents a **single-riveted lap joint;** that is, the plates are overlapped and joined with one row of rivets. The distance *p* from center to center of the rivet holes is called the **pitch** of the rivets.

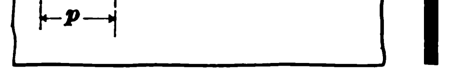

FIG. 158.

711. Fig. 159 shows a **double-riveted lap joint.** The rivets may be **staggered,** as shown in the figure, or placed one behind the other, similar to those of Fig. 160. In the latter case the joint is **chain-riveted.** In Fig. 159 the distance between the center of a rivet in the one row and the center of the nearest rivet in the other row is called the **diagonal pitch.**

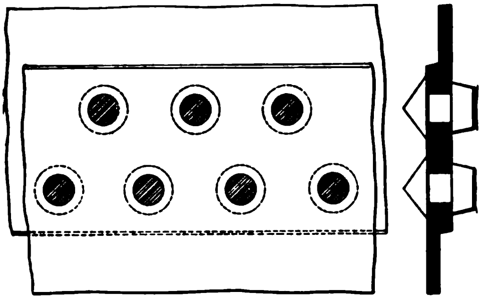

FIG. 159.

712. A **butt joint** with a single **butt strap** is shown in Fig. 160, while a butt joint with two butt straps is shown in Fig. 161. Either may be riveted with one, two, or more rows of rivets. Fig. 161 shows an example of chain-riveting. The butt joint with two straps is stronger than one with a single strap, since the rivets tend to shear off in two places instead of one, and may therefore be of smaller diameter. In other words, the rivets in the former case have double the shearing surface that they have in the latter. Butt joints

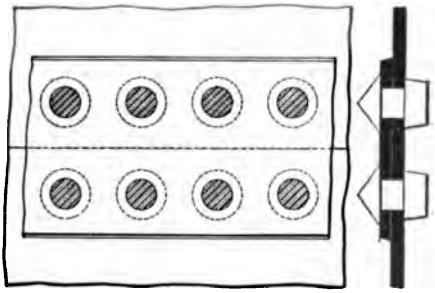

FIG. 160.

are generally used for plates over ½ inch thick, and are taking the place of lap joints in good designs of smaller work. When one butt strap is used on a butt joint, its thickness must in no case be less than the thickness of the plate; when two butt straps are used, the thickness of each must not be less than ⅝ of the thickness of the plate.

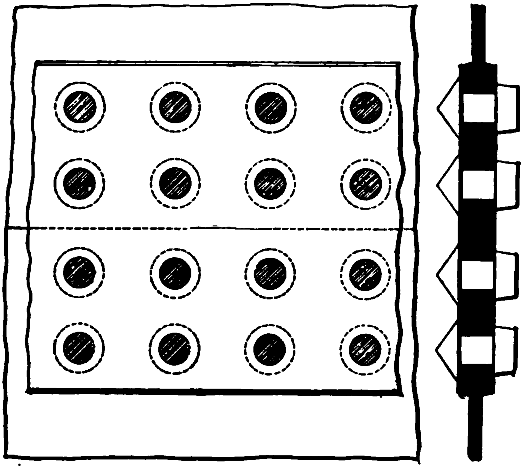

FIG. 161.

THE ARRANGEMENT OF JOINTS AND PLATES.

713. The seams of a boiler that run in the direction of its length are called the **longitudinal seams;** those running around the boiler are known as the **girth, circular,** or **transverse seams.**

Wherever a girth seam occurs, the longitudinal seams should break joint, as shown in Fig. 162. In order to make a tight joint where the three plates come together, the inner plate of the longitudinal joint must be hammered thin at the edge, as shown in Fig. 163.

In the construction of both vertical and horizontal shells, it is customary to have the inside lap facing *downwards*, since if it faces upwards a ledge is formed on which sediment may deposit.

As will be shown later on, the stresses in a boiler shell are greater at right angles to the boiler axis than in the direction of the axis. For this reason, since wrought-iron plates are stronger in the direction of the fiber, they should be arranged so that the fiber runs circumferentially around the shell, that is, in the direction of the girth seams.

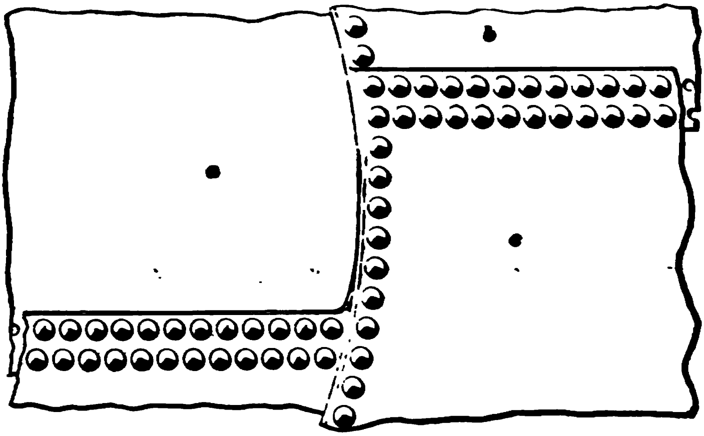

FIG. 162.

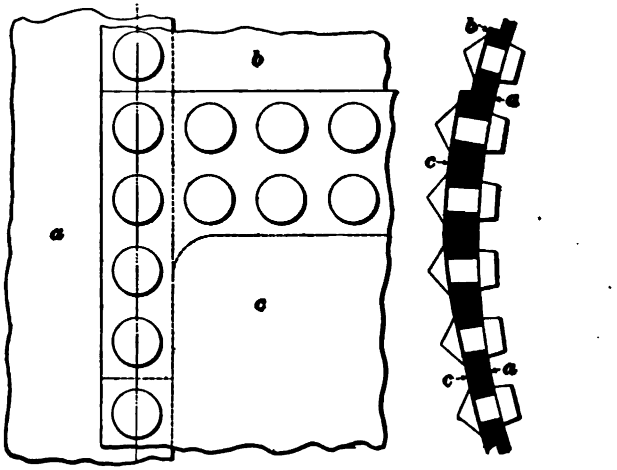

FIG. 163.

For the same reason a stronger riveted joint is employed for longitudinal seams than for girth seams, as, for instance, a double lap joint for the former and a single lap joint for the latter, as shown in Figs. 162 and 163, or butt joints for longitudinal seams and lap joints for the girth seams.

CONNECTING PLATES.

714. Different methods of connecting plates at right angles are shown in Figs. 164 to 167. In Fig. 164 the two plates are riveted to an angle iron. This construction is used sometimes for connecting the heads of a boiler to the shell. As shown in Figs. 165 and 166, the head is flanged and riveted to the shell, while in Fig. 167 the head and shell are connected by a flanged ring.

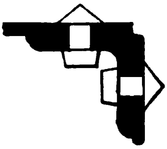

Fig. 164.

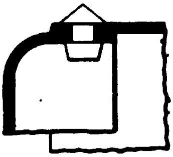

Fig. 165.

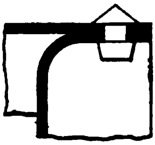

Fig. 166.

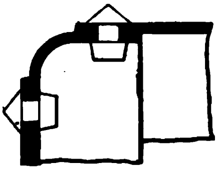

Fig. 167.

Iron for flanging should be of the best quality. The radius of the curve to which the head is flanged should be at least four times the thickness of the plate.

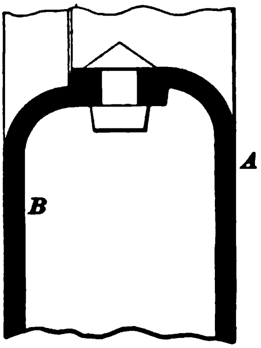

Fig. 168.

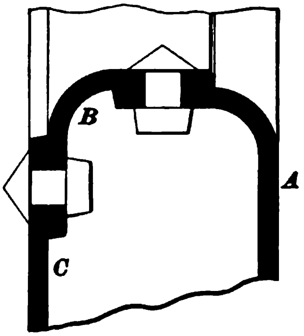

Fig. 169.

Some makers of large marine boilers prefer to flange the end plates of the shell to receive the head, which is consequently a flat disk.

715. Figs. 168 and 169 show two of the usual ways of connecting the sheet forming the head of a fire-box boiler to the inner fire-box sheet, so as to form the fire door. In Fig. 168 the two plates are flanged, *A* being the furnace sheet and *B* the front head of the boiler. Single riveting is shown here, although double riveting is frequently used. In Fig. 169 the furnace sheet *A* is flanged and connected to the front head *C* by the flanged ring *B*, in the manner shown. Around the lower ends of the water legs of dry-bottomed boilers, or around the bottom of the furnace and between the outside and inside plates, is riveted a wrought-iron ring. Instead of this construction, a flanged ring *D*, Fig. 170, is sometimes used, one of the flanges being riveted to the furnace plate *A* and the other to the shell *C*, as shown.

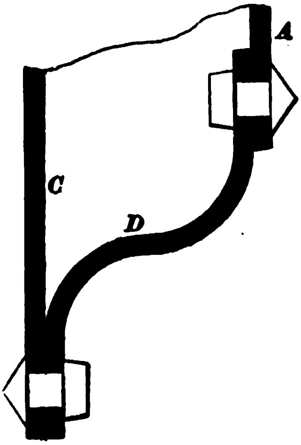

FIG. 170.

Rivet holes (for marine boilers) must always be drilled—the rules and regulations of the Board of Supervising Inspectors prohibiting punching.

716. Calking is an upsetting process applied to a riveted joint in order to make it steam-tight. The operation is shown in Fig. 171. A round nose calking tool is driven against the beveled edge of the upper plate, forcing the metal into close contact with the lower plate, thus effectually closing the seam. A tool with a sharp edge should never be used, as it is liable to score the under plate, and thus lead to grooving.

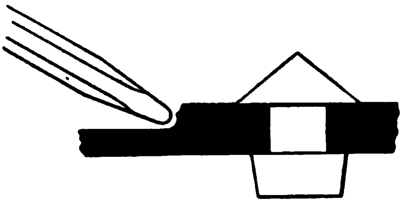

FIG. 171.

PROPORTIONS OF RIVETED JOINTS.

716*a*. The diameter of rivets, the pitch of rivets, distance between rows of rivets, the diagonal pitch, maximum pitch, and distance from center of rivets to the edge of the lap, both for single and double riveted lap joints in iron and steel marine boilers, must conform to the rules of the Board of Supervising Inspectors of Steam Vessels.

716*b*. In the following formulas the letters represent the values given below:

p = pitch of rivets in inches,
p_m = maximum pitch allowable;
n = number of rows of rivets;
p_d = diagonal pitch in inches;
d = diameter of rivets in inches;
T = thickness of plate in inches;
V = distance between rows of rivets in inches;
E = distance from edge of plate to center of rivet in inches.

Rule 132*a*.—*To find the proper diameter of iron rivets for iron boiler plates and a single-riveted lap joint, add $\frac{3}{8}$ of an inch to the thickness of the plate.*

That is, $d = T + \frac{3}{8}''$.

EXAMPLE.—What is the proper size of an iron rivet for an iron boiler plate $\frac{7}{16}$ of an inch in thickness, the lap joint being single riveted?

SOLUTION.—Applying rule **132*a***, we have

$$d = \tfrac{7}{16} + \tfrac{3}{8} = \tfrac{13}{16} \text{ of an inch. Ans.}$$

Rule 132*b*.—*To find the proper diameter of iron rivets for iron boiler plate and a double-riveted lap joint, add $\frac{5}{16}$ of an inch to the thickness of the plate.*

That is, $d = T + \frac{5}{16}''$.

EXAMPLE.—What is the proper size of an iron rivet for an iron boiler plate $\frac{13}{32}$ of an inch thick, the lap joint being double riveted?

SOLUTION.—Applying rule **132*b***, we have

$$d = \tfrac{13}{32} + \tfrac{5}{16} = \tfrac{23}{32} \text{ of an inch. Ans.}$$

Rule 132*c*.—*To find the proper diameter of steel rivets for steel boiler plates having a single-riveted lap joint, add $\frac{7}{16}$ of an inch to the thickness of the plate.*

That is, $d = T + \frac{7}{16}''$.

Example.—What is the proper size of a steel rivet to be used with a steel boiler plate $\frac{9}{16}$ of an inch thick and a single-riveted lap joint?

Solution.—Applying rule **132***c*, we have

$$d = \tfrac{9}{16} + \tfrac{7}{16} = 1 \text{ inch. Ans.}$$

Rule 132*d*.—*To find the proper diameter of steel rivets for steel boiler plates having double-riveted lap joints, add $\frac{3}{8}$ of an inch to the thickness of the plate.*

That is, $d = T + \frac{3}{8}''$.

Example.—Find the size of a steel rivet for a steel boiler plate $\frac{3}{4}$ of an inch thick, the joint being a double-riveted lap joint.

Solution.—Applying rule **132***d*, we get

$$d = \tfrac{3}{4} + \tfrac{3}{8} = 1\tfrac{1}{8} \text{ inches. Ans.}$$

Rule 132*e*.—*To find the proper pitch of rivets for iron plates and iron rivets, multiply the cross-sectional area of the rivet by the number of rows of rivets. Divide the product by the thickness of the plate, and add the diameter of the rivet to the quotient.*

That is, $p = \dfrac{.7854\, d^2 n}{T} + d$.

In a single-riveted lap joint, the number of rows of rivets is 1; hence $n = 1$. In a double-riveted lap joint, whether zigzag or chain riveted, the number of rows of rivets is 2; hence $n = 2$.

Example.—What is the proper pitch for a single-riveted joint, the plate being $\frac{5}{8}$ of an inch thick? Plate and rivets are iron.

Solution.—Applying rule **132***a*, it is found that the proper diameter of rivet is $\frac{5}{8} + \frac{3}{8} = 1$ inch. Then, applying rule **132***e*,

$$p = \frac{.7854 \times 1^2 \times 1}{\frac{5}{8}} + 1 = 2.257 \text{ inches. Ans.}$$

Example.—What is the proper pitch for a double-riveted joint, the plate being $\frac{5}{8}$ of an inch thick? Plate and rivets are iron.

Solution.—Applying rule **132***b*, the proper diameter of rivets is found to be $\frac{5}{8} + \frac{5}{16} = \frac{15}{16}$ of an inch. Then, applying rule **132*e***, and remembering that $n = 2$, we get

$$p = \frac{.7854 \times (\frac{15}{16})^2 \times 2}{\frac{5}{8}} + \frac{15}{16} = 3.146 \text{ inches. Ans.}$$

Rule 132*f*.—*To find the proper pitch of rivets for steel plates and steel rivets, multiply the constant number 23 by the area of the rivet. Multiply this product again by the number of rows of rivets. Divide this last product by the product of the constant number 28 and the thickness of the plate, and add the diameter of the rivet to the quotient.*

That is,
$$p = \frac{23 \times .7854\, d^2\, n}{28\, T} + d.$$

EXAMPLE.—What is the proper pitch of rivets for a $\frac{7}{16}$ of an inch steel plate having a single-riveted lap joint and steel rivets $\frac{11}{16}$ of an inch in diameter?

SOLUTION.—First of all it must be found whether the rivet has the lawful size. Applying rule **132*c***, it is seen that the diameter of the rivet must be $\frac{7}{16} + \frac{7}{16} = \frac{7}{8}$ of an inch. It is thus seen that the question as stated will not allow a correct answer to be given. In engineers' examinations, questions are occasionally worded this way in order to test an applicant's forethought. Substituting the correct rivet diameter and applying rule **132*f***, we get

$$p = \frac{23 \times .7854 \times (\frac{7}{8})^2 \times 1}{28 \times \frac{7}{16}} + \frac{7}{8} = 2.004 \text{ inches. Ans.}$$

EXAMPLE.—What is the proper pitch of rivets for a $\frac{5}{8}$ of an inch steel plate having a double-riveted lap joint and steel rivets?

SOLUTION.—By rule **132*d***, the proper diameter of rivets is $\frac{5}{8} + \frac{3}{8} = 1$ inch. Applying rule **132*f***, remembering that $n = 2$, we get

$$p = \frac{23 \times .7854 \times 1^2 \times 2}{28 \times \frac{5}{8}} + 1 = 3.064 \text{ inches. Ans.}$$

Rule 132*g*.—*To find the distance the center of a rivet must be from the edge of the plate, multiply the diameter of the rivet by 1.5.*

That is,
$$E = 1.5\, d.$$

EXAMPLE.—How far should a 1-inch rivet be placed from the edge of the plate?

SOLUTION.—Applying rule **132*g***, we have

$$E = 1.5 \times 1 = 1.5 \text{ inches. Ans.}$$

Rule 132*h*.—*To find the distance in inches between the center lines of the rows of rivets for double chain-riveted lap joints, add 1 to 4 times the diameter of the rivet and divide the sum by 2.*

That is, $$V = \frac{4d + 1}{2}.$$

Example.—The diameter of the rivet being $\frac{3}{4}$ of an inch, what should be the distance between the rows of rivets for a double chain-riveted lap joint?

Solution.—Applying rule **132*h***, we get

$$V = \frac{4 \times \frac{3}{4} + 1}{2} = 2 \text{ inches. Ans.}$$

Rule 132*i*.—*To find the distance in inches between the center lines of the rows of rivets for a double zigzag-riveted lap joint, multiply the sum of 11 times the pitch and 4 times the rivet diameter by the sum of the pitch and 4 times the rivet diameter. Extract the square root of the product and divide the root by 10.*

That is, $$V = \frac{\sqrt{(11p + 4d)(p + 4d)}}{10}.$$

Example.—Given, a rivet diameter of $\frac{3}{4}$ of an inch and a pitch of $2\frac{1}{2}$ inches; what should be the distance between the rows of rivets for a double zigzag-riveted lap joint?

Solution.—Applying rule **132*i***, we have

$$V = \frac{\sqrt{(11 \times 2.5 + 4 \times .75) \times (2.5 + 4 \times .75)}}{10} = 1.295 \text{ inches. Ans.}$$

Rule 132*j*.—*To find the diagonal pitch for a double zigzag-riveted lap joint, add 4 times the rivet diameter to 6 times the pitch. Divide the sum by 10.*

That is, $$p_a = \frac{6p + 4d}{10}.$$

Example.—Given, a rivet diameter of $\frac{3}{4}$ of an inch and a pitch of $2\frac{1}{2}$ inches; what should be the diagonal pitch?

Solution.—Applying rule **132*j***, we get

$$p_a = \frac{6 \times 2\frac{1}{2} + 4 \times \frac{3}{4}}{10} = 1.8 \text{ inches. Ans.}$$

716*c*. While the correct pitch of rivets is given by rules **132*e*** and **132*f***, practical considerations do not always allow the pitch thus found to be used. The rules and regulations of the Board of Supervising Inspectors of Steam Vessels make due allowance for this fact by providing for a maximum pitch for single and double riveted lap joints. The maximum pitch given by the following rules must in no case be exceeded.

Rule 132*k*.—*To find the maximum pitch of rivets allowable for a single-riveted lap joint, add 1⅝ inches to 1.31 times the thickness of the plate.*

That is, $p_m = 1.31\ T + 1\tfrac{5}{8}''$.

Example.—With a plate ¾ of an inch thick and a single-riveted lap joint, what is the maximum pitch of rivets that may be used?

Solution.—Applying rule **132*k***, we have

$$p_m = 1.31 \times \tfrac{3}{4} + 1\tfrac{5}{8}'' = 2.608 \text{ inches. Ans.}$$

Rule 132*l*.—*To find the maximum pitch of rivets allowable for a double-riveted lap joint, add 1⅝ inches to 2.62 times the thickness of the plate.*

That is, $p_m = 2.62\ T + 1\tfrac{5}{8}''$.

Example.—With a plate ¾ of an inch thick and a double-riveted lap joint, what is the maximum pitch of rivets allowable?

Solution.—Applying rule **132*l***, we get

$$p_m = 2.62 \times \tfrac{3}{4} + 1\tfrac{5}{8}'' = 3.59 \text{ inches. Ans.}$$

716*d*. It will be observed that none of the foregoing rules relate to butt joints. This is not to be regarded by the student as an omission. Butt joints with single cover-plates are somewhat stronger than lap joints, and butt joints with double cover-plates are still stronger. By treating butt joints as lap joints, as is the practice of the Board of Supervising Inspectors of Steam Vessels, an additional margin of safety is obtained. The student must not fall into the error of considering a butt joint like the one shown in Fig. 160 as double riveted, nor the one shown in Fig. 161 as quadruple riveted; while there are two rows of rivets in the first and four in the second instance, these joints are single and double riveted joints, respectively.

EXAMPLES FOR PRACTICE.

1. Find the correct diameter of an iron rivet for a $\frac{17}{32}$ of an inch iron boiler plate, the lap joint being double riveted. Ans. $\frac{27}{32}$".

2. Given, a single-riveted lap joint and a steel plate $\frac{15}{16}$ of an inch thick. What should be the diameter of the rivet? Ans. $1\frac{3}{8}$".

3. Find the proper pitch of rivets for an iron plate $\frac{3}{4}$ of an inch thick, the joint being single riveted. Ans. 2.45".

4. Find the proper pitch for a 1-inch steel plate having steel rivets and a double-riveted joint. Ans. 3.814".

5. How far should a $1\frac{3}{8}$-inch rivet be placed from the edge of the plate? Ans. $2\frac{1}{16}$".

6. Given, a steel plate $\frac{3}{4}$ of an inch thick. The joint is a double chain-riveted joint. What should be the distance between the center lines of the rows of rivets? Ans. 2.75".

7. Given, an iron plate $\frac{9}{16}$ of an inch thick, having a double zigzag-riveted butt joint. What is the proper distance between the center lines of the rows of rivets? Ans. 1.544".

8. A steel plate $\frac{5}{8}$ inch thick has a double zigzag-riveted lap joint. What is the diagonal pitch? Ans. 2.239".

9. For a single-riveted lap joint, what is the maximum pitch of rivets that may be used for a $\frac{7}{8}$ of an inch plate? Ans. 2.771".

10. What is the maximum pitch of rivets allowable with a double-riveted lap joint for a plate $\frac{19}{32}$ of an inch thick? Ans. 3.18".

STAYING OF FLAT SURFACES.

717. The surfaces of boiler shells are in general either cylindrical, hemispherical, or flat. A cylinder or sphere subjected to an internal steam pressure is *self-supporting;* that is, the steam pressure tends to maintain the cylindrical or spherical form of the vessel, and hinders distortion instead of producing it. If, on the contrary, the vessel is composed of flat surfaces, the steam pressure tends to distort it and give it an approximately spherical form. Hence flat surfaces are not self-supporting, and must be braced or stayed.

The flat surfaces commonly found in boiler construction are the flat heads of Scotch boilers and the sides, back, and top of the combustion chambers; in fire-box boilers the sides, back, and top of the combustion chamber, the crown of the furnace, the water legs, and the flat heads.

SCREW STAYS AND STAYBOLTS.

718. The most common form of a **screw stay** used in fire-box boilers is shown in Fig. 172. The bolt is threaded the entire length. It is screwed in place and the ends are headed over by hammering. A rather better form is shown in Fig. 173. The thread is turned off in the center, which increases the durability of the bolt.

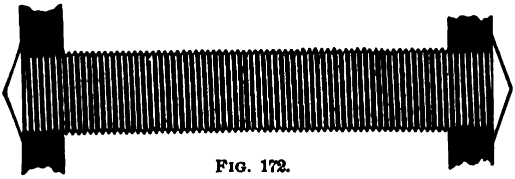

FIG. 172.

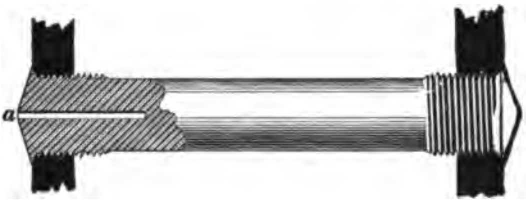

FIG. 173.

719. Fig. 174 shows a staybolt of the construction usually met with in Scotch boilers, used for staying the sides and back of the combustion chambers. It consists of a wrought-iron or steel bolt A, screwed into the two plates B and B_1, and secured by a nut at each end. An enlarged section of the nut is shown at C, Fig. 174. The face of the

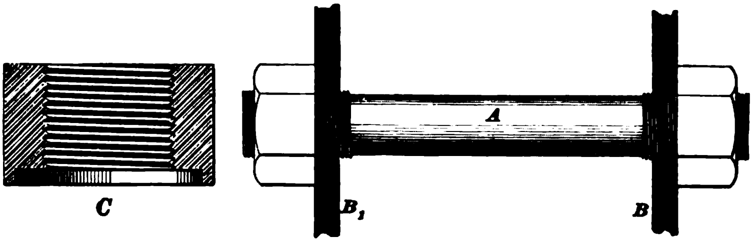

FIG. 174.

nut is recessed, the recess being filled with red-lead putty mixed with iron filings. The putty serves to make a steam-tight and water-tight joint.

All screw stays and staybolts for boilers using fresh water which are constructed after July 1, 1899, must be drilled on both ends with a central hole, as *a*, Fig. 173, having a diameter of not less than $\frac{1}{8}$ of an inch and a depth sufficient to extend at least $\frac{1}{2}$ of an inch beyond the inside surface of the sheet. Should a stay break, water will issue from this hole, thus giving notice of the break.

720. A **socket staybolt** is shown in Fig. 175. The socket consists of a tube expanded into the sheets *B* and *C*.

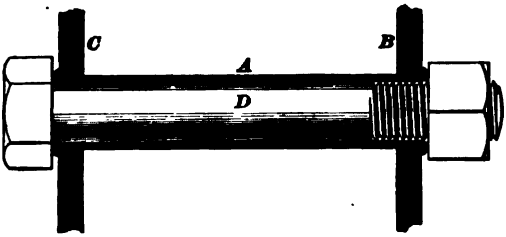

FIG. 175.

A bolt *D* fitting closely the inside of the tube *A* is passed through and secured by a nut. The ends of the socket project from the sheets and are beaded over. Sometimes the staybolt shown in Fig. 173 is protected by a socket. If salt water is used for the generation of steam, all screw staybolts must be provided with sockets to protect them from the corrosive effect of the sea-water.

STAY TUBES AND GIRDER STAYS.

721. **Stay tubes** are used to stay the tube-sheets. They are tubes of greater thickness than the ordinary tubes. The ends are enlarged and threaded, and the tube is screwed into the front and rear tube-sheets. They are secured either by a nut on the outside of the sheet, as shown in Fig. 176, or beaded over, as illustrated in Fig. 177.

Sometimes a nut on each side of the sheet is used. The top of the combustion chamber of Scotch boilers is usually supported by **girder stays,** the construction of which is shown in Fig. 178. Two girders *A*, *A'* are held apart by two distance pieces *a*, *a'*. Staybolts similar to the one shown in Fig. 174 are supported by the girders. To prevent spreading of the girders, washers *d* provided with

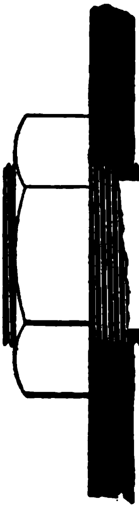

FIG. 176.

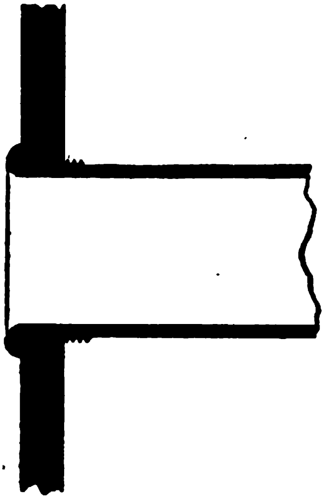

FIG. 177.

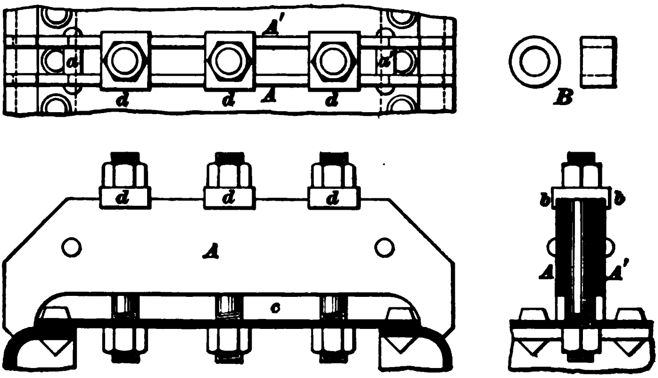

FIG. 178.

lugs *b*, *b* are used. To give access to the plate and to prevent local overheating of the plate, which would be likely to occur if the girders touched the whole length of the plate, a space *c*, generally about two inches in depth, is left between the girder and the plate. To prevent buckling of the plate by setting the staybolt too tight, a thimble or socket, shown at *B*, is sometimes placed over the staybolt and between the girders and the plate.

STAYRODS.

722. Stayrods are used to stay the flat heads of Scotch boilers; they pass through from head to head. Examples are shown in Figs. 179, 180, and 181. The end of the stayrod *B*, in Fig. 179, is enlarged and threaded and passes through the plate *A*. Two nuts and washers are provided. The larger washer is on the outside and riveted to the plate; it serves to distribute the supporting effect of the rod.

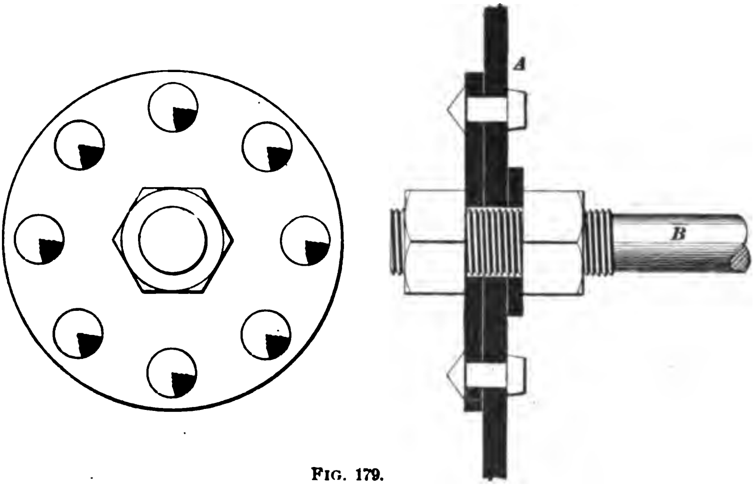

FIG. 179.

Sometimes, instead of the washer, a stiffening plate is used, covering the whole area to be braced, and placed either inside or outside. By means of the nuts, the tension of the stayrod may be adjusted, the nuts on the inside serving to lock the rod in position.

Sometimes stayrods without washers, as shown in Fig. 180, are employed. The nuts in this case are recessed, the same as those illustrated in Fig. 174.

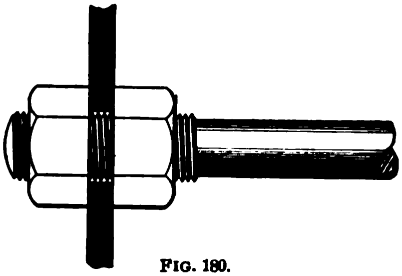

FIG. 180.

723. A somewhat different form of a stayrod is shown in Fig. 181. Two angle irons *A*, *A* are riveted to the plate. The end of the stayrod is enlarged and made square in cross-section. It is tapped to receive a bolt *C* passing through the plate between the angle irons. A leaf *D* is placed on the outside and helps to support the plate.

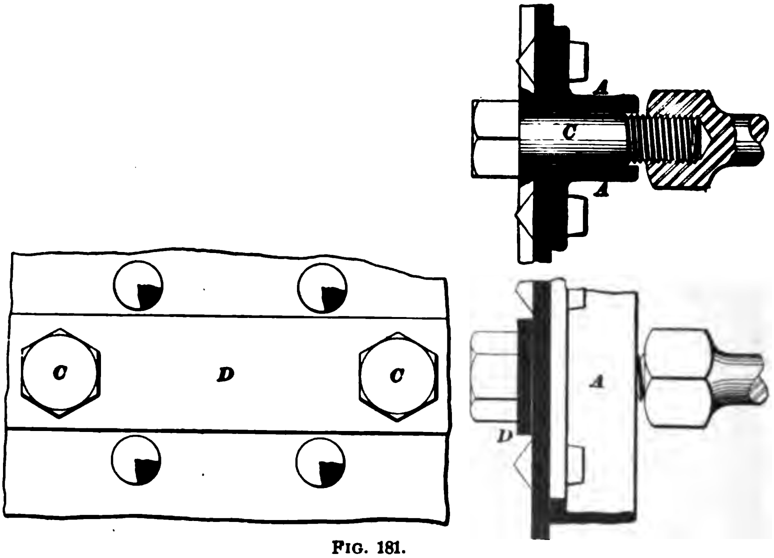

FIG. 181.

The diameters of through stayrods vary from 1¾ to 2¾ inches, according to the steam pressure.

724. Another method of connecting the ends of stayrods to the plate is shown in Fig. 182. Here the end of the stayrod *A*, instead of being threaded, is forked to receive the connection *B B*, which is riveted to the sheet to be stayed, and joined to the stayrod by a bolt *C*. The combined effective area of the two legs of connection *B B* should exceed the area of the stayrod. This connection is often used to support the lower part of the rear tube-sheets of Scotch boilers.

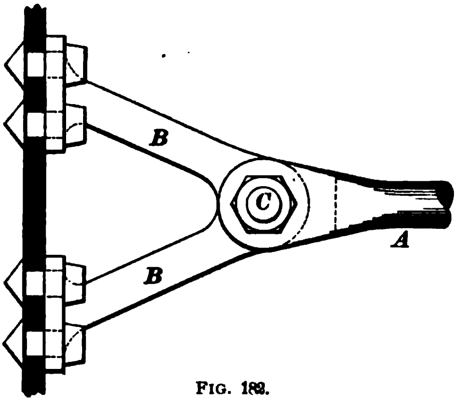

FIG. 182.

PALM STAYS AND GUSSET STAYS.

725. A **palm stay** is shown in Fig. 183. The end *B* is flattened and riveted to the shell. The other end *A* is threaded and supplied with a nut and taper washer on each side of the head, the washers having such a taper that when one of the faces is against the head, the other is parallel to the face of the nut. The hole through which the stay passes is not threaded, but is made sufficiently large to allow the stay to pass through.

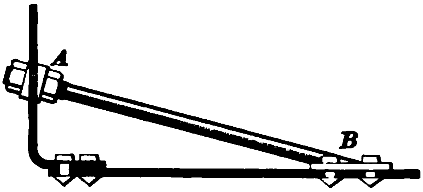

FIG. 183.

In some instances, the end *A* is bent, so as to pass through the plate at a right angle to it. This is done to do away with the use of taper washers.

Palm stays, or **diagonal stays,** as they are sometimes called, are used in locations prohibiting the use of a through stayrod. In Scotch boilers they are usually found supporting the lower part of the heads between the furnaces.

The combined area of the rivets attaching the stay to the shell should at least equal the area of the rod. The angle which a diagonal stay makes with the shell should not exceed 30°, and should be as much smaller as possible.

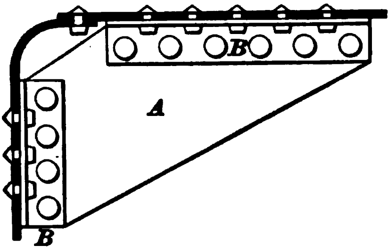

FIG. 184.

726. In Fig. 184 is shown a **gusset stay.** This stay consists of a wrought-iron or steel plate *A*, secured to the head and shell by either angle or **T** irons *B*, *B*.

Gusset stays are sometimes used for the same purpose as palm stays.

STAYING THE COMBUSTION CHAMBER.

726*a*. The lower part of the combustion chamber of Scotch boilers is often strengthened by several hoops made of angle iron and riveted to the plate. This arrangement is shown in Fig. 185.

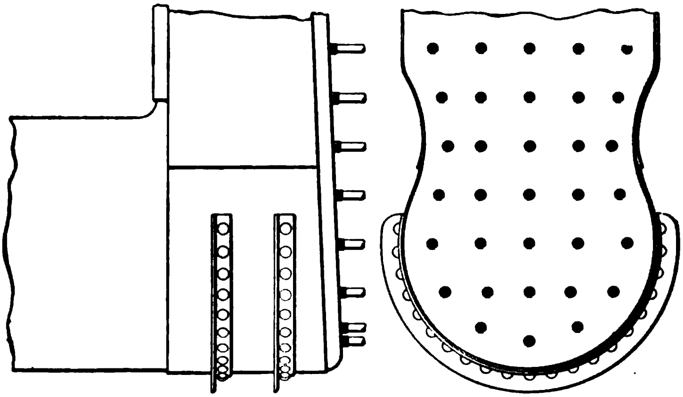

FIG. 185.

There are numerous other forms of braces and stays, but the ones illustrated here are those most commonly employed.

STRESS IN STAYS.

727. The area supported by one stay is found by multiplying the distance from center to center of stays in one direction by the distance from center to center in the other direction. The center-to-center distance is known as the *pitch* of the stays. Through stayrods are generally spaced about 15 inches from center to center; screw staybolts are generally spaced $6\frac{1}{2}$ inches center to center, the stays being arranged in vertical and horizontal rows. Their diameter varies from $\frac{3}{4}$ inch to $1\frac{5}{8}$ inches. Screw staybolts used for staying the flat surfaces of fire-boxes, furnaces, and back connections must in no case be spaced more than $10\frac{1}{2}$ inches center to center. No flat surface greater than 16 inches square must be left unsupported. In Fig. 186 the stays being spaced equidistant in both directions, the area supported by each stay is $a \times a = a^2$. If the pitch be 7 inches

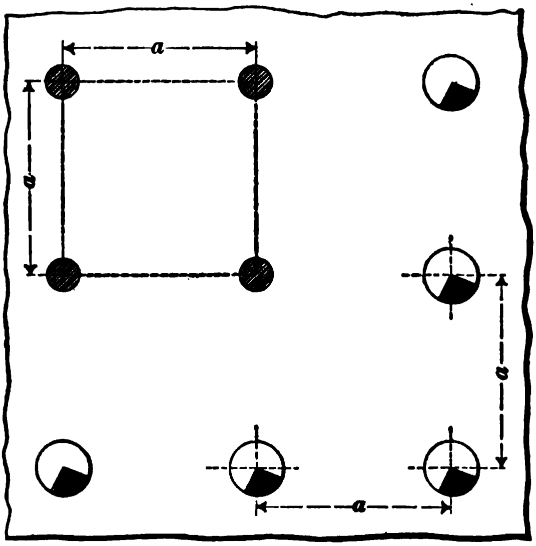

FIG. 186.

and 6 inches, respectively, the area supported by each stay will be 7 × 6 = 42 square inches.* Now, the area supported by each stay, in square inches, multiplied by the *gauge pressure* in pounds, constitutes the load borne by each stay. To bear the load safely, the stay must have a certain minimum area, which depends upon the maximum stress allowable per square inch of cross-section. The maximum stress in a wrought-iron brace or stay of any size, or steel stay up to 1¼ inches in diameter at the smallest part, is fixed at 6,000 pounds per square inch of cross-section of the stay or brace. Steel staybolts exceeding a diameter of 1¼ inches at the smallest part and not over 2½ inches in diameter may be allowed a stress not exceeding 8,000 pounds per square inch of cross-section. Steel stays above 2½ inches diameter may be allowed a stress not to exceed 9,000 pounds per square inch of cross-section. No steel stays are allowed to be forged or welded, but the ends of such stays may be upset to a sufficient thickness to allow for truing up and forming the threads.

By "area of a stay" is always meant the smallest cross-sectional area of the stay. In the case of a screw stay, the smallest cross-sectional area is that corresponding to the diameter over the bottom of the thread, unless some other part has a smaller diameter.

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* NOTE.—If extreme accuracy is required, the area of the staybolt should be subtracted from the area found by multiplying the pitch in one direction by the pitch in the other direction. Thus, in the foregoing example, assuming the area of the staybolt to be 1 sq. in., the area supported by the bolt would be 42 − 1 = 41 sq. in. This is a refinement of calculation rarely used in practice, but it is the mathematically correct way of calculating the area.

728. Let A = the area in square inches supported by a screw stay, staybolt, or stayrod;
B = the cross-sectional area of the stay in square inches;
P = the working steam pressure in pounds per square inch;
L = the load on the stay in pounds;

S = the stress per square inch of cross-section of the stay;
S_t = total allowable stress in a stay;
M = the lawful stress per square inch of cross-section of the stay;
A_t = total area to be supported, in square inches;
N = number of stays.

Rule 133.—*To find the load on a stay, multiply the area supported by the stay by the gauge pressure of the steam.*

That is, $$L = A\,P.$$

Example.—Find the load on a staybolt, the pitch of the staybolts being 8 inches either way and the steam pressure 85 pounds.

Solution.—Applying the rule just given, we have

$$L = 8^2 \times 85 = 5{,}440 \text{ pounds. Ans.}$$

Rule 134.—*To find the stress per square inch of cross-section in a stay, divide the load on the stay by the area of the stay.*

That is, $$S = \frac{A\,P}{B}.$$

Example.—Staybolts $1\frac{1}{4}$ inches in diameter are placed $8\frac{1}{2}$ inches from center to center. The steam pressure being 100 pounds, what would be the stress in the stay?

Solution.—The area of the staybolt $= 1.25^2 \times .7854 = 1.2272$ sq. in. Applying rule **134**,

$$S = \frac{8.5^2 \times 100}{1.2272} = 5{,}887.38 \text{ lb. Ans.}$$

Rule 134*a*.—*To find the total allowable stress in a stay, multiply the area of the stay by the lawful stress per square inch.*

That is, $$S_t = B\,M.$$

Example.—What is the total allowable stress in a rectangular wrought-iron stay measuring 3 in. × $\frac{1}{2}$ in.?

Solution.—According to Art. **727**, $M = 6{,}000$ lb. Then, applying the rule just given, we get

$$S_t = 3 \times \tfrac{1}{2} \times 6{,}000 = 9{,}000 \text{ lb. Ans.}$$

Rule 135.—*To find the working steam pressure allowable on a stay, multiply the area of the stay by the stress corresponding to the diameter and material of the stay. Divide this product by the area supported by the stay.*

That is, $P = \frac{BM}{A}$.

Example.—Steel stayrods 2 inches in diameter are placed 14 inches from center to center. What working steam pressure would be allowed for these stays?

Solution.—From the statement made in Art. **727**, a stress of 8,000 pounds per square inch may be allowed on a rod of this size. Applying rule **135**, we have

$$P = \frac{2^2 \times .7854 \times 8,000}{14^2} = 128.23 \text{ lb. Ans.}$$

Rule 135*a*.—*To find the number of stays required, multiply the area of the sheet to be stayed by the steam pressure, and divide by the total stress allowable in the given size of stay.*

That is, $N = \frac{A_t P}{S_t}$.

Example.—How many 1⅛-inch round staybolts are required for a sheet 40 × 50 inches to carry 90 lb. steam pressure?

Solution.—By Art. **727**, $M = 6,000$ lb. Then, by rule **134*a***, $S_t = (1\tfrac{1}{8})^2 \times .7854 \times 6,000 = 5,964$ lb. Applying rule **135*a*** and substituting values, we get

$$N = \frac{40 \times 50 \times 90}{5,964} = 30.2, \text{ say 31 stays.}$$

In an actual design, it is necessary to check this calculation after the stay centers have been plotted out on the drawing board, by rule **138**, in order to make sure that the pitch of the stays does not exceed the maximum pitch allowable for the given thickness of plate and steam pressure.

729. While the load on screw stays, staybolts, and stayrods is directly equal to the total pressure on the area supported by the stay, it is greater for palm stays.

The load on a palm stay may be found graphically. In Fig. 187 let $E\ F$ be the stay, the load on which it is desired to find. If the area of the head that is supported by this stay is 80 square inches and the steam pressure 120 pounds, the total pressure is $80 \times 120 = 9{,}600$ pounds.

Draw an indefinite line in a direction perpendicular to the head. On this line lay off the distance *A B* representing the total pressure. If a scale divided into thirty-seconds of an inch be used, each thirty-second representing 100 pounds, a line equal to $9{,}600 \div 100 = 96$ thirty-seconds of an inch $= \frac{96}{32}$ inches $= 3$ inches in length would represent 9,600 pounds. Through the point *B* draw an indefinite line *B D* perpendicular to *A B*. Through the point *A* draw a line parallel to the center line of the stay, intersecting the line *B D* at *C*. The length of the line *A C* represents the stress in the palm stay, measured to the same scale as the line *A B*. Measuring it, it is found to be $3\frac{1}{4}$ inches $=$ 104 thirty-seconds of an inch in length. Hence, the load on the stay $= 104 \times 100 = 10{,}400$ pounds.

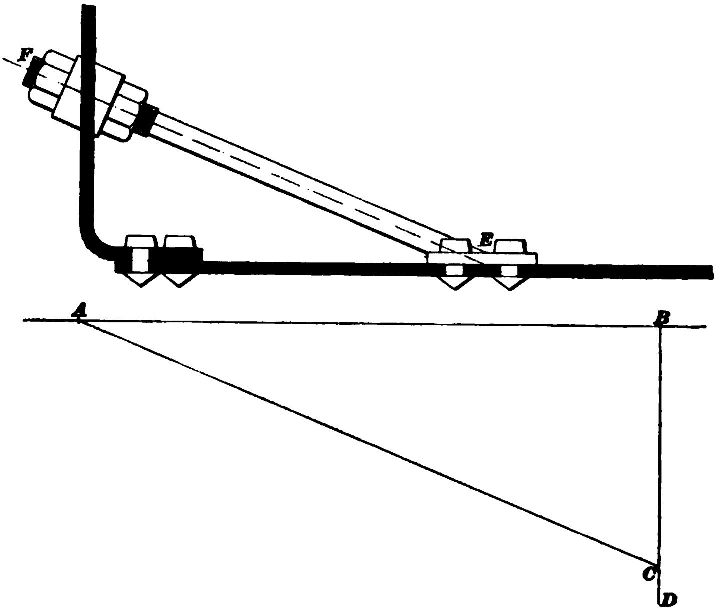

FIG. 187.

729*a*. The load on a palm stay may also be found by calculation, and will be more correct generally.

Let b = the distance in inches from the boiler head to the intersection of the center line of the palm stay with the shell;

l = length of the palm stay in inches, measured between head and shell along the center line of the stay;

$A P$ = total pressure in pounds on area supported by the stay;

L = load on the stay in pounds.

Rule 135*b*.—*To find the load on a palm stay, divide the length of the palm stay by the distance from the head to the intersection of the center line of the palm stay with the shell. Multiply the quotient by the total pressure.*

That is, $$L = \frac{l}{b} A P.$$

Example.—The palm stay of a boiler is 6 feet 6 inches long. The distance from the head to the point of intersection of the center line of the stay with the shell is 6 feet. If the area supported by the stay is 100 square inches and the steam pressure 80 pounds, what is the load on the stay?

Solution.—The total steam pressure is 100 × 80 = 8,000 lb. Then, applying rule **135*b***, we have

$$L = \frac{6\frac{1}{2}}{6} \times 8{,}000 = 8{,}666.67 \text{ lb. Ans.}$$

729*b*. The stress per square inch of cross-section is found for a palm stay in the same manner as for any other stay, that is, by rule **134**.

Example.—What is the stress per square inch if the stay in the previous example is 1⅝ inches in diameter?

Solution.—Applying rule **134**, we have

$$S = \frac{8{,}666.67}{(1\frac{5}{8})^2 \times .7854} = 4{,}178.83 \text{ lb. Ans.}$$

729*c*. The load for a given supported area of plate and steam pressure being greater for palm stays than for stay-rods, the allowable working pressure is conversely smaller.

Rule 135*c*.—*To find the working steam pressure allowable on a palm stay, multiply the result obtained by rule* **135** *by the quotient obtained by dividing the distance from the head to the intersection of the center line of the stay with the shell by the length.*

That is, $$P = \frac{b}{l} \times \frac{BM}{A}.$$

Example.—What working steam pressure would be allowed for the palm stay having the same dimensions as given in the last example, and made of iron?

Solution.—According to Art. **727**, a stress of 6,000 pounds per square inch may be allowed for iron stays. Applying rule **135*c***, we have

$$P = \frac{6}{6\frac{1}{2}} \times \frac{(1\frac{5}{8})^2 \times .7854 \times 6{,}000}{100} = 114.86 \text{ lb. Ans.}$$

Note.—The student is recommended to thoroughly familiarize himself with the principles and rules governing the load on staybolts and stayrods, as a candidate for a marine engineer's license **must be rejected** by the examining inspectors should he fail to answer any question similar to the ones given here.

STRENGTH OF STAYED SURFACES.

730. The working pressure on flat surfaces stayed by screw stays and staybolts, as shown in Figs. 172, 173, and 174, or socket bolts, as shown in Fig. 175, is determined by the following rule, provided the thickness of the plate does not exceed $\frac{7}{16}$ inch, and provided the surfaces are those of the fire-boxes, furnaces, and back connections.

Let P = the working pressure;
t = the thickness of the plate in sixteenths of an inch;*
a = the pitch of the stays.

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* Note.—In rules **136, 137, 138, 139,** and **140,** the expression "the square of the thickness of the plate in sixteenths of an inch" means the square of the numerator of the fraction expressing the thickness of the plate in sixteenths of an inch. For instance, if a plate is $\frac{3}{8}$ in. thick, the thickness in sixteenths of an inch is $\frac{6}{16}$, and the square of the thickness in sixteenths would be $6 \times 6 = 36$.

Rule 136.—*To find the working pressure allowed on the surfaces mentioned above, multiply the constant 112 by the square of the thickness of the plate in sixteenths of an inch. Divide this product by the square of the pitch of the stays.*

That is, $$P = \frac{112\,t^2}{a^2}.$$

Example.—Find the working pressure allowed on a flat plate $\frac{3}{8}$ inch in thickness, fitted with staybolts having a pitch of 7 inches.

Solution.—Since $\frac{3}{8} = \frac{6}{16}$, we have, applying the rule just given,

$$P = \frac{112 \times 6^2}{7^2} = 82.29 \text{ lb. per sq. in. Ans.}$$

For plates above $\frac{7}{16}$ in. thick, the working pressure may be found by substituting 120 for the constant 112.

Rule 137.—*To find the working pressure allowed on flat surfaces above $\frac{7}{16}$ inch in thickness, multiply the constant 120 by the square of the thickness of the plate in sixteenths of an inch. Divide this product by the square of the pitch of the stays.*

That is, $$P = \frac{120\,t^2}{a^2}.$$

Example.—A plate $\frac{3}{4}$ inch thick is supported by stays having a pitch of 10 inches. What working pressure would be allowed on this plate?

Solution.—Since $\frac{3}{4} = \frac{12}{16}$, we have, applying rule **137**,

$$P = \frac{120 \times 12^2}{10^2} = 172.8 \text{ lb. per sq. in. Ans.}$$

731. The pitch of the staybolts supporting the fireboxes, furnaces, and back connections must in no case exceed 10½ inches, nor be more than will be determined by the following rule, the thickness of the plate and the steam pressure being given:

Rule 138.—*To find the maximum pitch of staybolts, multiply the constant 112 by the square of the thickness of the plate in sixteenths of an inch. Divide this product by the steam pressure and extract the square root of the quotient. For plates exceeding $\frac{7}{16}$ inch in thickness, substitute the constant 120.*

That is, $a = \sqrt{\frac{112\,t^2}{P}}$ for plates $\frac{7}{16}$ inch or less.

$a = \sqrt{\frac{120\,t^2}{P}}$ for plates above $\frac{7}{16}$ inch.

EXAMPLE.—Find the proper pitch of the staybolts, the thickness of the plate being $\frac{3}{8}$ inch and the steam pressure 112 pounds.

SOLUTION.—Since $\frac{3}{8} = \frac{6}{16}$, we have, applying rule **138,**

$$a = \sqrt{\frac{112 \times 6^2}{112}} = 6 \text{ in. pitch. Ans.}$$

EXAMPLE.—Find the proper pitch of the staybolts, the thickness of the plate being $\frac{3}{4}$ inch and the steam pressure 172.8 pounds.

SOLUTION.—Since $\frac{3}{4} = \frac{12}{16}$, we have, applying rule **138,**

$$a = \sqrt{\frac{120 \times 12^2}{172.8}} = 10 \text{ in. pitch. Ans.}$$

732. The working pressure on plates stayed by rods with threaded ends and provided with nuts on the inside and on the outside of the plate, similar to that shown in Fig. 180, is determined by the following rule:

Rule 139.—*To find the working pressure allowed on flat surfaces fitted with stayrods provided with double nuts, multiply the constant 140 by the square of the thickness of the plate in sixteenths of an inch. Divide this product by the square of the pitch.*

That is, $P = \frac{140\,t^2}{a^2}.$

EXAMPLE.—What pressure would be allowed on a plate $\frac{13}{16}$ inch thick, fitted with stayrods as above, 14 inches from center to center?

SOLUTION.—Applying the rule just given, we have

$$P = \frac{140 \times 13^2}{14^2} = 120.7 \text{ lb. per sq. in. Ans.}$$

733. The working pressure on flat surfaces fitted with stayrods similar to the one shown in Fig. 179, that is, provided either with washers or a stiffening plate, may be determined by rule **140,** given below.

When a square washer is used riveted to the outside of the head, its thickness must be at least ½ the thickness of the head, and the length of its side must be at least equal to ⅞ of the pitch.

When a stiffening plate is used, covering the whole area to be braced, and placed either inside or outside, it must be firmly riveted to the head and be at least equal to ½ the thickness of the head.

Plates supported by double angle irons riveted to the plate, with stayrods between the two angle irons, as shown in Fig. 181, may also be allowed the working pressure computed by rule **140,** if the legs of the angle iron have a thickness at least equal to ⅔ the thickness of the plate and a depth of not less than ¼ the pitch of the stayrods.

For plates reenforced as above, the thickness of plate to be used in computing the working pressure allowed is to be taken as 80 per cent. of the combined thickness of plate and washer, or plate and stiffening plate, or plate and leg of angle, and in the rule given must be expressed in sixteenths and fractional parts of a sixteenth of an inch. For example, if the combined thickness equals 1⅛ inches, 80% of this would be $\frac{1\frac{1}{8} \times 80}{100} = \frac{9}{10}$ of an inch. But as the rule requires the thickness to be expressed in sixteenths of an inch, this fraction must be reduced to sixteenths. This may be done readily by multiplying the numerator of the fraction expressing the 80% of the combined thickness by 16, and dividing the product by the denominator of the same fraction. Thus, $\frac{9}{10}$ of an inch would be $\frac{9 \times 16}{10} = 14.4$ sixteenths of an inch.

Rule 140.—*To find the working pressure on flat surfaces stayed by stayrods, reenforced in conformity with Art.* **733,** *multiply the constant 200 by the square of the number of sixteenths of an inch representing 80% of the combined thickness of the plate and reenforcing washer, plate, or angle. Divide this product by the square of the pitch.*

That is, $P = \frac{200\,t^2}{a^2}$.

TABLE 2

Thickness of Material Required.	Greatest Length of Sections Allowable, 5 Feet.				Greatest Length of Sections Allowable, 3 Feet.												
	Least Thickness of Material Allowable.				Least Thickness of Material Allowable.												
	.18	.20	.21	.21	.22	.22	.23	.24	.25	.26	.27	.28	.29	.30	.31	.32	.33
	Diameter of Flues.																
	Over 6 and not over 7 in.	Over 7 and not over 8 in.	Over 8 and not over 9 in.	Over 9 and not over 10 in.	Over 10 and not over 11 in.	Over 11 and not over 12 in.	Over 12 and not over 13 in.	Over 13 and not over 14 in.	Over 14 and not over 15 in.	Over 15 and not over 16 in.	Over 16 and not over 17 in.	Over 17 and not over 18 in.	Over 18 and not over 19 in.	Over 19 and not over 20 in.	Over 20 and not over 21 in.	Over 21 and not over 22 in.	Over 22 and not over 23 in.
	Pounds Pressure Allowable.																
.18	189	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
.19	194	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
.20	199	184	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
.21	204	189	179	174	...	...	...	...	...	...	...	...	...	...	...	...	...
.22	...	194	184	179	172	158	...	...	...	...	...	...	...	...	...	...	...
.23	...	199	189	184	179	165	152	...	...	...	...	...	...	...	...	...	...
.24	...	204	194	189	187	172	158	147	...	...	...	...	...	...	...	...	...
.25	...	...	199	194	195	179	165	153	143	...	...	...	...	...	...	...	...
.26	...	...	204	199	203	186	172	159	148	139	...	...	...	...	...	...	...
.27	...	...	...	204	211	193	178	165	154	145	136	...	...	...	...	...	...
.28	...	...	...	...	...	200	185	172	160	150	141	134	...	...	...	...	...
.29	...	...	...	...	...	207	191	178	166	155	146	138	131	...	...	...	...
.30	...	...	...	...	...	...	198	184	172	161	152	143	135	129	...	...	...
.31	...	...	...	...	...	...	205	190	178	166	156	148	140	133	126	...	...
.32	...	...	...	...	...	...	...	196	183	171	161	153	145	137	131	125	...
.33	...	...	...	...	...	...	...	202	189	177	167	157	149	141	135	129	122
.34	...	...	...	...	...	...	...	...	194	182	172	162	154	146	139	132	126
.35	...	...	...	...	...	...	...	...	200	187	177	167	159	150	143	137	130
.36	...	...	...	...	...	...	...	...	...	193	182	172	163	154	147	141	134
.37	...	...	...	...	...	...	...	...	...	...	187	177	168	159	151	145	138
.38	...	...	...	...	...	...	...	...	...	...	...	182	172	163	155	149	142
.39	...	...	...	...	...	...	...	...	...	...	...	...	176	167	159	152	146
.40	...	...	...	...	...	...	...	...	...	...	...	...	...	172	163	156	150
.41	...	...	...	...	...	...	...	...	...	...	...	...	...	...	167	160	153
.42	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	164	157
.43	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	160

Thickness of Material Required.

.34
.35
.36
.37
.38
.39
.40
.41
.42
.43
.44
.45
.46
.47
.48
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.56
.57
.58
.59
.60

Thickness of Material Required.	Greatest Length of Sections Allowable, 30 Inches.																
	Least Thickness of Material Allowable.																
	.34	.35	.36	.37	.38	.39	.40	.41	.42	.43	.44	.45	.46	.47	.48	.49	.50
	Diameter of Flues.																
	Over 23 and not over 24 in.	Over 24 and not over 25 in.	Over 25 and not over 26 in.	Over 26 and not over 27 in.	Over 27 and not over 28 in.	Over 28 and not over 29 in.	Over 29 and not over 30 in.	Over 30 and not over 31 in.	Over 31 and not over 32 in.	Over 32 and not over 33 in.	Over 33 and not over 34 in.	Over 34 and not over 35 in.	Over 35 and not over 36 in.	Over 36 and not over 37 in.	Over 37 and not over 38 in.	Over 38 and not over 39 in.	Over 39 and not over 40 in.
	Pounds Pressure Allowable.																
.34	121	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
.35	125	120	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
.36	129	123	119	...	...	...	...	...	...	...	...	...	...	...	...	...	...
.37	132	127	122	117	...	...	...	...	...	...	...	...	...	...	...	...	...
.38	136	130	125	121	116	...	...	...	...	...	...	...	...	...	...	...	...
.39	139	133	129	124	119	115	...	...	...	...	...	...	...	...	...	...	...
.40	143	137	132	127	123	118	115	...	...	...	...	...	...	...	...	...	...
.41	147	141	135	130	126	121	118	114	...	...	...	...	...	...	...	...	...
.42	151	144	139	133	129	124	120	117	112	...	...	...	...	...	...	...	...
.43	154	147	142	137	132	127	123	119	115	112	...	...	...	...	...	...	...
.44	157	151	145	140	135	130	126	122	118	115	110	...	...	...	...	...	...
.45	...	155	148	143	138	133	129	124	121	117	113	110	...	...	...	...	...
.46	...	...	151	146	141	136	132	127	124	120	116	112	109	...	...	...	...
.47	...	...	...	149	144	139	135	130	126	122	118	115	112	109	...	...	...
.48	...	...	...	...	147	142	138	133	129	125	120	117	114	111	108	...	...
.49	...	...	...	...	...	145	140	135	131	127	123	120	116	113	111	108	...
.50	...	...	...	...	...	...	143	138	134	130	126	122	119	116	113	110	107
.51	...	...	...	...	...	...	...	141	137	133	129	125	121	118	115	112	109
.52	...	...	...	...	...	...	...	...	140	135	131	127	123	120	117	114	111
.53	...	...	...	...	...	...	...	...	...	138	134	129	126	123	119	116	113
.54	...	...	...	...	...	...	...	...	...	...	137	132	128	125	122	118	116
.55	...	...	...	...	...	...	...	...	...	...	...	135	131	127	124	121	118
.56	...	...	...	...	...	...	...	...	...	...	...	...	133	130	126	123	120
.57	...	...	...	...	...	...	...	...	...	...	...	...	...	132	128	125	122
.58	...	...	...	...	...	...	...	...	...	...	...	...	...	...	131	127	124
.59	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	130	126
.60	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	129

Example.—Find the working pressure allowed on a $\frac{3}{4}$ of an inch plate supported by stayrods spaced 15 inches center to center, a stiffening plate $\frac{1}{2}$ of an inch thick being riveted to the boiler head.

Solution.—The combined thickness of the two plates is $\frac{3}{4} + \frac{1}{2} = 1\frac{1}{4}$ inches. Then, $\frac{1\frac{1}{4} \times 80}{100} = 1$ inch, or 16 sixteenths. Now, applying rule **140,** we get

$$P = \frac{200 \times 16^2}{15^2} = 227.56 \text{ pounds per square inch. Ans.}$$

Example.—With stayrods spaced 14 inches center to center, and a $\frac{5}{8}$ of an inch plate reenforced by washers $\frac{3}{8}$ of an inch thick, of proper size and securely riveted, what working pressure would be allowed?

Solution.—The combined thickness of plate and washer is $\frac{5}{8} + \frac{3}{8} = 1$ inch. 80% of this is $\frac{1 \times 80}{100} = \frac{8}{10}$ of an inch. Reducing this fraction to sixteenths, we get 12.8 sixteenths. Then, by applying rule **140,** we have

$$P = \frac{200 \times 12.8^2}{14^2} = 167.18 \text{ pounds per square inch. Ans.}$$

734. In all the foregoing rules it has been assumed that the bracing is of the proper size; that is, that the stress per square inch of section does not exceed the lawful limit. Should the stress be more, the pressure must be reduced or the size of the stays increased until the stress comes within the prescribed limit. This applies to all cases, and is independent of the size of the plate.

FLUES.

735. Riveted smoke flues are usually made as shown in Fig. 188. They are made in sections, fitted one into the other and substantially riveted. Table 27 gives the working pressure, the thickness of material, the minimum

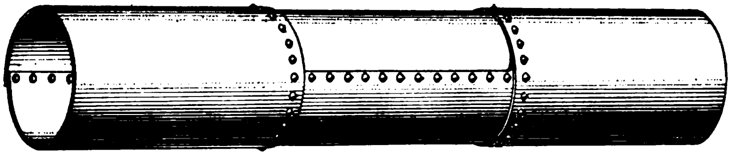

FIG. 188.

thickness of material for any diameter, and the maximum length of sections of both riveted and lap-welded flues over 6 inches in diameter and not over 40 inches in external diameter. The length of the sections is to be measured from center line to center line of the rivets in the circular seams of the flue.

The requirements laid down in Table 27, in regard to length of sections, may be deviated from in the following cases, and according to the rules given in conjunction with the same.

Lap-welded flues above 6 inches in diameter and not exceeding 16 inches diameter need not be made in sections, provided the steam pressure does not exceed 60 pounds per square inch or the length of the flue 18 feet. If the pressure is over 60 pounds and not exceeding 120 pounds, such flues may be made in sections not exceeding 5 feet in length. The thickness of these flues may be taken from Table 28, which, as will be noted, is carried up to 16 inches diameter.

736. Riveted and lap-welded flues of any thickness of material, diameter, and length of section prescribed in Table 27 may be made in sections of any desired length exceeding the maximum length allowed in the table, provided the working pressure is reduced in proportion to the increased length of section, according to the following rule:

Rule 141.—*To find the working pressure on flues as described above, multiply the pressure in the table allowed for any prescribed thickness of material and diameter of flue by the greatest length, in feet, of sections allowable for such flue, and divide the product by the desired length of sections in feet.*

Example.—According to the table, the working pressure allowed on a flue 30 inches in diameter constructed of material .40 inch in thickness and made in sections of 2½ feet, is 115 pounds. Find the working pressure if the length of the section is increased to 4 feet.

Solution.—Using rule **141**, we have

$$\frac{115 \times 2.5}{4} = 71.875 \text{ lb. per sq. in.} \quad \text{Ans.}$$

TUBES.

737. Tubes, as the smaller sizes of smoke flues are generally called, are used to give increased heating surface to boilers. They are usually made of charcoal iron and are lap-welded, although lately solid drawn steel tubes have come into extensive use. The tubes are driven in from the head of the boiler through both tube-sheets until they project about ¼ inch into the back connection, their length being such that about ⅜ to ½ inch will be left outside the front tube-sheet. By means of a tube expander the tubes are expanded into the sheets, this serving to make a tight joint. The ends of the tubes projecting into the combustion chamber are then beaded over nearly flush with the sheet, the front ends of the tubes being merely expanded, so the tubes may be driven inwardly a little and expanded and beaded anew should they leak.

738. A **"Dudgeon" roller-tube expander,** shown in Fig. 189, is extensively used for expanding tubes. It consists of a body *A* provided with three slots for the reception of the rollers *a*, *a*, *a*. A plate *B* fastened to the body by the three screws *c*, *c*, *c* prevents any longitudinal movement of the rollers. The rollers are forced outwards and rotated by the taper pin *C*, which is provided with a head perforated with two holes at right angles to each other. To make the expander adjustable for different thicknesses of tube-sheets, the hood *D* may be moved longitudinally and may be locked in any desired position by means of a small taper pin *d*, which is driven inwards to lock the hood. The

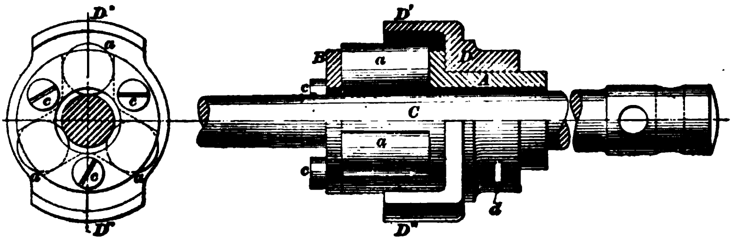

FIG. 189.

operation of the expander is as follows: The expander is pushed into the tube, the projections D', D'' limiting the depth to which the tool may be inserted. A sharp blow with a copper hammer is struck on the head of the pin *C*, forcing it inwards, and, hence, the rollers outwards. Next, a bar is inserted into one of the holes in the head and the pin rotated; the friction between the pin and the rollers causes the rollers, and consequently the whole tool, to rotate. This operation expands the tube.

THE THICKNESS OF BOILER TUBES.

739. The thickness of boiler tubes, as required by the Board of Supervising Inspectors, is given in the table below:

TABLE 28.

Outside Diameter.	Thickness.	Outside Diameter.	Thickness.
1	.072	4½	.134
1¼	.072	5	.148
1½	.083	6	.165
1¾	.095	7	.165
2	.095	8	.165
2¼	.095	9	.180
2½	.109	10	.203
2¾	.109	11	.220
3	.109	12	.229
3¼	.120	13	.238
3½	.120	14	.248
3¾	.120	15	.259
4	.134	16	.270

The table gives the minimum thickness of boiler tubes up to 6 inches in diameter, and of any required length, to carry a working pressure not to exceed 225 pounds per square inch, if deemed safe by the inspectors.

FURNACE FLUES.

740. Furnace flues may be made in different ways, as shown in Fig. 190. In *A*, Fig. 190, the furnace flue is made in sections of not more than 8 feet in length. Each section is flanged to a depth of not less than 2½ inches, and the sections are riveted together with a wrought-iron ring (shown at *a*) between the flanges. The thickness of the ring must not be less than ½ inch nor its width less than 2½ inches.

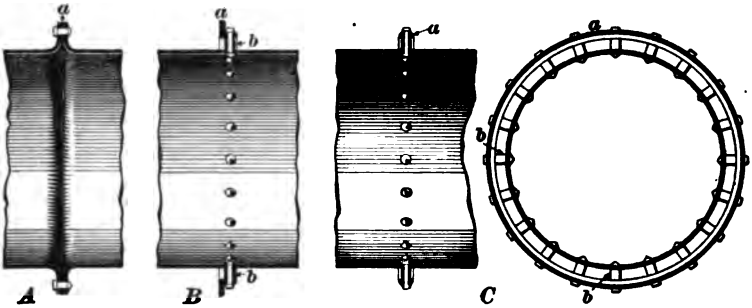

FIG. 190.

741. A different construction is shown at *B*, Fig. 190. Angle-iron rings (shown at *a*) are employed, serving to stiffen the flue. The thickness of the material of the ring must not be less than double that of the flue, and the depth must not be less than 2½ inches. The rings are held in position by rivets passing through wrought-iron thimbles *b*, *b*, placed between the inner surface of the strengthening ring and the outer surface of the flue. The length of the thimbles must not be more than 2 inches, nor the diameter of the rivets less than 1½ times the thickness of the material in the flue. The pitch of the rivets, measured at the outer surface of the flue, is not to be more than 6 inches.

742. At *C*, Fig. 190, the strengthening ring shown at *a* is made of half round iron. The proper area of the ring may be found by multiplying the thickness of the flue in decimals of an inch by the constant 9.6; the product will be the area in square inches. The ring is held in position by rivets passing through wrought-iron thimbles *b* not more than 2 in. in height. If rivets $\frac{7}{8}$ in. diameter and over are used, the pitch must not exceed 8 in.; for rivets $\frac{3}{4}$ in. diameter, 6 in., or for rivets $\frac{5}{8}$ in. diameter, 4 in., the pitch to be measured at the outer surface of the flue. No rivets smaller than $\frac{5}{8}$ inch are to be used for securing strengthening rings.

The distance center to center of flanges or strengthening rings is to be taken as the length of the flue in computing the working pressure allowable.

743. The working pressure allowed on furnace flues with flanged ends, as *A*, Fig. 190, or with angle-iron rings, as *B*, Fig. 190, is determined by the following rule, which also applies to the furnaces of vertical boilers, provided the diameter of the furnace does not exceed 42 in. and the height 40 in.:

Let $D =$ diameter of flue in inches;
$T =$ thickness of flue in decimals of an inch;
$L =$ length of flue in feet, not to exceed 8 feet;
$P =$ the working pressure in pounds per square inch;
$89{,}600 =$ a constant;
$67{,}200 =$ a constant.

Rule 142.—*To find the working pressure allowed on furnace flues with flanged joints, and with angle-iron rings, and on furnaces of vertical boilers, multiply the square of the thickness of the flue by the constant 89,600. Divide the product by the product of the length in feet and the diameter in inches.*

That is, $$P = \frac{89{,}600\, T^2}{L D}.$$

EXAMPLE.—A furnace flue 8 feet long, 38 inches in diameter, and constructed of material .5 inch in thickness, is fitted with an angle-iron strengthening ring at the middle of its length. Find the working pressure allowable.

SOLUTION.—As the furnace flue has a strengthening ring at the middle, the length of flue to be used in applying the rule will be $8 \div 2 = 4$ feet. Then, applying rule **142,** we get

$$P = \frac{89{,}600 \times .5^2}{4 \times 38} = 147.36 \text{ lb. per sq. in.} \quad \text{Ans.}$$

Rule 143.—*To find the working pressure allowed on furnace flues fitted with half-round strengthening rings, multiply the square of the thickness of the flue by the constant 67,200. Divide this product by the product of the length of a section in feet and the diameter in inches.*

That is, $$P = \frac{67{,}200\,T^2}{L\,D}.$$

EXAMPLE.—A furnace flue 40 inches in diameter and .5 in. thick is fitted with half-round iron rings 3 feet center to center. What working pressure would be allowed on this flue?

SOLUTION.—Applying rule **143,** we have

$$P = \frac{67{,}200 \times .5^2}{3 \times 40} = 140 \text{ lb. per sq. in.} \quad \text{Ans.}$$

744. Corrugated furnace flues are used very extensively for Scotch boilers, the corrugations serving to strengthen the flue considerably. In Fig. 191 a common construction of such a flue is shown. The end marked *A* in the figure is cylindrical and is attached by rivets to the head of the boiler, which is flanged either inwardly or outwardly to fit closely over the cylindrical part of the furnace flue. The other end of the flue is flanged in the manner shown; the back tube-sheet is riveted to the flange *C*, and the plates forming the sides of the combustion chamber are riveted to the flanges *D*, *D*. The depth of the corrugations must not be less than $1\frac{1}{2}$ inches, the distance from center to center of corrugations not more than 8 inches, the plain part at the ends not more than 9 inches, and the thickness not less than $\frac{5}{16}$ inch.

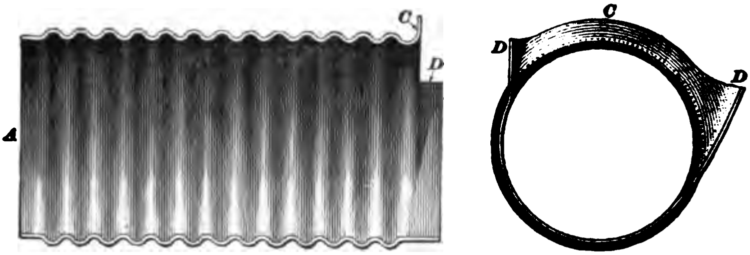

FIG. 191.

745. A **Purves ribbed furnace flue** is shown in Fig. 192. The height of the ribs is made $1\frac{3}{8}$ inches, the distance from center to center of ribs 9 inches. The thickness of the flue must not be less than $\frac{7}{16}$ inch, and the length of the plain part at the ends must not exceed 9 inches.

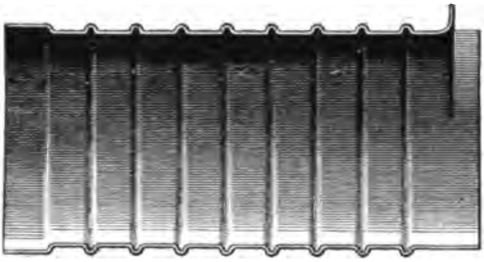

FIG. 192.

746. Both corrugated and ribbed flues may be used for the uptakes of boilers having a wet uptake. The working pressure allowed on corrugated and ribbed furnace flues may be found as follows:

Let T = the thickness of the flue in decimals of an inch;
D = the mean diameter in inches;
P = the steam pressure;
14,000 = a constant.

The mean diameter of a corrugated flue is to be taken as the outside diameter measured at the bottom of the corrugations. The mean diameter of a ribbed flue is to be taken as the outside diameter over the flat part of the flue.

Rule 144.—*To find the working pressure allowed on corrugated and ribbed furnace flues, multiply the thickness of the flue by the constant 14,000. Divide this product by the mean diameter.*

That is,
$$P = \frac{14{,}000\,T}{D}.$$

EXAMPLE.—Find the working pressure allowed on a corrugated furnace flue 36 inches mean diameter and constructed of $\frac{3}{8}$-inch material.

SOLUTION.—Applying rule **144**, we have

$$P = \frac{14{,}000 \times .375}{36} = 145.83 \text{ lb. per sq. in.} \quad \text{Ans.}$$

Example.—Find the working pressure allowed on a ribbed furnace flue 38 inches in diameter over the plain part and ½ inch thick.

Solution.—Applying rule **144,** we have

$$P = \frac{14,000 \times .5}{38} = 184.21 \text{ lb. per sq. in.} \quad \text{Ans.}$$

746a. A **Morison suspension furnace flue** is shown in Fig. 192*a*. This flue somewhat resembles the corrugated flue shown in Fig. 191. The outer corrugations, as *a*, *a* in Fig. 192*a*, are made to a small radius, and are joined by a curve of about double the radius of the outer corrugations. When this flue is made with corrugations 8 inches center to center, of material not less than $\frac{5}{16}$ of an inch thick, with the plain part at the ends not over 9 inches long, and the radius of the outer corrugations not more than one-half of the radius of the curve joining them, the working pressure is determined by rule **144***a*. In this rule the letters have the same value as in rule **144,** and the constant is 15,000 instead of 14,000.

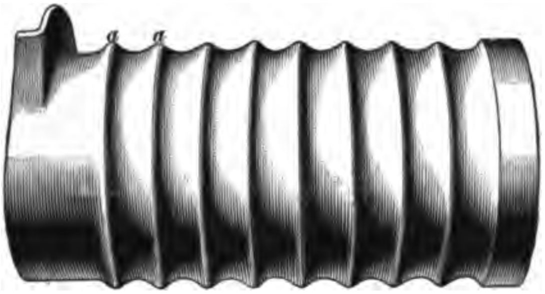

FIG. 192*a*.

Rule 144*a*.—*To find the working pressure allowed on Morison supension furnace flues, multiply the thickness of the flue by the constant 15,000. Divide this product by the mean diameter.*

That is, $$P = \frac{15,000\,T}{D}.$$

The mean diameter is to be taken as the outside diameter over the bottom of the corrugations.

Example.—What working pressure would be allowed on a Morison furnace flue 42 inches mean diameter and $\frac{3}{8}$ of an inch thick?

Solution.—Applying rule **144*a***, we have

$$P = \frac{15{,}000 \times \frac{3}{8}}{42} = 133.93 \text{ lb. per sq. in.} \quad \text{Ans.}$$

HEADS.

747. A **bumped head,** which is often used in the construction of steam drums, is shown at (*a*), Fig. 193. As shown in the figure, it is flanged and riveted to the shell in such a manner that the concave part of the head receives the steam pressure.

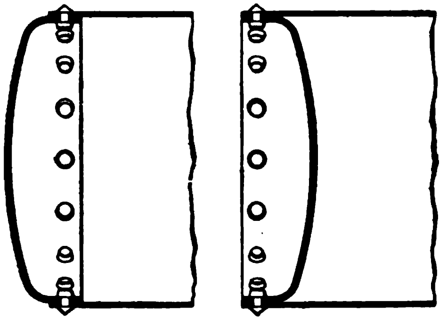
(*a*) (*b*)

Fig. 193.

Let $P =$ the steam pressure;
$T =$ thickness in inches;
$R =$ the radius to which the head is bumped in inches;
$S =$ the tensile strength of the material in pounds.

Rule 145.—*To find the pressure allowed on bumped heads when single riveted to the shell, multiply the tensile strength of the material by its thickness. Divide the product by 3 times the radius to which the head is bumped. When the head is double riveted to the shell, divide the product of the tensile strength of the material and its thickness by 2.5 times the radius to which the head is bumped.*

That is, $$P = \frac{TS}{3R} \text{ for single-riveted heads,}$$

and $$P = \frac{TS}{2.5R} \text{ for double-riveted heads.}$$

EXAMPLE.—The head of a steam drum is bumped to a radius of 36 inches. The thickness of the head being ½ of an inch and the tensile strength of the material 55,000 pounds, what would be the working pressure allowed (*a*) if single riveted? (*b*) if double riveted?

SOLUTION.—Applying rule **145** to question (*a*), we have

$$P = \frac{\frac{1}{2} \times 55{,}000}{3 \times 36} = 254.63 \text{ lb. per sq. in.} \quad \text{Ans.}$$

Applying rule **145** to question (*b*), we get

$$P = \frac{\frac{1}{2} \times 55{,}000}{2.5 \times 36} = 305.56 \text{ lb. per sq. in.} \quad \text{Ans.}$$

To find the radius to which the head is bumped, square half the diameter of the head and divide the product by the height of the bump. Add the height of the bump to the quotient and divide the sum by 2.

748. A **concaved head,** shown at (*b*), Fig. 193, is merely a bumped head fastened into the drum in such a manner that the convex surface receives the steam pressure. The working pressure allowed is given by the following rule:

Rule 146.—*To find the pressure allowed on concaved heads single riveted to the shell, multiply the tensile strength of the material by its thickness, and divide this product by 5 times the radius to which the head is bumped. For double-riveted concaved heads divide the product of the tensile strength of the material and its thickness by 4⅙ times the radius to which the head is bumped.*

That is, $P = \frac{TS}{5R}$ for single-riveted heads,

and $P = \frac{TS}{4\frac{1}{6}R}$ for double-riveted heads.

EXAMPLE.—Given, a concaved head bumped to a radius of 36 inches, the material being ½ of an inch thick and having a tensile strength of 55,000 pounds; what working pressure would be allowed (*a*) if single riveted? (*b*) if double riveted?

SOLUTION.—Applying rule **146** to question (*a*), we have

$$P = \frac{\frac{1}{2} \times 55{,}000}{5 \times 36} = 152.78 \text{ lb. per sq. in.} \quad \text{Ans.}$$

Applying rule **146** to question (*b*), we get

$$P = \frac{\frac{1}{2} \times 55{,}000}{4\frac{1}{6} \times 36} = 183.33 \text{ lb. per sq. in. Ans.}$$

749. The pressure allowed on *unstayed flat heads* of steam drums or shells of boilers when flanged and made of wrought iron, steel, or cast steel is determined by the following rule:

Let $P =$ working pressure in lb. per sq. in.;
$T =$ thickness in inches;
$S =$ tensile strength of the material;
$A =$ area of the head in square inches.

Rule 147.—*To find the pressure allowed on unstayed flat heads, multiply the thickness of the head by its tensile strength. Divide this product by fifty-four hundredths of the area of the head.*

That is,
$$P = \frac{TS}{.54\,A}.$$

EXAMPLE.—The diameter of the flat head of a steam drum being 24 inches, its thickness ½ inch, and its tensile strength 48,000 pounds, find the working pressure allowed on this head.

SOLUTION.—Applying rule **147**, we have

$$P = \frac{.5 \times 48{,}000}{.54 \times 24^2 \times .7854} = 98.24 \text{ lb. per sq. in. Ans.}$$

STRESSES IN BOILER SHELLS.

750. If the cylindrical shell shown in Fig. 194 be subjected to an internal pressure, there will be two forces tending to rupture it. One force, indicated by the arrows A, A, acting in the direction of the length, tends to tear the shell

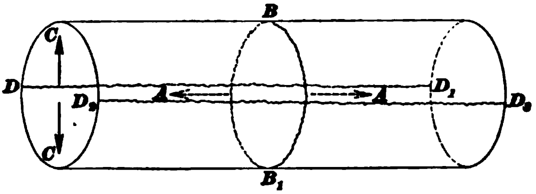

FIG. 194.

in a transverse plane, as $B\ B_1$. The other force, indicated by the arrows C, C, acting perpendicular to the axis, tends to rupture the boiler in a longitudinal plane passing through the axis, as $D D_1 D_2 D_3$. These two forces are opposed by the tenacity of the material of which the shell is composed. It is easy to see that the magnitude of the force tending to rupture the shell in a transverse plane is equal to the area of the head in square inches times the steam pressure per square inch. As this force is resisted by the tenacity of the material, the magnitude of the tenacity being measured by the sectional area, the stress per square inch of section of the material is

$$\frac{\text{area of the head} \times \text{pressure}}{\text{area of section}}.$$

It can be proven, both by mathematics and by experiment, that the magnitude of the force tending to rupture the shell in Fig. 194 in a longitudinal plane is equal to the internal diameter times the length times the pressure.

To resist this force, we have the combined sectional area of the material of the two sides of the shell. Hence the stress per square inch of section equals

$$\frac{\text{the internal diameter} \times \text{the length} \times \text{the pressure}}{\text{combined sectional area}}.$$

751. Suppose we have a plain cylindrical shell constructed of any convenient material. Let the inside diameter be 36 inches, the length 120 inches, the thickness of the shell $\frac{1}{4}$ inch, and the internal pressure to which it is subjected 100 pounds per square inch.

The pressure on the head and, consequently, the magnitude of the force acting in the direction of the length is $36^2 \times .7854 \times 100 = 101{,}787.8$ pounds. This force is resisted by the tenacity of $36.5^2 \times .7854 - 36^2 \times .7854 = 28.471$ square inches of material. Hence the stress per square inch of section is $101{,}787.8 \div 28.471 = 3{,}575.14$ pounds. The magnitude of the force acting perpendicular to the axis equals $36 \times 120 \times 100 = 432{,}000$ pounds. The area of the material resisting this force equals 120 × .25 × 2 = 60 square inches; hence the unit stress is 432,000 ÷ 60 = 7,200 pounds per square inch. This shows that there is 7,200 ÷ 3,575.14 = say about twice as much resistance to transverse rupture as there is to rupture in a longitudinal plane. Hence it follows that if the material is proportioned to withstand the force perpendicular to the axis, it will possess ample strength in the other direction. For convenience in calculation, the length of the shell is taken as 1 inch. If a boiler is constructed of plates varying in thickness and tensile strength, the least thickness and the lowest tensile strength must be used in calculating the strength of the boiler.

752. In a cylinder that is on the point of bursting, the resistance of the material to rupture must be equal to the force tending to cause rupture. Hence a cylinder is on the point of bursting if the product of the diameter and pressure equals the product of twice the thickness of the cylinder and the ultimate tensile strength of the material of which it is composed. It will be noticed that the length of the cylinder has not been taken into account; the length has been assumed to be 1 inch, for the reason previously given. From arithmetic it should be plain that the bursting pressure equals

$$\frac{\text{twice the thickness} \times \text{the ultimate tensile strength}}{\text{diameter}}.$$

This may be simplified by using the radius of the cylinder instead of the diameter. Then, as the radius is one-half the diameter, the bursting pressure would be

$$\frac{\text{the thickness} \times \text{the ultimate tensile strength}}{\text{radius}}.$$

Let T = thickness of the shell of a cylinder or boiler;
R = internal radius;
S = ultimate tensile strength of the material;
P_b = bursting pressure;
P = safe working pressure.

Rule 148.—*To find the bursting pressure of a cylindrical shell, neglecting the weakening effect of the seam, divide the product of the thickness of the shell and the ultimate tensile strength of the material by the internal radius.*

That is, $$P_b = \frac{TS}{R}.$$

EXAMPLE.—A boiler 48 inches in diameter is built of wrought-iron plates .375 inch thick. The iron having an ultimate tensile strength of 50,000 pounds, find the pressure at which the boiler would be about to burst, neglecting the weakening effect of the seam.

SOLUTION.—Applying rule **148,** we have

$$P_b = \frac{.375 \times 50,000}{24} = 781.25 \text{ lb. per sq. in. Ans.}$$

753. The safe working pressure of a marine boiler having the longitudinal seams single riveted is fixed by law at one-sixth the bursting pressure of a cylindrical shell having the same dimensions as the boiler.

Rule 149.—*To find the safe working pressure of a cylindrical boiler having single-riveted seams, divide the product of the thickness of the shell and the ultimate tensile strength of the material by 6 times the radius.*

That is, $$P = \frac{TS}{6R}.$$

EXAMPLE.—Find the safe working pressure of the boiler given in the previous example, the longitudinal seams to be single riveted.

SOLUTION.—Applying rule **149,** we have

$$P = \frac{.375 \times 50,000}{6 \times 24} = 130.2 \text{ lb. per sq. in. Ans.}$$

754. For boilers having double-riveted longitudinal seams, the safe working pressure is 20% additional to that allowed for single riveting.

Rule 150.—*To find the safe working pressure of a cylindrical boiler having double-riveted longitudinal seams, divide the product of the thickness of the shell and the ultimate tensile strength of the material by 5 times the radius.*

That is, $$P = \frac{TS}{5R}.$$

EXAMPLE.—A cylindrical boiler has the following dimensions: Diameter, 36 inches; thickness of shell, .25 inch; ultimate tensile strength of the material, 45,000 pounds per square inch. The seams being double riveted, find the safe working pressure.

SOLUTION.—Applying rule **150,** we have

$$P = \frac{.25 \times 45,000}{5 \times 18} = 125 \text{ lb. per sq. in. Ans.}$$

Rules **149** and **150** give the safe working pressures allowed, provided that no rivet holes are punched. The rivet holes must be *drilled* fairly; that is, they must be drilled so they will match.

754*a*. The different rules given in the chapter on boiler construction conform with those prescribed by the Board of Supervising Inspectors of Steam Vessels for the guidance of their inspectors. A number of the rules have been put into a more convenient form in order to simplify the solution of examples, but in all cases the result will be the same as that obtained from using the original rules.

The attention of the student is called to the fact that the rules and regulations are amended from time to time. Notices of the amendments appear in the various technical journals devoted to marine interests. The rules given are from the regulations in force March 1, 1900.

754*b*. The rules and formulas governing the construction of marine boilers given in this volume apply only to American steam vessels. Most foreign governments have passed laws providing for the licensing of engineers and the inspection of marine boilers. As might be expected, the rules prescribed by foreign governments frequently differ from those given in the United States Statutes. Students employed on vessels under a foreign flag are therefore advised to obtain copies of the rules and regulations in force in the country in which the vessel is registered. In Canada the inspection of steamboats and the licensing of marine engineers is under the supervision of the Minister of Marine and Fisheries, Ottawa, Canada. In England the British Board of Trade, London, is charged with the supervision of steam vessels. Students on foreign vessels should substitute the rules in force in their respective countries for those here given when taking an examination for a marine engineer's license. When answering the questions in this volume, the student must use the rules here given.

754*c*. When applying the rules governing the construction of a marine boiler to a boiler for the purpose of determining the safe working pressure, it is not sufficient to merely calculate the pressure that would be allowed on the shell. Since the strength of a boiler is the strength of its weakest part, it is necessary to compute the working pressures allowable on the different parts of which the boiler is composed by the rules prescribed. Then, the working pressure allowable on the weakest part found will be the pressure the boiler is allowed to carry. For illustration, suppose the boiler shell to be safe for a pressure of 150 pounds per square inch, but that on calculating the stress in the staybolts or stayrods it is found that the legal stresses will be exceeded with a steam pressure of 150 pounds. The pressure would then have to be reduced until the stress per square inch of section of the stays comes within the legal limit. Likewise if the pressure allowed on, say, the head of the steam drum falls below that allowable on the boiler itself, it will be necessary to reduce the working pressure.

EXAMPLES FOR PRACTICE.

1. Find the load on a staybolt, the pitch being 6½ inches and the steam pressure 125 pounds. Ans. 5,281.25 lb.

2. Find the stress per square inch of section of a staybolt 1 inch in diameter, the pitch being 5½ inches and the steam pressure 120 pounds. Ans. 4,621.85 lb. per sq. in.

3. What is the stress per square inch of section in a palm stay 7 feet 6 inches long? The diameter of the stay is 2 inches, the area supported by the stay is 156 square inches, and the steam pressure 120 pounds. The distance from the head to the point of intersection of the center line of the stay with the shell is 6 feet 9 inches. Ans. 6,620.83 lb.

4. A plate ⅜ of an inch thick is supported by staybolts having a pitch of 8 inches. Find the working pressure allowable. Ans. 63 lb. per sq. in.

5. The pitch of the staybolts fitted to a plate ½ inch thick is 7 inches. Find the working pressure. Ans. 156.73 lb. per sq. in.

6. A plate ⅞ of an inch thick is fitted with stayrods spaced 15 inches center to center. The plate is reenforced by double angle irons, the legs of which are 4 inches deep and ⅝ of an inch thick. What pressure would be allowed on this plate? Ans. 327.68 lb. per sq. in.

7. Find the proper pitch for staybolts staying a plate ½ inch thick, to carry a working pressure of 75 pounds. Ans. 10.1 in.

8. Find the working pressure allowed on a plate ¾ in. thick fitted with stayrods with double nuts, stayrods 15 inches center to center.
Ans. 89.6 lb. per sq. in.

9. A furnace flue 7 feet long is strengthened by a ring and flanges at the middle of its length. The diameter being 41 inches and the thickness ½ inch, find the working pressure allowable.
Ans. 156.1 lb. per sq. in.

10. If a half-round iron strengthening ring is employed, what would be the working pressure allowable on the flue in example 9?
Ans. 117.1 lb. per sq. in.

11. Find the working pressure allowable on a corrugated furnace flue 44 inches mean diameter and ⅝ inch thick.
Ans. 198.86 lb. per sq. in.

12. What pressure would be allowed on a Morison suspension furnace flue ½ of an inch thick and 40 inches mean diameter?
Ans. 187.5 lb. per sq. in.

13. The head of a steam drum is bumped to a radius of 60 in. Its thickness being ¾ inch and its tensile strength 50,000 pounds, find (*a*) the pressure allowable if used as a bumped head and single riveted; (*b*) the pressure allowable if used as a concaved head and double riveted.
Ans. { (*a*) 208.33 lb. per sq. in.
(*b*) 150 lb. per sq. in.

14. The flat, unstayed head of a steam drum is 23 inches in diameter. Its thickness being ⅝ inch and its tensile strength 57,000 pounds, find the working pressure allowable. Ans. 158.78 lb. per sq. in.

15. Find the working pressure allowed on a boiler shell 66 inches in diameter and .21 inch thick. Longitudinal seams are single riveted. The tensile strength of the material is 45,000 pounds.
Ans. 47.72 lb. per sq. in.

16. Find the working pressure allowed if the longitudinal seams of the boiler in example 15 are double riveted. Ans. 57.27 lb. per sq. in.

17. Find the bursting pressure of the same boiler, leaving the strength of the seam out of consideration. Ans. 286.36 lb. per sq. in.

FITTINGS OF MARINE BOILERS.

755. The fittings of a marine boiler are: The safety valve; the steam gauge; the gauge-cocks and glass water gauge; the fusible plugs; the bottom and surface blow-off cocks; the manhole and handhole plates; the main and donkey feed check-valves; the steam pipe and stop valve; the steam pipe and stop valve to the hold; the donkey valve; the furnace fittings, such as the grate-bars, bearing bars, bridge, furnace front; furnace doors and ash-pit doors; the front connection; .uptake and smokestack, including the smokestack damper.

Other appliances used in connection with marine boilers are the steam drum; the dry pipe; the superheater; the steam chimney, and the separator.

SAFETY VALVES.

756. A **safety valve** is a device used to prevent the steam within the boiler from exceeding a certain pressure. This is accomplished by the steam overcoming the downward force on the valve and opening a passage for the steam to escape through into the atmosphere. The valve remains open sufficiently long to reduce the steam to a somewhat lower pressure. This relieves the boiler.

FORMS OF SAFETY VALVES.

757. A common **lever safety valve,** as applied to boilers of steam vessels, is shown is Fig. 195. The lower end of the casing *A* communicates with the boiler, and is bored out to receive the gun-metal bushing *B*. This bushing is secured to the casing, its upper edge *b* being chamfered, and forming the seat for the valve *V*. Four arms on the inside of the bushing support the boss *b'*, which is bored out central with the seat, and forms a guide for the lug on the lower face of the valve. Attached to the upper end of the valve is the stem *S*, passing through a gun-metal bushing in the bonnet *A'* and guided at its upper end by a bushing in the yoke *Y*. The stem is slotted below the yoke and above the bonnet, and is provided with knife-edge bearings for the lever *L*; this lever having its fulcrum at the upper end of the link *l* pivoted at *a*. A weight *W* is placed at a certain distance from the fulcrum; this weight acts upon the valve stem, and, consequently, presses the valve to its seat, forming the resistance, or downward force, which the steam acting upon the under side of the valve has to overcome. On the steam pressure reaching a certain point, it will balance the downward force, and a slight increase of the steam pressure will cause the valve to lift from its seat, thus forming an annular opening through which the steam escapes into the passage *P*, and thence through a pipe (not shown in the figure) into the atmosphere. As the steam pressure becomes less, a point is soon reached at which the downward force balances the steam pressure, and on the pressure becoming still further reduced, the valve closes. A small drain pipe should be fitted to the casing just above the valve seat, to carry away any water that may collect, and which, by its weight, would add to the external load on the valve.

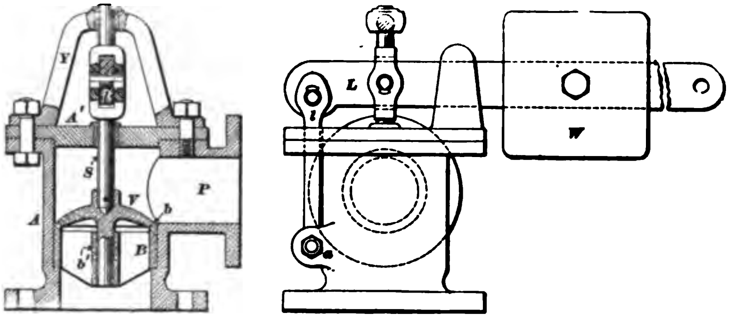

FIG. 195.

758. **A dead-weight safety valve** is shown in Fig. 196. In this construction the external load on the valve is formed by cast-iron or leaden weights *W*, *W*, piled upon a plate *P* attached to the valve stem. The action of this valve under pressure is the same as that of the valve previously described. Attached to the upper end of the valve stem *S*, by means of the cotter *C*, and fitting the stem loosely, is a crown cap, or hood *D*, provided with two rings cast upon it, and also with a handle. The depth of the slot in the stem *S*, which works in a bush *F* fixed in the cover, is such that the valve can move upwards a certain distance without lifting the hood, while the hood can not be raised without raising the valve with it. By means of the forked lever *L* the valve can be lifted and its freedom of action tested. The handle attached to the hood is for the purpose of turning the valve around on its seat occasionally, thus crushing any scale or other sediment that may lodge upon it.

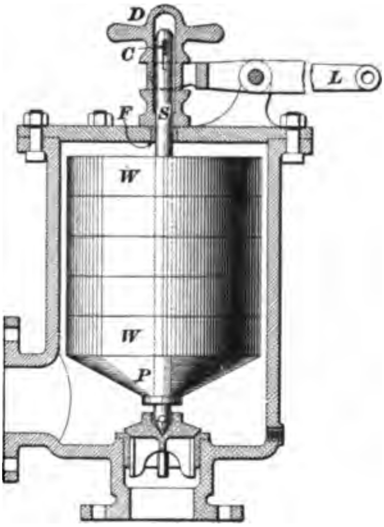

Fig. 196.

759. A spring-loaded safety valve is illustrated in Fig. 197. The external load is that due to the compression of the coiled spring *T*. By means of the threaded bushing *G* the tension of the spring can be regulated by screwing the bushing up or down, thus providing a simple method of adjusting the downward force to the pressure at which the valve is to open. The hood *D*, which fits loosely on the upper part of the bonnet or casing *B*, is keyed to the valve stem *S* by means of the cotter *C*, in which is the padlock hole *d*. The hood, and, consequently, the valve, may be lifted by means of the lever *L*. At the same time the valve is at liberty to open without lifting the lever. The bushing *G*, which is provided with a locknut G', forms a guide for the upper end of the valve stem. When the steam pressure lifts the valve off its seat, the spring is compressed; this increases the resistance of the spring, and so increases the force acting downwards upon the valve. This prevents the valve from lifting any higher, and gives but a limited area for the steam to escape. To overcome the increasing resistance of the spring, the so-called **pop safety valve** has been designed, and has superseded at present all other forms of spring-loaded safety valves.

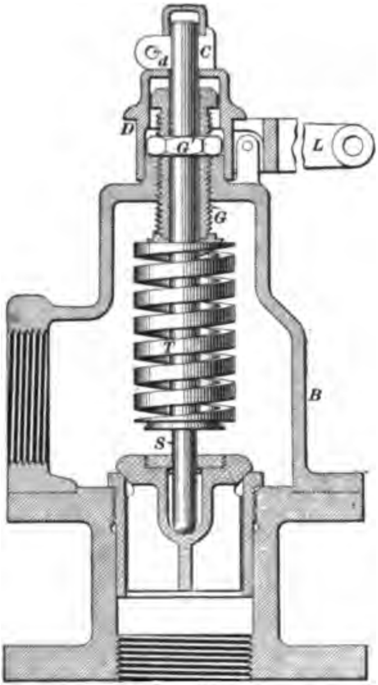

FIG. 197.

760. A **pop valve** is shown in Fig. 198. A gunmetal bushing *B* is fitted to the lower end of the valve casing *A*. The bushing is threaded and fitted with a movable ring *R* of the cross-section shown in the figure. A threaded bushing *G*, provided with a locknut and forming a guide for the valve stem *S*, is used to adjust the external load on the valve. An annular recess *r* is cut into the face of the valve *V*. The other parts *C*, *d*, *D*, *L*, and *T*, are the same as in Fig. 197. Upon the steam pressure lifting the valve, the steam flows through the opening between the valve and its seat and through the annular passage *b b* into the casing, and thence into the atmosphere. Its free escape being somewhat restricted by the narrow passage *b b*, a slight pressure accumulates under the face of the valve, and this pressure acting upon the increased surface presented, causes the valve to open promptly and allows the steam to escape freely. This additional pressure can be adjusted by raising or lowering the ring *R*, thus reducing or increasing the area of the passage *b b*. The less the area of this passage, the faster the pressure will accumulate and the higher the valve will lift; conversely, the larger the area of the passage, the slower will the pressure accumulate, and the longer will be the time required for the full opening of the valve. A pop valve closes very promptly, thus preventing an undue loss of steam.

The use of the first two safety valves described is confined almost exclusively to vessels navigating still waters.

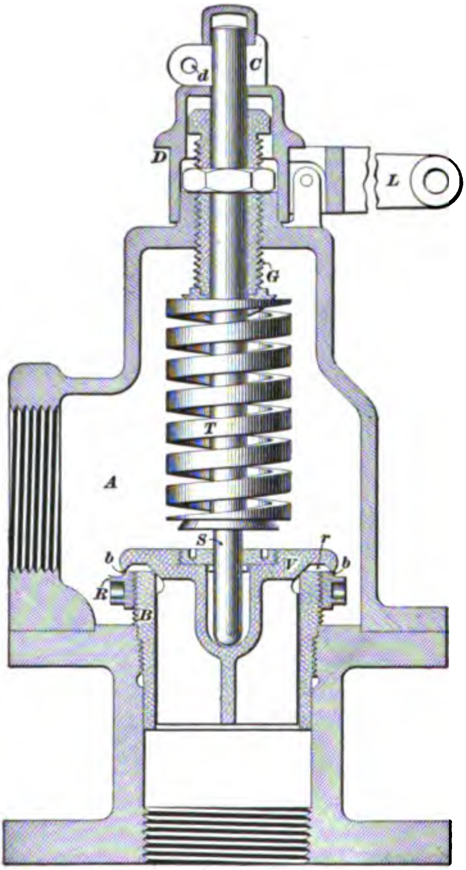

Fig. 198.

761. As before mentioned in Art. **756,** a safety valve is intended to relieve a boiler of all the surplus steam generated. To accomplish this object, this valve must have a certain area of opening, and this area is fixed by the rules and regulations of the Board of Supervising Inspectors of Steam Vessels to be, for lever safety valves, at least 1 square inch to every 2 square feet of grate surface of the boiler to which the valve is attached. For spring-loaded safety valves constructed so as to give an increased lift by the operation of steam after being raised from their seats, or any spring-loaded safety valve constructed in any other manner, or so as to give an effective area equal to the first mentioned spring-loaded valve, the area of opening must be at least 1 square inch to every 3 square feet of grate surface.

All spring-loaded safety valves for water-tube, coil, and sectional boilers required to carry a steam pressure exceeding 175 pounds per square inch, must have an area of not less than one square inch to every six square feet of grate surface of the boiler.

NOTE.—The term "area of opening" refers to the area corresponding to the internal diameter of the valve. The term "effective area" always refers to the area of the annular opening between the valve and its seat, through which the steam escapes upon the valve being raised.

762. The safety valves of a battery of boilers must be arranged in such a manner that each boiler shall have one separate safety valve, unless the arrangement is such as to preclude the possibility of shutting off the communication of any boiler with the safety valve or valves employed. This also applies to safety valves provided with an attachment preventing any but an authorized person to increase the downward force on the valve. A valve constructed in such a manner is known as a **lock-up safety valve.** Referring to Figs. 197 and 198, if a padlock were attached to the cotter *C* at *d*, the cotter could not be withdrawn unless the padlock were removed first; and as the hood *D* covering the adjusting bushing can not be removed without withdrawing the cotter, nobody but the person having the key of the padlock can adjust the compression of the spring.

763. All spring-loaded safety valves must be provided with a lever that will raise the valve from its seat a distance of not less than one-eighth the diameter of the valve opening. The seats of any size of safety valve used on a marine boiler must have an angle of inclination to the center line of 45 degrees. The area of opening of the connection between the safety valve and the boiler must be at least equal in area to the area of the valve.

The foregoing rules apply to all safety valves used on marine boilers.

764. Very often one lock-up safety valve is provided for every common safety valve. The valves (two or more in number) are usually placed on a separate fitting, shown of at *A*, Fig. 199. This fitting may be attached to the shell of the boiler or the shell of the steam drum. It avoids the necessity of cutting a separate hole in the shell for each valve. *B* and *C* are the safety valves, and *b* and *c* their respective escape pipes leading the steam blown off to the main escape pipe. The drain pipes *b'*, *c'* prevent the accumulation of water in the valve casing.

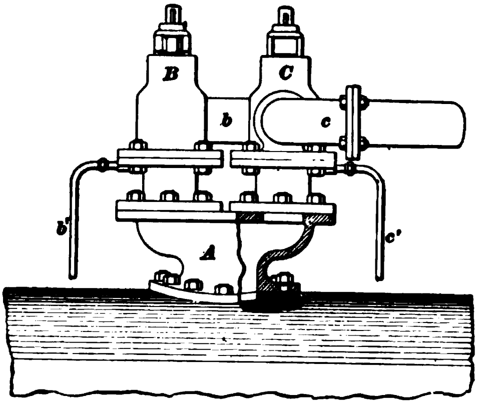

FIG. 199.

No safety valve can open without a slight increase of pressure above what it is set for; since, in order to lift the valve, the pressure on the under side of the valve, which we may call the *internal, or upward, force*, must exceed the *external, or downward, force* on the valve plus the friction of the mechanism of the valve. If the internal and the external forces upon the valve are equal, the valve will be in equilibrium (balanced), and an increase of the internal force will cause it to open. A safety valve will not close until the pressure has been reduced somewhat below the pressure at which the valve opened.

CALCULATIONS RELATING TO SAFETY VALVES.

765. The point at which a safety valve will blow off depends upon the external force upon the valve. To be in equilibrium, the external load exerting a downward pressure upon the valve must be equal to the internal force exerting an upward pressure on the under face of the valve. Evidently, the upward pressure is equal to the area of the valve multiplied by the pressure per unit of area.

Whenever the word "pressure" is used in relation to calculations pertaining to safety valves, the **gauge pressure** is meant, unless otherwise stated.

NOTE.—By "the area of a safety valve" is meant the area of that part which is exposed to the steam pressure when the valve is seated.

766. Suppose we have a dead-weight safety valve, as shown in Fig. 196, having a diameter of 4 inches and an external load or force consisting of the valve and stem, the supporting plate and the weights, equal to 815.8 lb. It is desired to know the pressure at which the valve will open. Since the internal and external forces must balance, it is evident that $815.8 = 4^2 \times .7854 \times$ steam pressure in pounds per sq. in.

From this we get $\frac{815.8}{4^2 \times .7854} = 65$ lb. per sq. in., nearly, the pressure at which the valve is about to open.

In the lever safety valve shown in Fig. 195, the external load depends upon the position of the weight *W* on the lever *L*. Here the same general law holds good; the external and the internal forces must be equal before the valve is about to open. The internal force, as stated before, is the area of the valve times the steam pressure. The downward force upon the valve may be found as follows: Suppose we place the weight *P*, Fig. 200, weighing 100 pounds, directly on top of the valve stem *C*. Evidently, the downward force is now equal to the weight of the weight *P*. Suppose, now, that the weight is removed to the position shown in the figure, the weight's distance *d* from the fulcrum being 6 times greater than the distance *a* from the fulcrum to the center line of the valve. Evidently, the effect of the weight on the valve stem will now be six times greater; that is, the downward force will be $6 \times 100 = 600$ pounds. Hence, to find

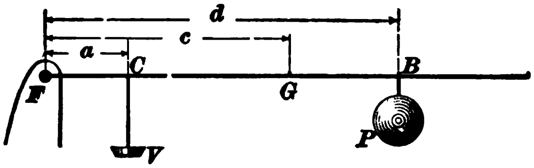

FIG. 200.

the downward force, we divide the distance *d* by the distance *a*, and multiply the quotient by the weight of the weight *P*.

As the valve and stem have a certain weight, the external force is increased an amount equal to that of the weight of the valve and stem in pounds. Furthermore, the lever has a certain weight, and this, acting at the center of gravity of the lever, adds a certain amount to the downward force. This amount is equal to the product of the distance from the fulcrum of the lever to its center of gravity and the weight of the lever, divided by the distance from the fulcrum to the center line of the valve.

The distance to the center of gravity of the lever may be found by balancing the lever on a knife edge and measuring the distance from the center of the fulcrum to the knife edge. If this should not be feasible, the center of gravity must be found by calculation. In engineers' examinations, the lever is usually given as straight and parallel, in which case the distance from the fulcrum to the center of gravity of the lever should be taken as equal to one-half the length of the lever.

The amount of the downward force on the valve due to the weight of the lever may be found directly by attaching a spring balance by a cord to the lever at the point at which it acts upon the valve stem. The spring balance will indicate the correct downward force in pounds.

Now, to have the valve balance, the area of the valve times the steam pressure (the upward force) must equal the weight times the distance from the fulcrum to the weight divided by the distance from the fulcrum to the center line of the valve; to this downward force must be added the additional downward force due to the weight of the valve, stem and lever, the sum of these two downward forces constituting the external force.

767. How to find the pressure per square inch at which a safety valve is about to blow off, may best be explained by the following example: Suppose we have a safety valve of the following dimensions: Let the area of the valve be 12.566 sq. in.; the distance from the fulcrum to the center line of the valve, 4 in.; the weight is to weigh 135.2 lb.; the length of the lever, between fulcrum and weight, is to be 36 in.; the weight of the valve and stem, 9.2 lb., and the downward force due to the weight of the lever, as found by one of the three methods previously explained, 150 lb. From what has been explained above, it should be clear that the valve balances, or is in equilibrium, if the steam pressure $\times$ 12.566 $= 135.2 \times 36 \div 4 + 9.2 + 150$. That is, the steam pressure $\times$ 12.566 $= 1{,}376$.

From arithmetic, it should be plain that

$$\text{the steam pressure} = \frac{1{,}376}{12.566} = 109.5 \text{ lb. per sq. in.}$$

Let A = the area of the valve in square inches;
D = the distance from the center line of the valve to the fulcrum, measured in inches;
L = the distance of the weight from the fulcrum in inches;
P = the steam pressure in pounds per square inch;
W = the weight in pounds of the load or weight on the lever;
w = the weight of the valve and stem in pounds plus the downward pressure due to the weight of the lever.

Rule 151.—*To find the pressure at which a safety valve is about to blow off, multiply the weight by the length of the lever and divide this product by the distance from the fulcrum to the center line of the valve. To the quotient add the downward pressure on the valve due to the weight of the valve, stem, and lever, and divide the sum by the area of the valve.*

That is,
$$P = \frac{\frac{WL}{D} + w}{A}.$$

EXAMPLE.—The area of a lever safety valve is 11 square inches; the distance from the center line of valve to the fulcrum, 4½ inches; the distance of the weight from the fulcrum, 35 inches; its weight, 125 pounds; the weight of valve and stem plus the downward pressure due to the weight of the lever equals 137 pounds. Find the pressure per square inch at which the valve is about to open.

SOLUTION.—Applying the rule just given, we have

$$P = \frac{\frac{125 \times 35}{4.5} + 137}{11} = 100.84 \text{ lb. per sq. in. Ans.}$$

To explain how to find where a given weight must be placed on the lever in order that the safety valve may be about to blow off at a given pressure, we will take the example previously made use of in the beginning of this article. The pressure was found to be 109.5 lb. per sq. in.; hence, the total upward force is $109.5 \times 12.566 = 1{,}375.977$, say 1,376 pounds. This force is partially balanced by the weight of the valve and stem, and the downward force due to the weight of the lever. Consequently, the total upward force $= 1{,}376 - (150 + 9.2) = 1{,}216.8$ pounds, is to be balanced by the downward force. As the downward force, as previously explained, is the weight times the length of the lever divided by the distance from the fulcrum to the center line of the valve, it should be plain that the valve will be in equilibrium again if $1{,}216.8 = \frac{135.2 \times \text{the length of lever}}{4}$. That is, $1{,}216.8 = 33.8 \times$ the length of lever.

Hence, the lever $= \frac{1{,}216.8}{33.8} = 36$ inches.

Rule 152.—*To find the distance from the fulcrum to the point at which the weight must act in order to have the valve blow off at a given pressure, subtract the downward force due to the weight of the valve, stem, and lever from the product of the area and the steam pressure. Multiply the remainder by the distance from the fulcrum to the center line of the valve and divide this product by the weight.*

That is, $$L = \frac{(A\,P - w)\,D}{W}.$$

EXAMPLE.—At what distance from the fulcrum must a weight of 150 pounds act in order that the valve may be about to blow off at 100 pounds pressure; the diameter of the valve being 3¾ inches; the distance from the fulcrum to the center line of the valve 4½ inches, and the downward force due to the weight of valve, stem, and lever 125 pounds?

Solution.—Applying the rule just given, we get, since area $= (3\frac{3}{4})^2 \times .7854 = 11.04$ sq. in.,

$$L = \frac{(11.04 \times 100 - 125) \times 4.5}{150} = 29.37 \text{ in. Ans.}$$

768. Suppose it is desired to find the weight which must be placed on a lever to have the valve blow off at a given pressure:

Using the example given in the beginning of Art. **767**, the unbalanced upward force, as previously found, is 1,216.8 pounds.

The valve then will balance if $1,216.8 = \frac{\text{weight} \times 36}{4}$; that is, if $1,216.8 = \text{weight} \times 9$; whence, the weight $= \frac{1,216.8}{9} = 135.2$ pounds.

Rule 153.—*To find the weight which must act on a lever at a given distance from the fulcrum so that the valve is about to blow off at a given pressure, subtract the downward force due to the weight of the valve, stem, and lever from the product of the area and the steam pressure. Multiply the remainder by the distance from the fulcrum to the center line of the valve, and divide this product by the distance from the fulcrum at which the weight is to act.*

That is, $$W = \frac{(A\,P - w)\,D}{L}.$$

Example.—A safety valve has the following dimensions: Area of the valve, 15.7 square inches; length of lever, 48 inches; distance from fulcrum to center line of the valve, 5 inches; the downward force due to the weight of the valve, stem, and lever is 182 pounds. Find the weight if the valve is about to blow off at 64 pounds pressure.

Solution.—Applying rule **153**, we get

$$W = \frac{(15.7 \times 64 - 182) \times 5}{48} = 85.71 \text{ lb. Ans.}$$

Note.—The student should thoroughly familiarize himself with the calculations pertaining to lever safety valves, as a candidate for a marine engineer's license must be *rejected* by the examining inspectors should he fail to solve safety-valve problems similar to those given in the examples in this course.

Spring-loaded and pop safety valves are adjusted under pressure by comparison with an accurate steam gauge, and no rules are required by which to calculate the point at which they will blow off.

EXAMPLES FOR PRACTICE.

1. A dead-weight safety valve having an area of 12 square inches is to be on the point of blowing off at 75 pounds pressure, absolute; find the weight. Ans. 723.6 lb.

2. What area of safety-valve opening is required for a water-tube boiler carrying steam at 180 pounds pressure, the grate surface being 48 square feet? Ans. 8 sq. in.

3. At what pressure will a safety valve of the following dimensions blow off: Area of valve, 10 square inches; distance from the valve to the fulcrum, 3 inches; length of lever (the distance from the fulcrum to the point where the weight acts), 30 in.; weight of the weight, 83.1 pounds; weight of valve and stem, 5 pounds; weight of lever, 12 pounds; total length of lever, 32 in.? The lever to be straight and parallel. Ans. 90 lb.

4. Suppose all the quantities to remain the same as in the last example, except that the valve is to blow off at 75 pounds pressure. At what distance from the fulcrum must the weight be placed? Ans. 24.58 in.

5. All quantities remaining the same as in example 3, except that the valve is to blow off at 82 pounds pressure, find the weight that must be placed on the lever. Ans. 75.1 lb.

STEAM GAUGES.

769. The **steam gauge** indicates the pressure of the steam contained in the boiler. The most common form is the **Bourdon pressure gauge,** Fig. 201. It consists of a tube *a*, of elliptical cross-section, which is filled with water and connected at *b* with a pipe leading to the boiler. The two ends *c* are closed and are attached to a link which is in turn connected with a rack *e*. This rack gears with a pinion *f* on the index pointer *g*. When the water contained in the elliptical tube is subjected to pressure, the tube tends to take a circular form, and the tube as a whole straightens out, throwing the free end out a distance proportional to the pressure. The movement of the free end is transmitted to the pointer by the link, rack, and pinion, and the pressure is thus recorded on the graduated dial.

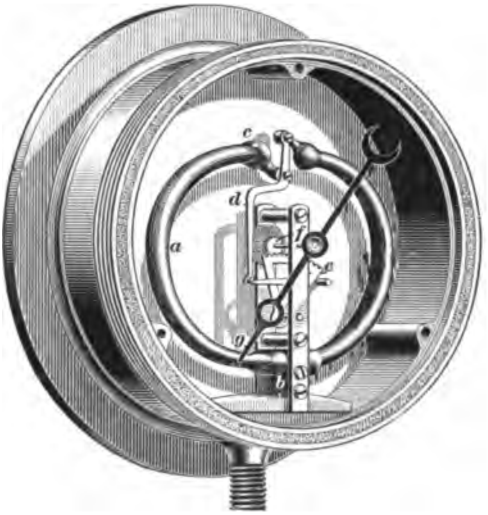

FIG. 201.

770. Mercurial gauges, in which a column of mercury was forced up into a closed glass tube by the steam pressure compressing the air above the mercury, were formerly used, but were superseded by a gauge of the construction shown in Fig. 201, and which is called **a metallic, or dial, gauge.**

771. Steam gauges are placed where they are immediately within sight of the man in charge of the boiler, but out of reach of rough usage.

The steam pipe for the gauge is generally connected to the top of the boiler, sometimes to the steam drum, and, in some instances, the gauge is placed on top of the water column. A so-called **siphon** should be placed directly under the gauge. This siphon is formed by carrying the steam pipe to a lower level than the gauge and then bending it upwards, thus forming a **U** which is filled with water to prevent the heat of the steam from injuring the spring and the other mechanism of the gauge, or distorting its action by the expansion of its parts. A small drip cock, shown at *d* in Fig. 202, is fitted to the lowest point of the "siphon" and serves to let the water out of the leg of the siphon connected with the boiler. If no cock were fitted, the water accumulating in this leg by the condensation of the steam would, by its weight, cause the gauge to indicate a higher pressure than that due to the steam.

By means of the cock shown at *a* the steam may be shut off from the gauge. When the vessel is under way, it will be noticed that the pointer of the steam gauge vibrates considerably. This is due to the vibration of the vessel caused by the motion of the machinery, and may be prevented by closing the cock *a* partially. Great care must be taken not to shut it entirely.

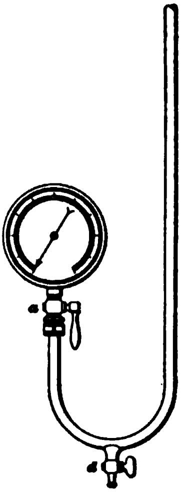

FIG. 202.

772. The pointer of a steam gauge will stick occasionally, and it is customary to jar the gauge slightly before accepting its indication as correct. Steam gauges require testing occasionally, as the spring will lose its elasticity in the course of time. They are tested by comparison with an accurate test gauge, both gauges being attached to the same vessel in which the pressure is raised by a small force pump. The Board of Supervising Inspectors of Steam Vessels requires all boilers or sets of boilers used on steam vessels to have attached to them at least one gauge that will correctly indicate a pressure of steam equal to 80 per cent. of the hydrostatic pressure applied by the Inspectors.

GAUGE-COCKS AND GLASS WATER GAUGES.

773. In order to be able to tell whether the water within the boiler is at its proper level, valves or cocks, called **gauge-cocks,** are employed.

Three gauge cocks *a*, *b*, *c*, are shown in Fig. 203, screwed into the front head of a Scotch boiler.

The middle gauge-cock *b* is at the proper water level, generally about 8 or 9 inches above the top of the combustion chamber.

The lower gauge-cock *c* is about 5 inches below the middle cock, and the top gauge-cock *a* 5 inches above it. Should the top gauge-cock be opened while the boiler is steaming, steam will issue from it. On opening the middle gauge-cock, a mixture of water and steam will issue, and pure water will come out of the lowest cock. When gauge-cocks are fitted in the position shown in the figure, a drip pan *D* is fitted below the cocks to prevent the water from the cocks coming in contact with the head of the boiler and corroding and dirtying it. The nozzles of the cocks are placed in such a direction that the jet of steam or water issuing from one cock can not strike the one below it and scald the attendant. Gauge-cocks are often placed on a separate fitting, consisting of a hollow tube with its ends connecting with the steam and water spaces of the boiler, sufficiently below and above the water level to be out of reach of the violent ebullition going on at the surface of the water. If the cocks are connected directly to the head or the side of the boiler, this violent ebullition may cause the gauge-cocks to indicate a wrong water level.

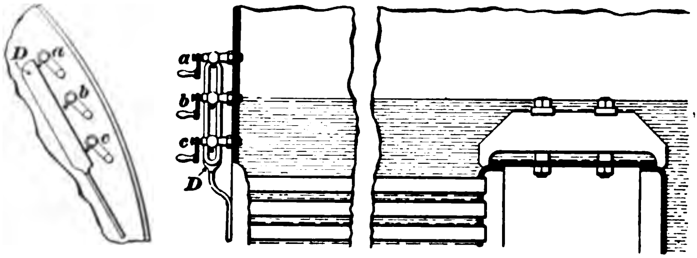

FIG. 208.

GLASS WATER GAUGES.

774. Glass water gauges are usually fitted to the boilers in addition to the gauge-cocks, and frequently the gauge-cocks, steam gauge, and glass water gauge are mounted on one fitting. The whole arrangement, which is known as a **water column,** is shown in Fig. 204. The top and bottom ends of the column *A*, which is simply a cast-iron tube, are connected to the steam and water space of the boiler by the pipes *B* and *C*, respectively. Fitted to this column are the gauge-cocks *a*, *a*, *a*. Attached to the top of it is the steam gauge *S*. It will be noticed that a siphon different from the one previously described is used below the steam gauge. No drip cock needs to be fitted to this kind of siphon, as no water can collect and disturb the indication of the gauge. The construction of the glass water gauge is as follows: Two fittings *b*, *b*, which are provided with cocks *e*, *e*, are screwed into the water column. These fittings connect with the top and bottom of a glass tube *f* open at both ends. Each fitting is provided with a stuffing-box *d* to make a steam and water-tight joint between the fitting and the glass tube. To the lower fitting a drain cock *g*, provided with a waste pipe, is attached. This cock is used to empty the glass tube. The water column is attached to the boiler at such a height that the water, when at its proper level, will show in the middle of the glass tube. It will be seen that the water level in this case is self-indicating, and always in plain sight of the attendant. The communication of the tube with the steam and water may be shut off by means of the cocks *e*, *e*,

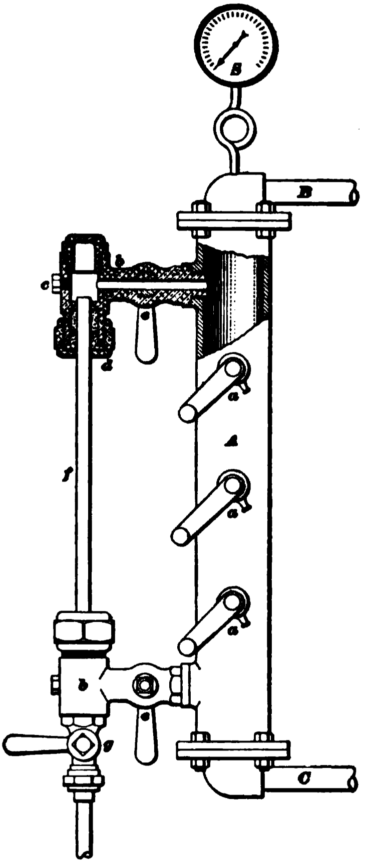

FIG. 204.

in case the tube breaks. The drain cock *g* may be used to test the working of the glass water gauge. Should the water in the glass tube fail to return immediately to its former level upon closing the drain cock, it would show that the lower fitting or pipe connection was choked with sediment or scale, and that the gauge could not be relied upon until the obstruction was removed. Should the water fail to leave the gauge glass upon opening the drain cock, or leave it very slowly, it would show the upper fitting or pipe connection to be choked, provided, however, the waste pipe is not choked. To clean the passages of the fittings, they are provided with a removable cleaning plug, shown at *c*.

775. Glass water gauges are often attached to the boiler directly; in that case, the same objection that was previously shown to exist with gauge-cocks attached to the boiler directly, applies with equal force to the glass water gauge.

The use of glass water gauges is optional, but of gauge-cocks, compulsory. The rules and regulations of the Board of Supervising Inspectors of Steam Vessels provide that all steam vessels having one or two boilers shall have three gauge-cocks in each boiler. Those having three or more boilers in a battery shall have three gauge-cocks in each outside boiler, and two in each remaining boiler in the battery, and the middle gauge-cocks in all boilers shall not be less than four inches above the top of the flues, tubes, or crown of the fire-box.

FUSIBLE PLUGS.

776. A **fusible plug** is shown in Fig. 205. It consists of a brass shell *P* screwed into the highest part of the boiler exposed to the fire and projecting about two inches above the sheet to which it is fitted. A plug *R* made of pure Banca tin and covered by a thin copper cap *S* is held in place by a brass nut *U* screwed over the shell. The bottom of this plug is exposed to the fire. When the water is at its proper level in the boiler, the plug is about 7 inches below the water level, and its melting is prevented by the water above it. Should the water become low within the boiler, it will uncover the plug, but still leave a sufficient quantity of water above the sheet to prevent its getting overheated. The heat of the fire will then melt the plug, and the steam will blow the copper cap out and rush into the furnace, thus giving timely warning to the attendant. The copper cap prevents the water from coming in contact with the plug, thus preventing any chemical changes in the material of which the plug is composed. The rules and regulations of the Board of Supervising Inspectors provide that all steam vessels shall have plugs of Banca tin inserted in their boilers, at least one-half inch in diameter at the smallest end of the internal opening. These plugs must be inserted in the following manner: In fire-box boilers one plug must be inserted in the crown of the back connection, or in the highest fire service of the boiler. All upright tubular boilers used for marine purposes must have one plug inserted in one of the tubes at a point at least two inches below the lowest gauge-cock, or said plug may be placed in the upper head sheet if deemed advisable by the inspector. All fusible plugs must have an external diameter not less than that of a one-inch gas or steam pipe screw tap, except when such plugs are used in the tubes of upright boilers, when they may have an external diameter equal to that of a ⅜-inch gas or steam pipe screw tap. All plugs used must conform in construction with plugs authorized to be used by the Board.

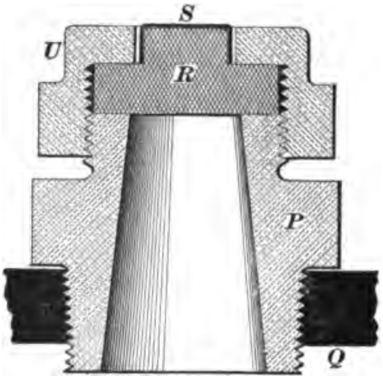

FIG. 205.

BLOW-OFF APPARATUS.

777. In order that a boiler may be emptied either partially or entirely, a cock called the **bottom blow-off cock** is attached by a short pipe to the lowest point of the boiler and is connected with a waste pipe discharging overboard. Fig. 206 shows the manner in which this cock is often constructed.

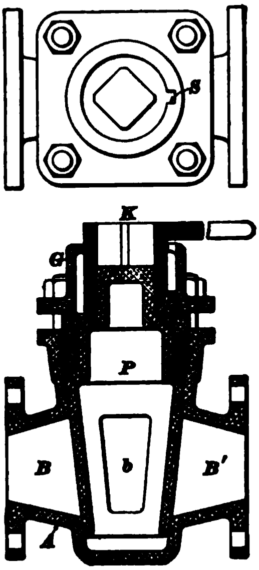

FIG. 206.

The shell *A* of the cock is provided with two rectangular passages *B B'*, and has fitted to it a taper plug *P*, pierced by the trapezoidal hole *b*. The upper end of this plug is cylindrical and somewhat smaller in diameter than the shell, thus leaving a space for the insertion of packing. With the plug in the position shown in the figure, communication between the passages *B* and *B'* is shut off, but on the cock being given a quarter turn, the hole *b* in the plug will be brought in line with the passages *B* and *B'*, thus allowing the steam or liquid in one passage to pass to the other, and from thence to its destination.

A cock in which the passages are directly opposite, like in the one shown, is called a **straight-way cock.** If the passages are at an angle to each other, the cock is called an **angle cock.**

The upper end of the gland *G* forms a cap, which is bored out large enough to admit the body of the spanner, or key, *K*, and is provided with a slot *S*, clearly shown in the plan view, in such a position that in removing the spanner from the square end of the plug, the tongue *t* of the spanner can not pass through the slot *S* unless the cock is fully closed. It is the custom in American steam vessels to put this spanner in the care of the engineer on watch or in charge, thus preventing any person but himself, or a person authorized by him, from opening the blow-off cock. After closing the cock, the spanner is removed and returned to the engineer.

Of late years the use of cocks has been gradually abandoned, the tendency being to substitute gate valves or globe valves, both for the bottom and the surface blow-off.

778. The waste pipe, or blow-off pipe, connects with an angle cock attached to the side of the vessel. The construction of this cock is similar to the one attached to the boiler. When it is desired to blow the boiler off, this cock is opened first, and then the cock attached to the boiler; this operation is reversed when the boiler has been emptied or blown out sufficiently.

Should the waste pipe break while the boiler is blowing off, or at any other time, the water from the sea could flow into the ship if no cock were fitted to the ship's side. It will thus be seen, that the fitting of this cock is merely a precautionary measure. A pipe *F* (see Fig. 207) with one end of it forming a scoop or ladle *G*, extends into the boiler at such a height that the scoop is about 3 inches below the proper water level. This pipe is attached to an angle cock bolted

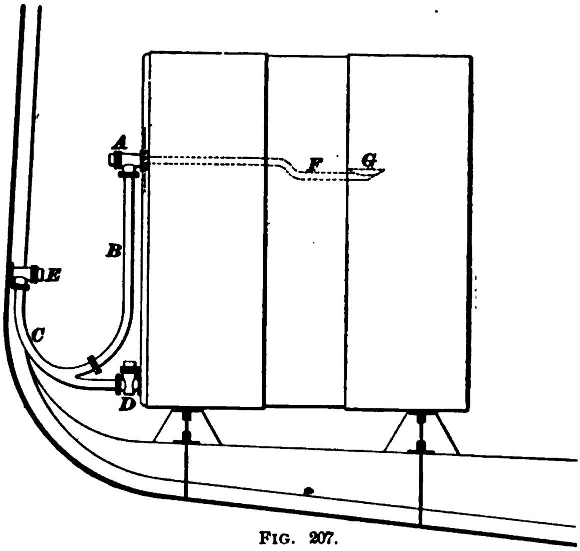

FIG. 207.

to the boiler; the cock has a waste pipe connected with the bottom blow-off pipe, between the bottom blow-off cock and the cock on the side of the vessel. This cock, attached near the surface of the water in the boiler, is known as the **surface blow-off cock,** or **scum cock.** It is used to remove the grease and oil and other impurities floating on the surface of the water. The ladle at the end of the pipe serves merely as a funnel for the collection of these impurities. Sometimes a trough is fitted, running across or from the front to the back of the boilers, having the surface blow-off connected to the bottom of the trough. The ordinary arrangement of the blow-off apparatus is clearly shown in Fig. 207. At *A* the surface blow-off cock is shown; its waste pipe *B* connects to the waste pipe *C* of the bottom blow-off cock *D*. The water discharges overboard through the cock *E*.

MANHOLES AND HANDHOLES.

779. Fig. 208 shows a common construction of a manhole and its cover. An elliptical hole is cut into the head or shell of the boiler, its size being such that it will admit the body of an ordinary man. A wrought-iron or steel ring *R*, called a compensation ring, is riveted to the plate *P*, generally on the outside, for the purpose of strengthening the plate, which is weakened considerably by the cutting of such a large hole through it. A cover *N*, made of wrought-iron or steel, is fitted to the hole, inside of the boiler, and is pro-

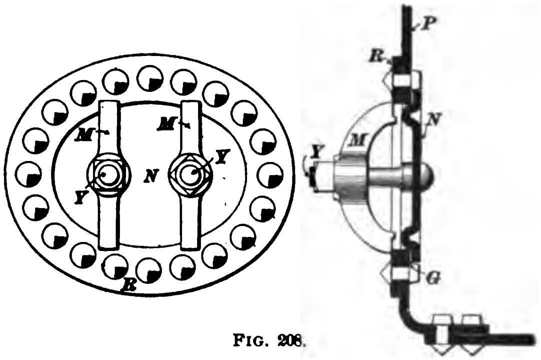

FIG. 208.

vided with two studs *Y*, *Y* riveted to it. This cover is flanged and overlaps the edges of the plate about one inch or more all around its perimeter. A yoke *M* is slipped over each stud, its two extremities resting on the end of the compensation ring. A ring *G*, or *gasket*, as it is commonly called, made of sheet rubber or any other pliable water-proof material, is placed between the plate and the cover and serves to make a water-tight joint.

780. Manholes are made elliptical to allow the cover to be passed through the hole. The smallest diameter of the cover is somewhat less than the largest diameter of the manhole, thus allowing the cover to pass freely through the manhole. It is then turned one-quarter around inside of the boiler, the gasket placed on the flange and put in position. Sometimes a flanged compensation ring is riveted to the inside of the boiler, as shown in Fig. 209, and sometimes the plate itself is flanged inwardly and faced on its edge. The cover is also faced and carefully fitted to it, thus making a "metallic joint." The rules and regulations of the Board of Supervising Inspectors provide that all manholes for the shell of the boilers shall have an opening not less in diameter than 11 × 15 inches in the clear, except that boilers less than 34 inches diameter of shell shall have a clear opening in the manholes of not less than 9 × 14¼ inches; all boiler shells between 34 and 38 inches diameter shall have an opening not less than 9 × 16 inches, and all boiler shells between 38 and 48 inches in diameter an opening not less than 11 × 15¼ inches.

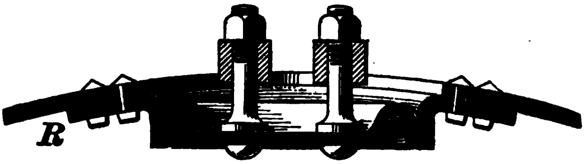

FIG. 209.

Handholes are small elliptical holes cut into boilers to allow removal of dirt or sediment. Their construction is the same as that of manholes, except that but one stud and yoke is fitted to the cover.

MAIN AND DONKEY FEED CHECK-VALVES.

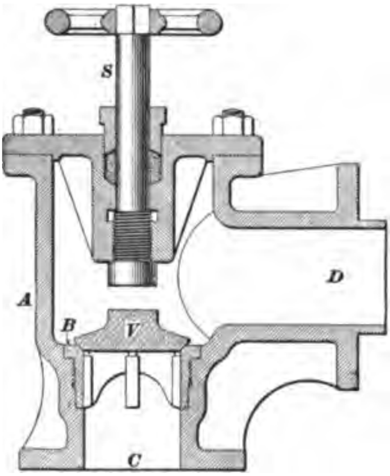

FIG. 210.

781. A common construction of a feed check-valve is shown in Fig. 210. It consists of a body or casing *A* bored out at the lower end to receive the gun-metal bushing *B* forming the seat, and a guide for the valve *V*, which is provided with four wings, as shown. The feed-pipe is bolted to the flange shown at *C*, while the passage *D* connects with the boiler. When water flows through the feed-pipe into the passage *C*, it raises the valve and flows through the annular opening between the valve and seat into passage *D*, and thence through a stop-valve (not shown in the figure) into the boiler. The valve *V* allows the water to pass only in one direction; should the direction of the flow of the water be reversed, the valve will return to its seat and shut off communication with the passage *C*. A valve constructed in such a manner as to allow any liquid to pass through it in one direction only is called a **check-valve.** By means of the screw *S* the height of the lift of the valve can be regulated, thus increasing or diminishing the quantity of water passing through the valve. The object of this arrangement will be explained in the chapter on feed apparatus. The screw *S* is commonly made of Muntz metal, to prevent corrosion.

782. The check-valve fitted to the main feed-pipe is called the **main check-valve,** and the one fitted to the auxiliary feed-pipe is called the **auxiliary,** or **donkey check-valve.** The donkey check-valve does not differ in any respect from the main check-valve; it is merely so named from its location.

STEAM PIPE AND STOP-VALVE.

THE STOP-VALVE.

783. The steam generated in the boiler is carried to its destination by the steam pipe. This pipe is provided with a stop-valve, the construction of which is shown in Fig. 211. A cast-iron casing or body *A* is bolted to a nozzle riveted to the boiler or steam drum. The lower part of the casing is fitted with the gun-metal ring *B*, forming the seat for the valve *V*, which is guided by wings as shown in the figure. The hand wheel *H* is fixed on the end of the valve stem *S*, which works inside a brass nut screwed into the yoke of the bonnet or cover *C*; the valve stem passes through the stuffing-box (with its gland and neck ring as shown), and is so attached to the valve *V* that the latter can be raised or lowered without turning round when the valve stem is revolved by the wheel *H*. To prevent any movement of the brass nut in the yoke, it is locked in position by means of a small pin shown at *D*, which passes through the yoke and nut. The steam pipe is bolted to the flange *F*. When the valve is raised the steam flows through the annular opening between the valve and the seat, and thence into the steam pipe.

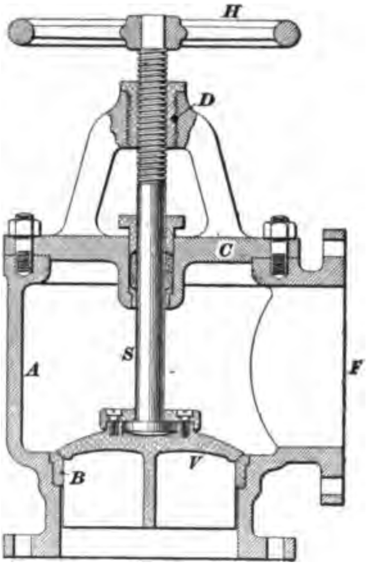

Fig. 211.

EXPANSION JOINTS.

784. Where more than one boiler is used on a steam vessel, all boilers are connected by short branch steam pipes to one large pipe called the **main steam pipe.** Each boiler is provided with a stop valve, by means of which the communication of the boiler with the main steam pipe may be shut off, if desired. The main steam pipe is also provided with a stop-valve close to the engine, for the purpose of shutting off all communication of the steam with the engine. As the heat of the steam expands the metal of which the main steam pipe is composed, and, consequently, increases its length, means must be provided by which this increase in length may be taken into account. This is usually accomplished by means of the expansion joint shown in. Fig. 212.

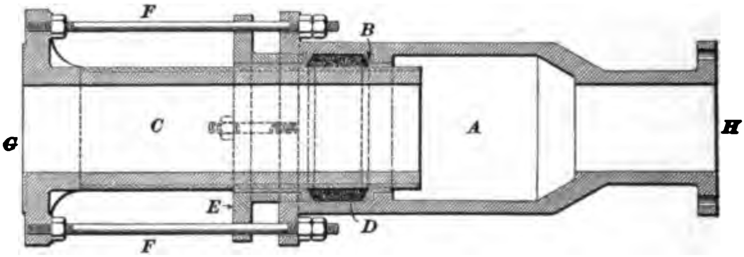

FIG. 212.

A is the body into which the brass bushing *B* is forced. Into the latter is fitted a sliding tube *C*. Packing is placed in the stuffing-box *D*, and is held in position by the gland *E*, the gland being screwed down by means of bolts, one of which is shown in dotted lines. The packing is put in to prevent the leakage of steam. *F*, *F* are studs fitted with nuts and check nuts, and limit the amount of movement of the sliding tube *C*. The flanges *G* and *H* are bolted to the flanges of the pipe; consequently, when the pipe expands, the body *A* of the expansion joint is forced to the left, and the sliding tube *C* to the right, and when the pipe contracts they are forced in opposite directions. A small drain pipe, not shown in the figure, is usually fitted to the lowest point of the body *A*, and is provided with a stop-cock. This pipe is used to drain the water formed by condensation of the steam from the joint. If no provision is made for carrying off this water, it will soon corrode the body, unless it be made of brass or gun metal, as is sometimes the case.

785. Slip joints are used where the expansion of a straight line of large pipe has to be provided for. In a

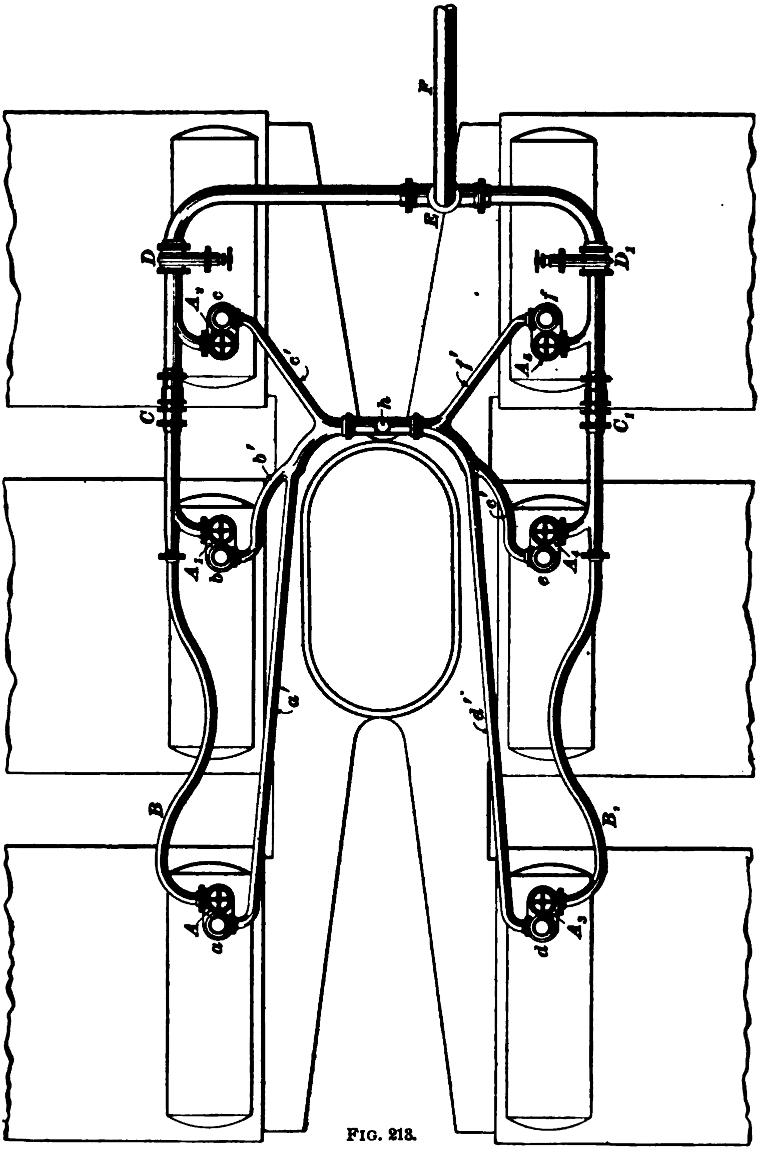

FIG. 213.

great many instances expansion may be provided for by bending the pipes in the manner shown in Fig. 213, the figure illustrating the arrangement of the steam pipes of the steamship Yumuri. There are six boilers standing athwartship, provided with steam drums. Attached to the top of the drums are the stop-valves A, A_1, A_2, etc. The steam pipes B, B_1 are bent to the shape shown, and increase in length by merely bending the pipes. Expansion between A_1 and A_2, and A_4 and A_5 is provided for by the slip joints C and C_1. Each battery has a separate main steam stop-valve, shown at D and D_1. The two branch mains join at E, from whence the main steam pipe F leads to the engine. Fig. 213 also shows the general arrangement of the safety-valve escape pipes. At a, b, c, d, e, and f, the safety valves are shown, a', b', c', d', e', and f' being their respective escape pipes which deliver into the main escape pipe h. Steam pipes are usually made of copper, each section being provided with flanges, by means of which the different sections are bolted together. Wrought-iron pipes are sometimes employed for the smaller sizes, and, lately, welded steel pipes have been used, especially in naval vessels. Copper pipes are sometimes strengthened by winding them with one or more layers of square steel wire, their weak spot generally being the brazed joints. Cast iron was formerly very extensively used for steam pipes, but, owing to numerous accidents due to the treacherous nature of the metal, it has been superseded by copper, wrought-iron, and steel pipes. Steam pipes are generally covered with some non-heat-conducting material.

DONKEY VALVES.

786. As the donkey boiler is used only when the vessel is in port, the steam required to work the auxiliary machinery at sea must be taken from the main boiler or boilers, and this is done by connecting one of the main boilers, where several are used, with the main steam pipe leading to the auxiliary machinery. The pipe connecting the boiler with this steam pipe is provided with a stop-valve similar in construction to the steam stop-valve previously described. This stop-valve is known as the **donkey valve.** Attached to one of the boilers, or sometimes to the main steam pipe, is a pipe leading to the whistle. A stop-valve, known as the **whistle valve,** is fitted to this pipe and serves to admit steam to the whistle.

FIRE APPARATUS.

787. Connected to the main steam pipe or to one of the main boilers is a pipe with branches leading to the different compartments of the vessel, each branch having a separate stop-valve. Usually, all the branches are connected to one fitting, called a manifold, shown in Fig. 214. In case of fire in any of the compartments of the vessel, the compartment is closed, and steam from the boilers is led to the burning compartment by the pipes mentioned above, the steam driving out the air, and thus smothering the fire.

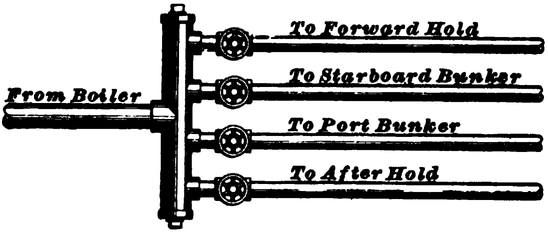

FIG. 214.

The rules and regulations of the Board of Supervising Inspectors of Steam Vessels provide that these pipes shall not be less than 1½ inches in diameter, except on steamers employed on Western rivers, where the branch pipes must not be less then three-quarter inch diameter. Each branch pipe must be supplied with a stop-valve, the handle of which must be marked to indicate the compartment or part of the vessel it leads to, and, if feasible, the whole arrangement is to be enclosed in a suitable box and plainly marked "fire apparatus."

FURNACE FITTINGS.

788. The furnace fittings of a marine boiler are shown in Fig. 215. A furnace front *A*, made of cast iron, is fitted to the front of the furnace. It is held in position by studs screwed into the head of the boiler. A fire door *B* is fitted to the furnace front. It is generally made of the shape shown, and is about 18″ × 15″ in size. A baffle plate *b* is attached to the doors by means of long bolts, distance pieces *b*′ made of iron pipe serving to keep the plate in place. This baffle plate increases the durability of the fire door and prevents to some extent the radiation of heat, as it absorbs most of the radiant heat of the burning fuel, and thus prevents its coming in contact with the fire door. The fire door is often provided with a small hinged door *B*′ to allow the slice bar to be pushed into the furnace and the fire treated without opening the fire door. The inrush of cold air which always accompanies the opening of the door proper is thus avoided. Means for admitting air above the grate are often provided. The fire door may be perforated and supplied with a suitable arrangement by which the admission of air may be regulated. The baffle plate *b* may also be perforated to aid the distribution of the air. The lower end of the furnace front forms what is known as the **dead plate,** extending clear across the furnace.

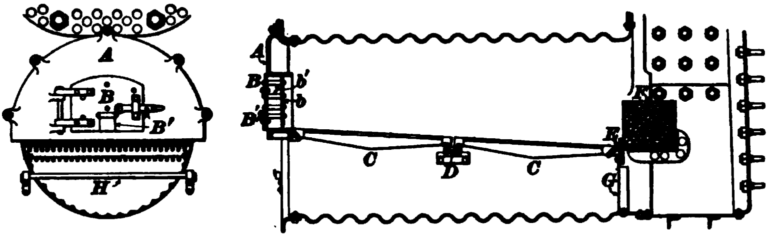

FIG. 215.

The grate, which is generally made in two sections, *C*, *C*, is composed of cast-iron grate bars, supported by a bearing bar *D* in the center of the furnace, and by the dead plate and the bridge bar *E* at their other ends. The grate bars are generally made from 3′ to 3′ 6″ in length, a greater length being awkward to handle. In order to facilitate the access of air, the fall of ashes, and the cleaning of the fire from below, they are made somewhat thinner at the bottom. The usual dimensions are: Width on top, ¾″; width on bottom, ⅜″; depth in the center, about 3″. They are provided with distance pieces cast in one with the bar at both ends, and sometimes also in the center. These keep the bars apart.

789. The opening between the bars varies somewhat with the kind of coal burned; it is from $\frac{7}{16}$″ to ¾″, a soft, bituminous coking coal requiring the larger space. The grate is usually placed lower at the back of the furnace, the inclination being about ¾″ for each foot of length of the grate. This facilitates cleaning and also permits free access of the air to the back of the grate. The bridge *F* is built up of fire-brick on a bearing bar of the cross-section shown. Its object is to retard the escape of the gases from the furnace, and thus promote a more perfect combustion. The distance from the top of the bridge to the top of the furnace varies from 13″ to 16″. The best height is determined by actual trial, as it depends upon the intensity of the draft, the thickness of the fire, the kind of coal used, and the quantity of air admitted. Roughly speaking, the area over the bridge is equal to ⅙ the area of the grate. The cast-iron bar on which the bridge is built is attached to brackets made of angle iron riveted or secured by studs to the side plates of the combustion chamber. The lower half of the furnace flue, that is, the part below the grate which forms the ash pit, is usually lined with sheet iron. Sometimes a thin coating of cement is applied. A baffle plate *G* is secured by tap bolts to the bridge bar *E* and prevents the free access of air to the combustion chamber. Sometimes this plate is perforated, as in some cases it is of advantage to admit air behind the bridge. As will be explained in the section on combustion, if insufficient oxygen is mixed with the fuel, incomplete combustion will result. But by mixing oxygen with the products of combustion, after they have left the furnace proper, and while they are still in a red-hot state, complete combustion can be obtained. Sheet-iron ash-pit dampers are usually provided for the furnaces. They are not attached to the furnace, but are kept in a convenient place until required, when they are merely placed in position, two hooks riveted to the dampers serving to keep them in place. A wrought-iron bar *H* extends across the furnace, about 12″ below the dead plate. This bar serves as a rest for the fire tools when cleaning the grates or ash pit.

FRONT CONNECTION AND SMOKESTACK.

790. The products of combustion, after leaving the furnace, pass through the combustion chamber into the tubes and thence into the front connection. This is made of suitable shape, for instance, as shown in Fig. 216, and is built up of sheet iron about $\frac{3}{16}''$ thick. It is provided with a large door *A* which is fitted with a baffle plate *B* to prevent radiation. This door gives access to the tubes and front tube-sheet.

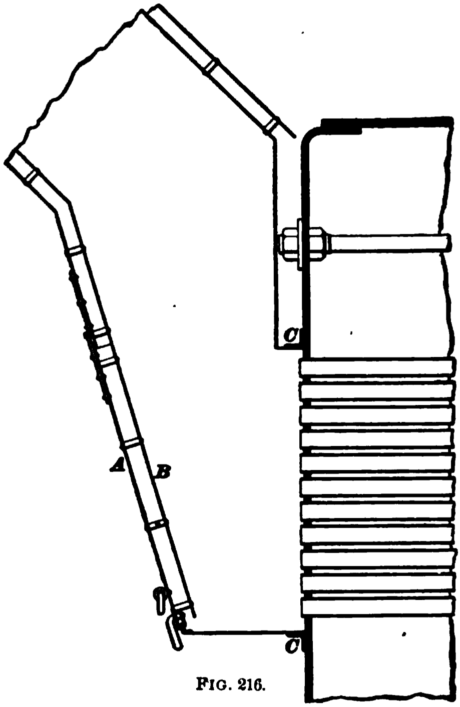

Fig. 216.

The front connection is usually attached to angle irons *C*, *C*, bent to the required shape and secured to the head of the boiler either by studs or by riveting. The front connection, the uptake, and the smokestack are usually made with an air casing around them; that is, they are made double, leaving an air space of about 3″ or more between the inner and the outer plates. The lower end of the smokestack is steadied by a cast or wrought iron ring secured to the upper deck of the vessel. The top of the smokestack is steadied by guy ropes, usually wire ropes, which may be tightened by means of turnbuckles. The area of the uptake should not be less than the combined area of all the tubes discharging into it. A damper is always fitted in the smokestack. The simplest construction of a damper is shown in Fig. 217. A shaft *A* carried in two bearings, one at each side of the stack, has riveted to it a flat wrought-iron plate *B*, elliptical in shape, fitting loosely into the stack. A lever *C* is keyed to one end of the shaft, and provided with an endless chain leading to the engine room, by means of which it may be opened or closed, thus increasing or decreasing the area of opening of the stack, and, hence, regulating the draft. The chain passes over rollers *E*, *F*, *G*, and *H*, and is connected to a lever *D*, working on a stud *d*. A sector *I*, provided with holes, serves to keep the lever in position, a pin *k* being inserted into a hole in the lever and a hole in the sector. With this arrangement the damper is not affected by the rolling or pitching of the vessel.

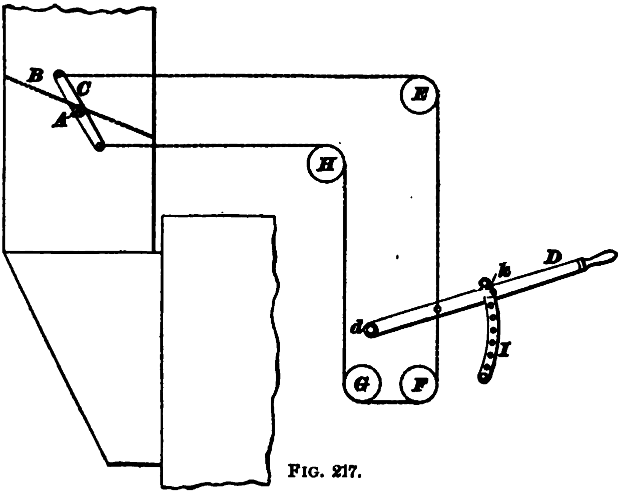

FIG. 217.

SUPERHEATERS.

791. A **superheater** is shown in Fig. 218. It consists of a cylindrical shell *S*, having a number of large tubes *T*, *T* passing through it. It is usually fitted into the uptake or the base of the smokestack, and is provided with a safety valve shown at *B*, and braced in the same manner as a boiler. It is used for drying the steam on its passage from the boilers to the engine. This is done by the hot gases of combustion passing through the tubes and around the shell, thus heating the steam in the superheater above the temperature due to its pressure. A drain pipe *C* is provided. The steam enters the superheater through the pipe *A* and leaves it through the pipe *A'*.

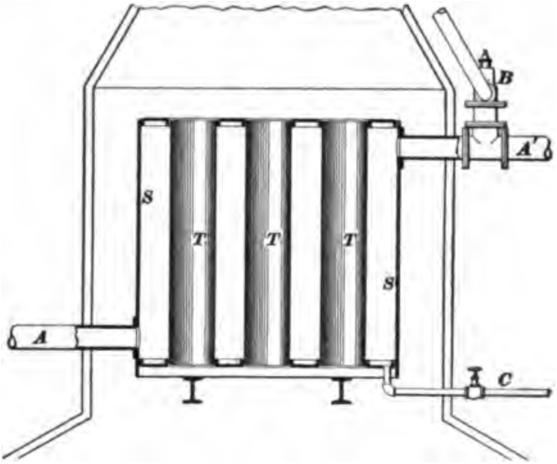

FIG. 218.

792. The connection to the boiler is made in the following manner: The steam pipes *A*, *A'* (see Fig. 219) leading from the several boilers are all joined to the pipe *A"* connected to the bottom of the superheater. The steam enters the superheater *I* through this pipe and passes out at the top in the main steam pipe *G*. Should it be desired to dispense with the use of the superheater, it may be done by opening the valve *C* on the by-pass pipe *D*, and closing the valves *E* and *F*. At *H* the safety valve is shown.

By means of these valves the superheated and the saturated steam may be mixed, if so desired. This is done by opening the three valves, when part of the steam will pass through the by-pass pipe and part through the superheater, the superheated and saturated steam mixing in the main steam pipe. The superheated steam will have a temperature but little less than that of the gases of combustion, say about 650° F., and this high temperature would soon dry out the packing used about the engine and carbonize the oil used for lubricating the cylinder. To avoid this the superheated and saturated steam are often mixed in the manner just described, the superheated steam expending part of its heat in drying the saturated steam. The temperature of the mixture will thus be reduced considerably, and dry steam furnished to the engine. The ratio of the heating surface of the superheater to the heating surface of the boilers is generally made about 1:10.

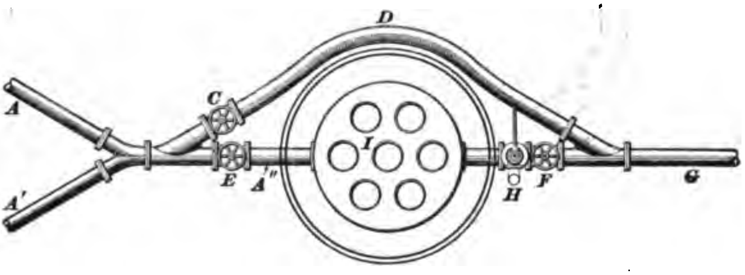

FIG. 219.

793. Superheaters of the shape shown in Fig. 218 have not found much favor in the United States. The **steam chimney,** shown in Fig. 220, is used instead. This, in effect, is nothing but a jacket surrounding the smokestack at its base. The steam enters the steam chimney through the stop-valve *A*, is superheated by coming in contact with the inner lining of the steam chimney, which is heated by the hot gases of combustion, and passes to its destination through the main steam pipe *B*. As usually arranged, the steam chimney can not be shut off, but all the steam on its passage to the engine must pass through the steam chimney. It is braced by staybolts of suitable size and pitch, and must be provided with its own safety valve, which is shown at *C*; also, a drain, shown at *a*, by means of which it may be emptied of all the water formed by the condensation of the steam. To allow of inspection and repair, it is provided with one or more manholes, according to the size, one of which is shown at *m*.

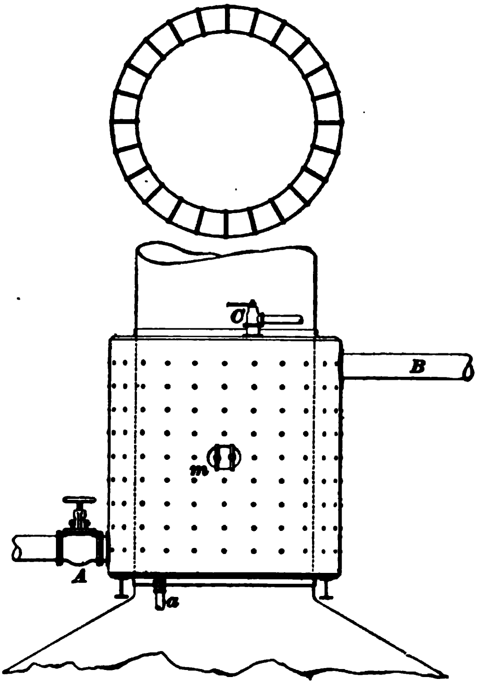

FIG. 220.

STEAM DRUM AND DRY PIPES.

794. A steam drum is a cylindrical vessel connected to the boiler by one or more passages, and placed on top of the boiler to increase the steam space and also to prevent priming. It is supposed that by taking the steam from the boiler at a considerable height above the water level, the steam will be drier. Where steam drums are used, the steam pipe is connected to the drum, and often the safety valves are placed on top of it. Lately, the use of steam drums has been almost abandoned in favor of a so-called **dry pipe.** This is a pipe provided with a number of slots, or perforations, shown at *a*, *a*, etc., in Fig. 221. It is placed inside the boiler at the highest point, and is supported at the ends by iron straps *b b*, bolted to the boiler shell. It is connected to the steam pipe in such a manner that the steam can only enter the steam pipe by passing through the slots, the ends of the pipe being closed. The combined area of the slots, or perforations, in the dry pipe is usually made equal to that of the steam pipe connected to it. A small hole should be drilled into the bottom of the pipe to allow any water entering the dry pipe to drain out of it.

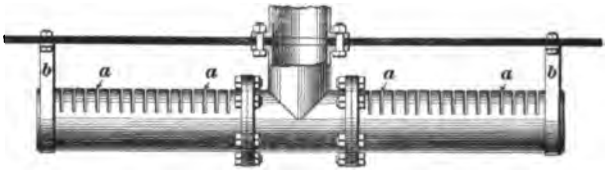

FIG. 221.

SEPARATORS.

705. A **separator** is an apparatus designed to remove the entrained water, or the oil, dirt, or other impurities from a current of steam flowing through a pipe. When the separator is intended to free the steam from water simply, it is placed on the main pipe leading from the boiler to the engine, and as close as possible to the latter. When it is desired to remove the grease and dirt from the exhaust steam before condensing it and feeding it back to the boiler, the separator is placed in the exhaust pipe leading from the engine to the condenser.

796. The Stratton separator is shown in Fig. 222. It consists of a chamber with a steam inlet and outlet, and containing a vertical pipe *a*. Steam enters by the inlet *c*, and is deflected by a curved partition, which gives it a spiral motion about the pipe *a*. The particles of water or oil are thrown off by centrifugal action, and run down the walls to the bottom of the chamber. The steam passes up through the pipe *a* and out of the outlet *d* in a practically dry condition. The separator is provided with a drip pipe *h* for the removal of the water, and a gauge glass *g*. The wings *b*, *b* are four in number, and are for the purpose of destroying the centrifugal effect of the steam after it has reached the bottom of the separator. They likewise offer additional surface for the water particles to adhere to. Were it not for these wings, the steam would keep up its rotative motion while passing up the pipe *a*, and thus necessarily carry some of the entrained water along with it.

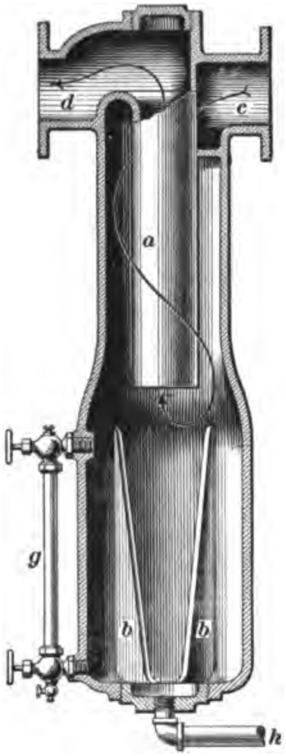

FIG. 222.

797. There are many other makes of separators, all, however, operating on practically the same principle. What is required of a separator is to check the velocity of the steam and change the direction of the current. The particles of water will continue in the original direction of the current by reason of their inertia, while the dry steam passes off in another direction.

STEAM AND STEAM BOILERS.

(CONTINUED.)

BOILER FITTINGS—Continued.

FEED APPARATUS.

798. The Surface Condenser.—The steam generated in the boiler, after performing its work in the engine, is usually discharged into a vessel known as the condenser. The surface condenser will be treated of here only in so far as it relates to the feed apparatus, but will be further dealt with in the section on Steam Engines when considering the general question of condensers. (See Arts. **1085** to **1097**.)

799. The condenser shown in Fig. 223 consists of a closed vessel *B* provided with numerous small brass tubes *C*, *C* fitted air-tight into the tube-sheets *C'*, *C'*. The steam from the engine enters the condenser through the exhaust pipe *A*, strikes against the baffle plate *H*, and is distributed through the condenser, as shown by the arrows.

The steam comes in contact with the outside of the tubes, which are kept cool by water passing constantly through them. The cooling water is taken from the sea by the circulating pump, not shown in the figure, and forced through the main injection pipe *D* into the chamber *E*; thence through the tubes *C*, *C* into the chamber *F*, whence it returns to the chamber *E'*, passes into the outboard delivery pipe *G*, and is discharged overboard.

The exhaust steam coming into contact with the cold surface of the tubes is condensed, that is, turned into water again. This water flows by gravity down the sloping bottom *J* of the condenser and into the lower end of the air pump *K*.

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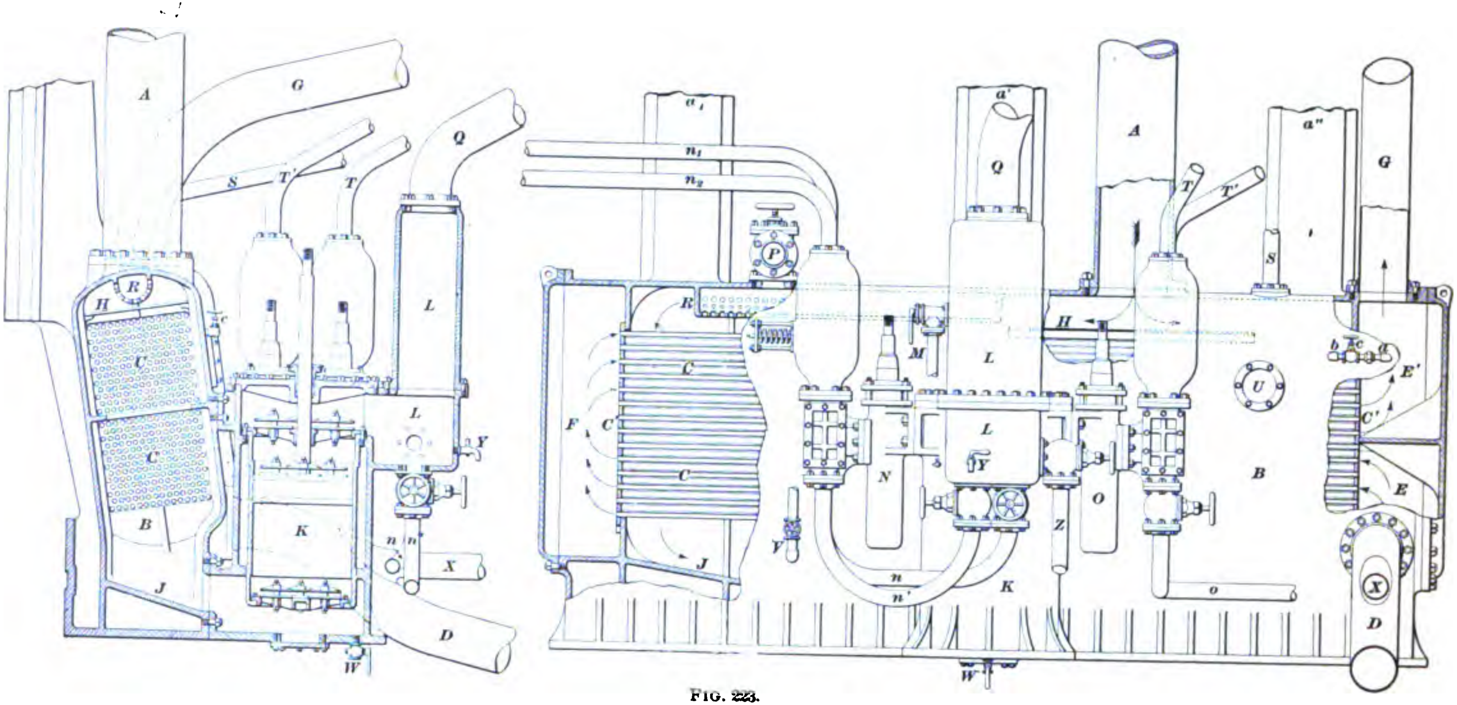
A
G
S
T'
T
Q
L
L
R
H
C
C
B
J
K
n
n'
X
Y
D
W
c
a,
n1
n2
Q
a'
A
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L
Y
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E'
C'
E
B
F
C
C
C
J
V
N
n
n'
K
Z
O
o
X
D
W'

FIG. 223.

This is usually a bucket pump and worked by means of a lever from the engine. Its construction and operation are described later. (See Art. **1094.**) Its purpose is to keep up the vacuum due to the condensation of the steam, which vacuum would otherwise soon be destroyed by the air and vapor arising from the hot water of condensation. It also removes the water from the condenser as fast as it is formed, delivering it into the hotwell *L*. Connected to the bottom of the hotwell are the suction pipes *n*, *n'* of the feed apparatus, which consists either of pumps, or injectors, or both. The feed apparatus is supposed to take the water from the hotwell as fast as it is pumped in; should it fail to do so, the hotwell will speedily fill. An escape for the water is formed by attaching an overflow pipe *M*, provided with a cock, to the side of the hotwell, the pipe leading the water to the bilges. Generally, one or more plunger feed-pumps, which are shown at *N*, are bolted to one side of the air pump and worked from the same connection from which the air pump is worked.

The feed-pumps are connected to the hotwell by their suction pipes *n* and *n'*, the feed-pipes being shown at n_1 and n_2. The suction pipes are provided with stop-valves or cocks, in order that each pump may be shut off, if desired. *Z* is the suction pipe for the auxiliary pump. There are two bilge pumps, one of which is shown at *O*, the pipe *o* leading from the bilge, and the bilge water being thrown overboard through the pipes *T*, *T'*.

That part of the condenser which is filled with the condensing water is called the **water end**; that part which is filled with vapor, or the water of condensation, is called the **steam end.**

800. The construction of the feed-pump is shown in Fig. 224. *A* is the plunger, made of Muntz metal. It is cored out to lighten it, and it passes through a stuffing-box and gland lined with white metal. The pump chamber *B* is bolted to the side of the air pump. A valve chamber *C*, containing the suction valve *D* and delivery valve *E*, is bolted to the pump chamber. These valves work in gun-metal bushings fitted to the valve chamber. Above each valve is a yoke (*d* and *e*) which serves to prevent the valve from rising too high. A small pet-cock *K* is fitted between the suction and

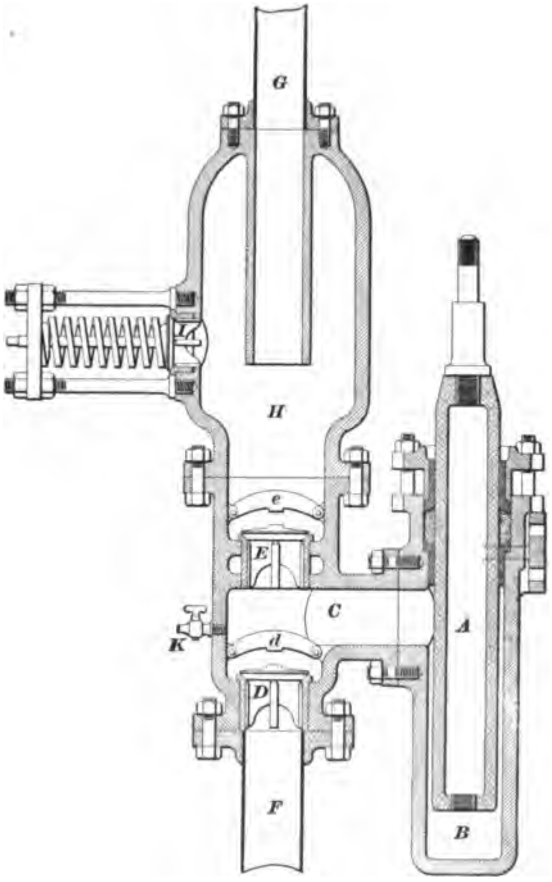


FIG. 224.

delivery valves. This may be used to test the working of the pump, and also to admit air to the air chamber. The air in the air chamber will sooner or later be absorbed by the water, in which case the valves will seat with a considerable shock. Opening the pet-cock will admit air on the up stroke of the plunger. On the down stroke water will issue if the pump is working properly. In some cases no pet-cock is fitted. When the valves seat heavily, or *slam* as it is called, loosening up the gland of the plunger will admit some air and cure the slamming. Bolted to the bottom of the valve chamber is the suction pipe *F*, which connects with the hotwell, and is provided with a stop-valve (not shown in the figure). For the purpose of examination, the valve chamber is provided with a removable cover, not shown in the figure. An air chamber *H* is fitted to the top of the valve chamber, and attached to this air chamber is the feed-pipe *G*, leading to the boiler. The operation of the pump is similar to that of the plunger pump shown in Fig. 131, Art. **612.**

801. There must be two independent sets of feed apparatus for marine boilers. In sea-going steam vessels every pump in the ship is usually provided with the necessary pipe connections to allow it to be used for feeding the boilers. Fig. 225 illustrates the way in which the feed-pipes are usually connected. For the sake of clearness, only the pipes relating to the feed apparatus are shown in the figure. The boiler is shown at *C*, the engine at *D*, the condenser at *E'*, the air pump at *F*, and the circulating pump at *G*. *A* and *A'* are the plunger feed-pumps, worked by the engine, taking their water supply through the suction pipes *o*, *o'* from the hotwell *B*, and discharging the water through the main feed-pipe *b* into the feed check-valve *C*, whence it passes through the stop-valve *D'* into the boiler.

802. On all marine boilers a bronze or brass seated stop-valve must be attached between the boiler and each feed check-valve to facilitate access to the check-valves while the boiler is under steam, and also as an additional safeguard to keep the water in the boiler. Check-valves will sometimes fail to seat properly, allowing the water in the boiler to leak past them while the pump is stopped, the water passing through the main feed-pipe into one of the other boilers in which the pressure may be somewhat less, thus allowing the water to become dangerously low in the first boiler. The piping just described constitutes the **main feed.**

803. The duplex pump *E* and single-acting pump *H* are connected to the hotwell *B* by the suction pipes *c'* and *c*, respectively. The pump *E* is provided with branch suction

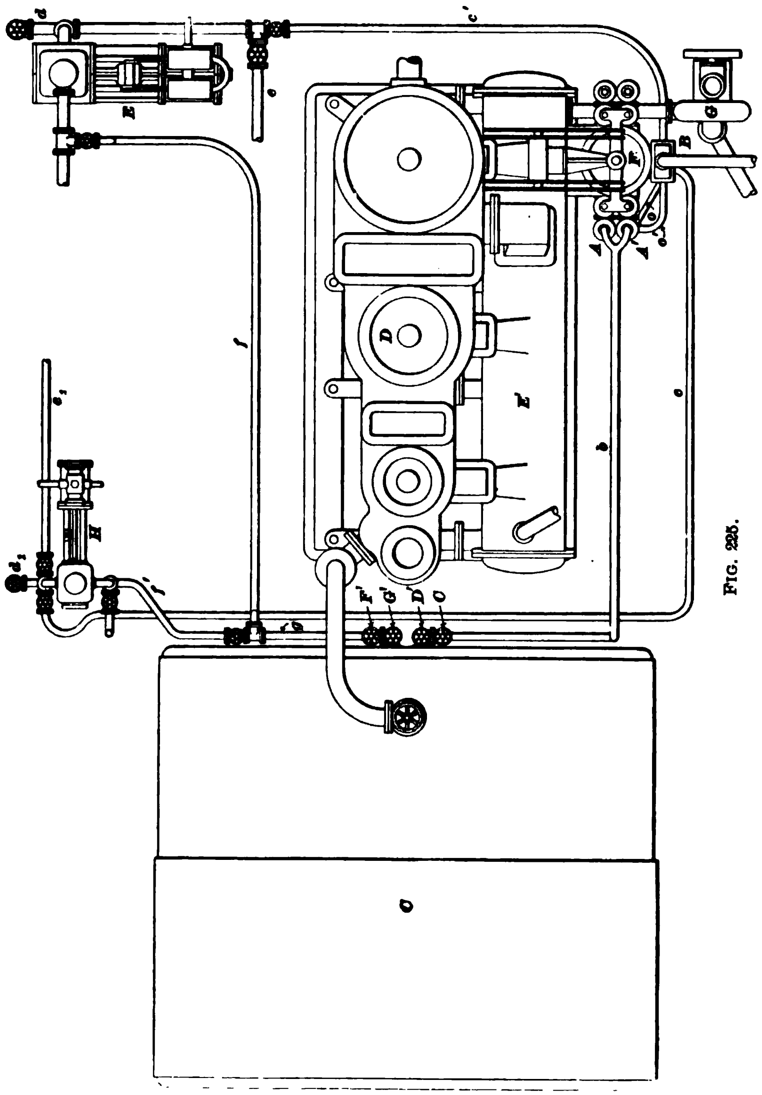


FIG. 225.

pipes *d* and *e*, leading to the sea and to the fresh-water ballast tank. Similar suction pipes d_1 and e_1 are fitted to the pump *H*. Other branch suction pipes leading to various other sources of water supply may be fitted to each pump as required. The pump *E* delivers through the pipe *f*, and the pump *H* through the pipe f', into the donkey feed-pipe *g*, whence the water passes through the donkey check-valve F' and the stop-valve G' into the boiler. These two pipe arrangements constitute the **donkey feed.** It should be clear that in this case there are three ways of feeding the boiler independently of each other. Either the main feed may be used, or, if desired, either one of the two pumps. Very often an injector is provided, taking its water supply from the same sources as the pumps, and discharging into the donkey feed-pipe.

The principle and working of injectors have been explained in Art. **606.**

From the foregoing it follows that it is only through carelessness that the water in a marine boiler can become too low.

804. The plunger pump *A*, Fig. 225, is working as long as the engine is running, and, consequently, is feeding the boiler or boilers continually. When two or more boilers are supplied from the same main feed-pipe, the amount of feed-water entering each boiler is regulated by adjusting the lift of the feed check-valves. If one boiler gets less than its proper amount of feed, as may be found by watching the water gauge, the amount may be increased by giving a higher lift to the check-valve, thus increasing the area of the passage to the boiler. Conversely, reducing the lift of the valve reduces the feed.

When an independent pump is used for feeding one boiler, the feed may be regulated by varying the speed of the pump; but with attached pumps having a constant speed, the check-valve must be used for that purpose.

805. Relief Valves.—Should either the check-valve *C* or stop-valve D', Fig. 225, be closed for some reason while the pump *A* is working, either the pump or the feed-pipe would burst. To prevent this, a relief valve which will open and allow the water to escape in case the pressure on the pipe exceeds the pressure on the boiler by say 15 pounds per square inch is fitted either to the pump chamber or to the air chamber. The usual construction of a relief valve is shown at *l*, Fig. 224. Its principle of action is the same as that of an ordinary spring loaded safety valve. The area of this valve is made equal to that of the feed-pipe.

806. Position of the Feed-Pipe.—The point at which the feed enters the boiler varies with the different builders. Some take the feed through the front head of the boiler, some through the rear head, and some through the shell. The common practice is to lead the feed-water to the coolest part of the boiler. This is done by a pipe secured to the inside of the boiler shell. When a large body of comparatively cool water is discharged on to a hot plate, severe local strains are set up in the plate, due to the contracting of the part cooled by the entering feed-water. These strains would soon destroy the plate, and, to obviate this, the end of the internal feed-pipe is often closed, and the entering feed-water distributed over a large area by numerous holes drilled into the pipe in such a position that the water will issue in a direction away from the plates composing the boiler. In Scotch boilers, in the best modern practice, the feed-water is discharged by a pipe lying on a level with the top row of tubes, the pipe being perforated and discharging downwards between two nests of tubes.

807. Feed-Water Filters.—Where a surface condenser is used, the oil or tallow used for lubricating the cylinder of the engine is carried by the exhaust steam into the condenser; thence, with the condensed steam, into the hotwell, whence it passes into the boiler. This greasy matter often leads to serious trouble. It appears to combine with some of the ingredients of the solid matter in the water, and adheres finally to the plates of the furnace and prevents the water from coming into contact with the plates; as it offers great resistance to the transmission of heat through the plates to the water, it has led to numerous cases of overheating, and to the consequent collapsing of the furnace flues. It also causes the boiler to foam or prime, which subject is treated of further on. Hence, it is very desirable to remove the oil and grease from the feed-water before it enters the boiler. This is done by filtering the feed-water. A common and very efficient method is to discharge the water taken by the air pump from the condenser into an open wooden box filled with hay, instead of discharging it into the hotwell. The suction pipe of the feed-pump is connected to the bottom of this box, the end of the suction pipe being covered with a perforated plate called a **strainer,** to prevent any hay from entering the pump. The oil and grease contained in the water will deposit on the hay, which is renewed occasionally.

808. A **Wass grease extractor,** as made by Cramp & Co., is illustrated in Fig. 226. It consists of a closed

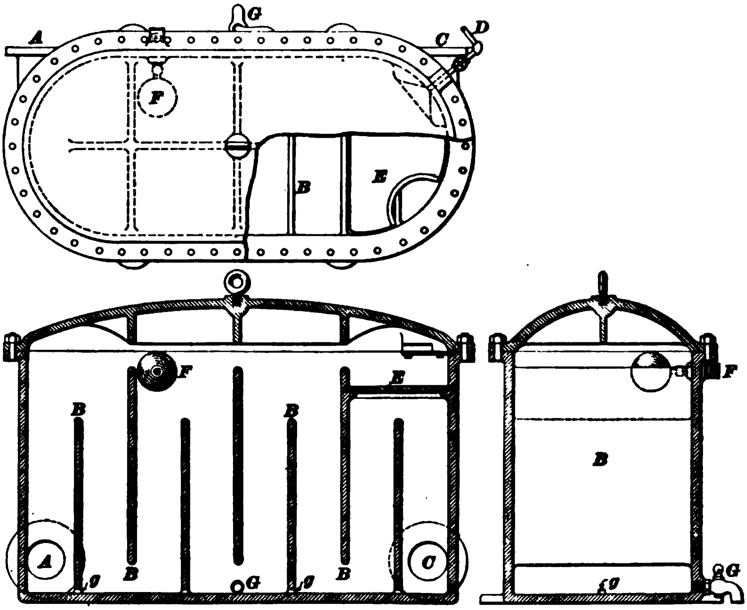


FIG. 226.

cast-iron vessel placed between the feed-pump and the boiler. The feed-water enters at *A* and passes over and under the alternate partitions *B*, *B*, leaving the extractor at *C*. The grease, on account of its lower specific gravity, rises to the surface in each of the compartments formed by the partitions, and since the level of the water is above the top of the higher partitions, the tendency of the grease on the surface is to flow towards the outlet end of the apparatus, where the grease outlet pipe *D* is located. The pressure exerted by the air confined between the surface of the liquid and the cover forces the grease out, on opening the valve in the pipe *D*. This is done at intervals varying with the amount of grease carried in. A plate *E* at the outlet end prevents the mixture of the outflowing grease and feed-water. A float valve *F*, similar in principle to the one described further on in connection with the steam trap, is provided. This valve is open at first, and allows the air to escape from the extractor when it is first filled. The valve is closed by means of the float, upon the water reaching a certain height, and the air remaining is compressed. By means of the drain cock *G* the extractor may be emptied for the purpose of examination or repair, the different compartments communicating with each other by a small hole *g* at the bottom of each partition. This style of extractor is extensively used in sea-going steam vessels having surface condensers, and has given universal satisfaction. The pipe connections are made in the same manner as those of the superheater previously described, in order that the extractor may be cut out, if desired, and the feed-water pass directly to the boiler.

809. Feed-Water Heaters.—For reasons which will be treated of in Art. **1090,** etc., the temperature of the water taken from the condenser does not exceed 150° F.

The higher the temperature of the feed-water, the less will be the amount of fuel required to evaporate it. Also, the higher the temperature of the feed-water entering the boiler, the less the various parts of the boiler will be strained by unequal expansion and contraction, and, consequently, the more durable the boiler will be. It is, therefore, desirable to heat the feed-water by some means before it enters the boiler.

The feed-water is sometimes heated by a jet of steam. This is done by connecting a small pipe to the bottom of the hotwell, this pipe taking live steam from some convenient steam pipe. Where this way of heating the feed-water is used, no fuel will be saved, but the life of the boiler will be increased.

Sometimes the feed-water is heated by passing it through a coil of pipe located in the base of the smokestack or in the uptake. The hot gases of combustion surround the coil of pipe on all sides, and heat the water passing through the coil.

810. Making Up for Loss of Feed-Water.—Where a surface condenser is used, a certain amount of water is evaporated into steam in the boiler, and turned into water again in the condenser, and taken from there to the boiler and again converted into steam, once more used in the engine, and again exhausted into the condenser, and so on. It will be seen that if there were no loss through leakage, etc., the same water could be used over and over again, no further supply being needed; but, as a matter of course, there is always a certain amount of leakage going on, both in the piping conveying the steam to the engines, in the engines themselves (at the glands, etc.), and also in the feed-pumps and piping, besides the loss due to blowing the whistle, and also using steam in the various auxiliary engines that do not exhaust into a condenser. All these losses have to be made good. This loss of water may be discovered by watching the water gauge of the boiler. Should the water gradually become less in the boiler, with the feed-pumps operating properly, it would show that there is a deficiency of water in the hotwell.

811. The most common way of making up for the loss consists in connecting the water end of the condenser with the steam end of the condenser. This may be done by means of the $\frac{3}{4}''$ pipe shown in Fig. 223, the end *a* communicating with the water end, and the end *b* communicating with the inside of the condenser; this pipe has a stop-valve *c* which is opened whenever required, and allowed to remain open for a certain length of time. Some of the cooling water will flow into the condenser and mingle with the condensed steam. This arrangement is known as the **salt feed.**

812. When a ship is fitted with ballast tanks filled with fresh water, a small pipe provided with a stop-valve may connect the inside of the condenser with the tank or tanks. When opening the stop-valve, the pressure of the air forces the water in the tanks into the condenser, as there is a partial vacuum within it.

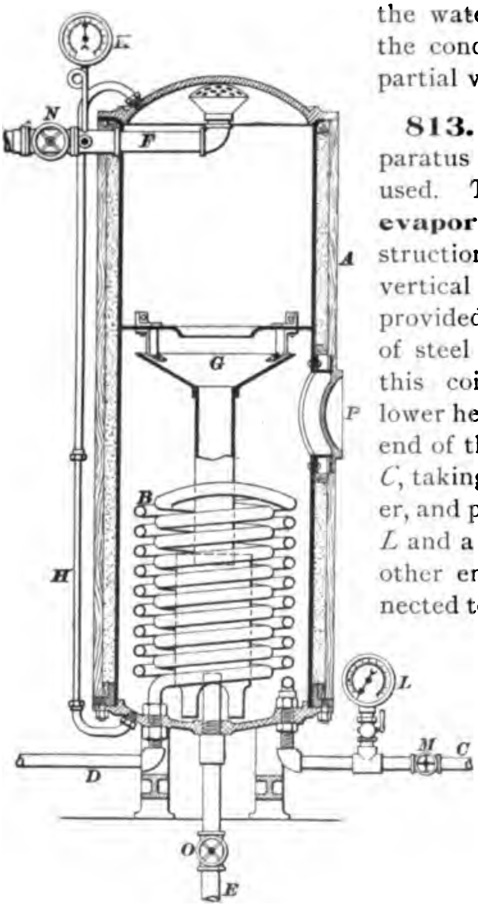


FIG. 227.

813. Sometimes the apparatus shown in Fig. 227 is used. This is the **Baird evaporator,** and its construction is as follows: A vertical cylindrical shell *A* is provided with a coil *B* made of steel pipe. Both ends of this coil pass through the lower head. Attached to one end of the coil is a steam pipe *C*, taking steam from the boiler, and provided with a gauge *L* and a stop-valve *M*. The other end of the coil is connected to a drain pipe *D* leading to the hotwell. A feed-pipe *E*, fitted with a stop-valve *O*, is screwed through the center of the lower head, and connects either with the sea or with the water end of the condenser. The pipe *F*, provided with a stop-valve *N* and a combined vacuum and pressure gauge *K*, is connected with the steam side of the condenser. Above the coil *B*, a separator *G* is fitted. This merely serves to keep the sea-water from entering the pipe *F* when the vessel is rolling or pitching. A manhole *P*, fitted with a cover, is cut into the shell, allowing examination of the inside.

814. The operation of the evaporator is as follows: On opening the stop-valve *N* a vacuum will be formed inside the evaporator, since the evaporator is connected with the condenser, in which there is a vacuum. Next, the stop-valve *O* is opened, and sea-water flows into the evaporator, the glass water gauge *H* showing the amount of water admitted. Enough sea-water is admitted to cover the coil *B* to a depth of several inches; then the water supply is shut off. Next, steam is admitted to the coil *B*, heating and evaporating the sea-water surrounding the coil. It has been shown (see Art. **673**) that water will boil at a temperature varying with the pressure. This principle is made use of in the evaporator by connecting it to the condenser and evaporating the sea-water in a partial vacuum. Consequently, somewhat less heat is required to evaporate the sea-water. The vapor from the sea-water rises and flows through the separator *G* into the pipe *F*, and thence into the condenser, where it mingles with the exhaust steam from the engine. The rate of evaporation is regulated by the stop-valve *M* on the steam pipe *C*, the pressure gauge indicating the pressure. The higher the pressure, the higher the temperature of the steam, and, consequently, the faster the rate at which the sea-water is evaporated. When the water gauge shows that most of the water has been evaporated, more is admitted by opening the stop-valve *O*. But the density of the water will soon increase, and, therefore, either some or all of it must be blown off occasionally. This is done by first closing the stop-valve *N*; this causes some of the sea-water to be evaporated into steam, and the pressure begins to rise, as shown by the gauge *K*. The stop-valve *O* of the feed-pipe is opened when a sufficient steam pressure has been reached, and the dense water in the evaporator is blown out. The evaporator is now ready for a fresh charge of sea-water, and the operation may be repeated.

815. An important part of the evaporator is a device called a **steam trap.** This device permits the water formed by the condensation of the steam in the coil *B* to drain into the hotwell, but it prevents the escape of steam, thus holding the pressure and temperature inside the coil, and, consequently, utilizing the latent heat of the steam in vaporizing sea-water fed to the evaporator. The construction of the trap usually furnished with the evaporator just described is shown in Fig. 228. Inside a cast-iron chamber is a lever *F*, pivoted at *H*, and working in a forked guide *E*. Attached rigidly to the lever *F* is a hollow copper ball *D*, known as a **float.** A piston valve *U* is attached by the link *C* to the lever *F*. The valve is provided with four ports, all of them, at a certain position of the valve, communicating with the passage in the valve chamber *K*, this passage leading to the drain pipe *I* which communicates to the hotwell. The inlet pipe *G* is connected to the drain pipe *D* of the evaporator. When steam is admitted to the evaporator, it flows through the coil and into the drain pipe, and thence into the steam

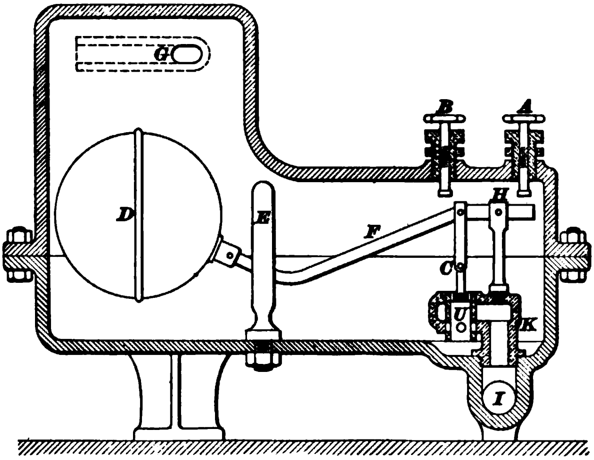


FIG. 228.

trap, where further escape is prevented, since the outlet to the drain pipe *I* is shut off by the valve *U*, the ball *D* being in the position shown. The steam in the coil and the trap gradually condenses, the water gradually collecting and rising in the trap and lifting the ball *D* until, by means of the connections shown, the valve *U* is opened, when the pressure of the steam will blow a certain quantity of the water through the four passages of the valve into the valve chamber, and thence into the drain pipe *I*. The ball *D* sinks as fast as the water is leaving the trap, until the communication to the drain is shut off, when the water will collect again into the trap and the operation will be repeated. By means of the adjusting screw *A*, which limits the drop of the ball, the quantity of water discharged may be regulated. The screw *B*, forming a stop for lever *F*, is used for regulating the amount of opening of the valve. Steam traps are made in a variety of forms, all of them acting upon the same principle as the one just described. They are often used on the return pipes of the steam-heating systems of steam vessels, and serve the same purpose as the one used in connection with the evaporator.

THE SALINOMETER.

816. Ordinary sea-water contains 1 pound of solid matter, about one-quarter of which is salt, in every 32 pounds of water. If sea-water be evaporated, the solid matter held in solution in the water will remain; that is, if 32 pounds of sea-water are evaporated, 31 pounds of steam will be formed and 1 pound of solid matter will remain. Should but part of the 32 pounds be evaporated, say 16 pounds, there will remain 16 pounds of salt water containing 1 pound of solid matter. Again, if one-half of this be evaporated, the 1 pound of solid matter would still be contained in the remaining 8 pounds of salt water. Suppose we have a vessel containing 32 pounds of sea-water, and that 16 pounds of water are evaporated and there remain in the vessel 16 pounds of water containing 1 pound of solid matter. Another 32 pounds of sea-water are put into the vessel, and the same number of pounds are evaporated. It is evident that there are now 2 pounds of solid matter contained in the 16 pounds of water that are left in the vessel. This shows that the more sea-water is added and evaporated, the more solid matter will be contained in the water remaining in the vessel. This is exactly what takes place in a marine boiler using sea-water, and it is evident that in order that the amount of contained solid matter may not exceed a certain amount, a portion of the water must be drawn off from the boiler occasionally. This is done by means of either the bottom or the surface blow-off cock, and is termed **blowing off.** It has been determined by practical experience that the amount of solid matter in the water of the boiler should not exceed 2 pounds in every 32 pounds of water.

817. The term **saturation** is used to denote the number of pounds of solid matter in every 32 pounds of water, and the saturation is usually expressed in the form of a fraction.

For instance, $\frac{3}{32}$ saturation means that there are 3 pounds of solid matter in 32 pounds of water. Fresh water at sea-level boils at 212° F., but if solid matter be added, the temperature of the boiling point will be raised. In Table 29 the exact boiling points of sea-water at different degrees of saturation are given. As water will boil at a temperature varying with the pressure of the atmosphere, the boiling points given in the table are correct for but one pressure, namely, 30 in. of mercury:

TABLE 29.

Saturation.	Boiling Point.	Saturation.	Boiling Point.
$\frac{0}{32}$	212.0° F.	$\frac{7}{32}$	220.3° F.
$\frac{1}{32}$	213.2° F.	$\frac{8}{32}$	221.5° F.
$\frac{2}{32}$	214.4° F.	$\frac{9}{32}$	222.7° F.
$\frac{3}{32}$	215.5° F.	$\frac{10}{32}$	223.8° F.
$\frac{4}{32}$	216.7° F.	$\frac{11}{32}$	225.0° F.
$\frac{5}{32}$	217.9° F.	$\frac{12}{32}$	226.1° F.
$\frac{6}{32}$	219.1° F.		

At $\frac{12}{32}$ saturation the water becomes saturated; that is, it will not dissolve any more matter.

It will be seen that the boiling point increases, practically, 1.2 degrees Fahrenheit for every pound of solid matter added. It has been determined that for every tenth of an inch variation in the height of the barometer, the boiling point of the water varies $\frac{16}{100}$ degree F. If the height of the barometer is less than 30 in., the water will boil at a lower temperature. For instance, if the height of the barometer is 29 in., water containing $\frac{2}{32}$ solid matter would boil at $214.4 - 10 \times .16 = 212.8°$. Conversely, if the height of the barometer were 30.4 in., the same water would boil at $214.4 + 4 \times .16 = 215.04°$ F.

Should it be desired to find the saturation of the water in the boiler, some water is drawn from it into an open vessel and is then heated to the boiling point. The temperature at which it boils is ascertained by means of a thermometer, and the boiling point, with the barometer standing at 30 in., is calculated. The saturation is found by the approximate rule given below:

Let A = boiling point of the water tested;
B = product of the number of tenths of an inch variation in the height of the barometer and .16, this product to be subtracted when the barometer is above 30 in., and to be added when below 30 in.

Rule 154.—*To find the degree of saturation by means of the thermometer, subtract 212° from the corrected boiling point of the water tested and divide the remainder by 1.2.*

That is,

$$S = \frac{(A \pm B) - 212}{1.2}.$$

Example.—A sample of water boils at 215.2° F., the height of the barometer being 30.5 in. What is the saturation of the water?

Solution.—Substituting values in the above formula, we have

$$S = \frac{(215.2 - 5 \times .16) - 212}{1.2} = 2 \text{ lb. of solid matter, or } \frac{2}{32} \text{ saturation.} \quad \text{Ans.}$$

It will be seen that we *subtracted* the value of B from that of A, because the barometric reading was *above* 30 in.

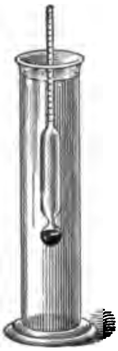


FIG. 229.

818. It is evident that, as the density of water increases, the more solid matter there is dissolved in it, and, consequently, by measuring the density of the water, the amount of solid matter may be readily found. This is done by means of a **hydrometer,** shown in Fig. 229. This instrument is often called a **salinometer;** that is, a salt measurer, as it is used for measuring the quantity of salt contained in water. It consists of a glass tube, near the bottom of which are two bulbs. The lower and smaller bulb is loaded with mercury or shot, so as to cause the instrument to remain in a vertical position when placed in the water. The upper bulb is filled with air, and its volume is such that the whole instrument is lighter than an equal volume of water.

It is graduated by placing it first in fresh water having a temperature of 200° F. The depth to which the instrument sinks in the water is marked on the glass tube. It is then placed in water having the same temperature and containing $\frac{1}{32}$ part of salt. The depth to which the instrument now sinks is the sea-water mark, and it is also marked on the tube. This operation is repeated with water containing $\frac{2}{32}$, $\frac{3}{32}$, etc., up to $\frac{12}{32}$ part of salt, always taking care that the temperature of the water is exactly 200° F. The marks on the tube are transferred to a paper scale, and this is pasted to the inside of the tube in exactly the same position as the marks on the tube. The distances between the marks are usually divided into halves, quarters, and eighths.

If the hydrometer is placed in a vessel containing some water drawn from the boiler and having a temperature of 200° F., it will sink to a depth corresponding to the density of the water, and the saturation may be read off on the scale. For instance, if the hydrometer sinks to the $2\frac{7}{8}$ graduation on the scale, it means that there are $2\frac{7}{8}$ pounds of solid matter in every 32 pounds of water.

Should the temperature of the water under test vary from the temperature at which the hydrometer was graduated, the indication of the hydrometer would not be correct. This is due to the difference in density of water at different temperatures. Allowance may be made for this error in the following manner: The indication of the hydrometer will vary $\frac{1}{80}$ part of 1 pound of solid matter for every degree the temperature of the water varies from 200° F.

Thus, if the temperature be 205° F., the hydrometer will show $\frac{5}{80}$ part of one pound of solid matter less than there really is in the water. For instance, the temperature of the water being 205° F., and the hydrometer indicating $1\frac{15}{16}$ pounds of salt, the actual amount of salt would be $\frac{5}{80}$ of one pound more, and as $\frac{5}{80} = \frac{1}{16}$, $1\frac{15}{16} + \frac{1}{16} = 2$ pounds would be the actual saturation.

Also, should the temperature of the water be less, the indication of the hydrometer will be too high. For example, the temperature of the water being 180° F., and the hydrometer indicating $2\frac{1}{4}$ pounds, the indication would be $\frac{20}{80}$ of a pound too high. Subtracting $\frac{20}{80} = \frac{1}{4}$ from $2\frac{1}{4}$, we get 2 pounds, the true amount of solid matter in every 32 pounds of the water drawn from the boiler.

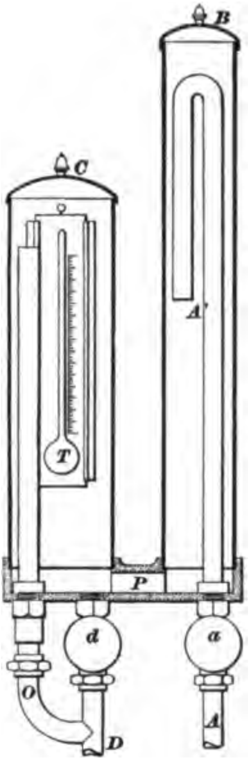


FIG. 230.

819. For convenience, a **salinometer pot,** shown in Fig. 230, is commonly used. It is attached to the boiler, or, should there be several boilers, it is put up in the engine room and connected by branch pipes with each boiler. It consists of two cylindrical brass vessels, one of larger diameter than the other, communicating with each other by a passage *P* in the base of the instrument. A removable cover *C* is fitted to the larger vessel. The pipe *A* connects with the water space of the boiler. Upon opening the stop-valve *a*, water flows into the smaller vessel, and through the passage *P* into the larger one. The water is prevented from overflowing by the overflow pipe *O*, which empties below the drain valve *d* into the drain pipe *D*. A thermometer *T* indicates the temperature of the water. If the temperature of the water is too low, the stop-valve *a* is opened until the temperature reaches 200° F.; if too high, the water is allowed to cool down. The water from the boiler is under high pressure, and, consequently, is liable to turn suddenly into steam when the pressure is reduced; hence, it is advisable to discharge the water first into the smaller vessel through the bent pipe *A′*, thus reducing the pressure gradually. Any vapor formed may escape through the perforated top *B*. When the water in the larger vessel is at the proper temperature, the density is ascertained by means of the hydrometer, and then the pot is drained by opening the drain valve *d*.

820. It is evident that to keep the water in the boiler at a certain degree of saturation, the solid matter carried with the feed-water into the boiler in a stated time must be removed from the boiler in the same length of time. That is, the number of pounds of feed-water multiplied by the amount of solid matter in 1 pound, must equal the number of pounds of water blown off multiplied by the amount of solid matter in 1 pound.

Let A = the weight of the water blown off, in pounds;
B = the numerator of the fraction expressing the saturation of A;
C = the weight of the feed-water, in pounds;
D = the numerator of the fraction expressing the saturation of C;
E = the weight of water evaporated, corresponding to C.

Rule 155.—*To find the amount of water to be blown off in a stated time, divide the product of the weight of the feed-water and its saturation by the saturation of the water blown off.*

That is, $$A = \frac{CD}{B}.$$

NOTE.—The volume of the water in gallons may be found approximately by dividing the number of pounds by 8.4. By *saturation* in rules **155** and **156** is meant the numerator of the fraction expressing the saturation.

EXAMPLE.—2,000 pounds of feed-water enter a boiler every hour at a saturation of $\frac{1}{32}$. How much water must be blown off every hour to keep the saturation at $\frac{2}{32}$?

SOLUTION.— $$A = \frac{CD}{B}.$$

Substituting, we have $A = \frac{2{,}000 \times 1}{2} = 1{,}000$ lb. Ans.

156.—*To find the amount of water evaporated into steam for a given weight of water blown off, divide the saturation of the water blown off by the saturation of the feed-water, subtract 1 from this quotient and multiply the remainder by the weight of the water blown off.*

That is, $$E = A\left(\frac{B}{D} - 1\right).$$

EXAMPLE.—The saturation of the feed-water being $\frac{1}{32}$, how much water is evaporated into steam for one pound of water blown off at $\frac{2}{32}$ saturation, the saturation to be kept constant?

SOLUTION.— $$E = A\left(\frac{B}{D} - 1\right).$$

Substituting, we have $E = 1\left(\frac{2}{1} - 1\right) = 1$ lb. Ans.

821. The following scheme shows the method of regulating the saturation in a boiler containing 100,000 pounds of sea-water. The saturation is not to exceed $\frac{2}{32}$.

	Number of Pounds of Water in Boiler.	Pounds of Solid Matter.
Evaporated into steam.......	100,000	3,125
Fed in to make up at $\frac{1}{32}$.....	100,000	3,125
Water in boiler..............	100,000	6,250
Blown off....................	20,000	1,250
Water in boiler..............	80,000	5,000
Fed in at $\frac{1}{32}$................	20,000	625
Water in boiler..............	100,000	5,625
Evaporated..................	20,000	Steam
Water in boiler..............	80,000	5,625
Fed in at $\frac{1}{32}$................	20,000	625
Water in boiler..............	100,000	6,250

By drawing hot water from the boiler and making up the deficiency with colder water, a certain amount of heat is lost. The amount is equal to the difference in temperature between that of the water in the boiler and that of the feed-water. For instance, if the temperature of the water in the boiler be 320°, and that of the feed-water 180°, then, $320° - 180° = 140°$ will be the amount of heat lost. This represents the number of B. T. U. lost for each pound of water blown off and replaced with a pound of cooler water.

822. The percentage of loss of heat due to blowing off is found by the following rule:

Let A = the total heat (reckoned above the temperature of the feed) imparted to the amount of water converted into steam for 1 pound of water blown off;

B = the number of B. T. U. lost in blowing off 1 pound of water;

L = the percentage of heat lost.

Rule 157.—*To find the percentage of loss of heat due to blowing off, multiply the number of B. T. U. lost in blowing off 1 pound of water by 100. Divide the product by the total heat above the temperature of the feed-water imparted to the amount of water converted into steam for 1 pound of* *water blown off plus the number of B. T. U. lost in blowing off 1 pound of water.*

That is, $$L = \frac{100\,B}{A+B}.$$

Example.—The saturation of the water in a certain boiler is to be kept at $\frac{3}{32}$. The temperature of the feed-water being 160° F., its saturation $\frac{1}{32}$, and the steam pressure 90 lb., absolute, what will be the percentage of loss due to blowing off?

Solution.— $$L = \frac{100\,B}{A+B}.$$

According to the steam table, the sensible heat of steam at 90 lb. pressure is 320.094° F., and 320.094° − 160° = 160.094°, which represents the difference of temperature, and, as explained before, the number of B. T. U. lost in each pound of water blown off.

In order to find the total heat mentioned in the rule, the amount of water converted into steam for one pound of water blown off has first to be found from rule **156.** This amount is found to be 2 pounds. The total heat above 32° F. of a pound of steam at 90 lb. pressure is 1,179.569 B. T. U., and above 160° F. it is 1,179.569 − (160 − 32) = 1,051.569 B. T. U. Consequently, for 2 pounds the total heat is 1,051.569 × 2 = 2,103.138 B. T. U. For each pound of water blown off, 1 pound will have to be fed into the boiler, in addition to the amount evaporated, and the amount of heat carried into the boiler by this pound of feed-water must be added to the total heat. Now, substituting the values found, we have

$$\frac{160.094 \times 100}{2{,}103.138 + 160.094} = 7.07\%, \text{ nearly. Ans.}$$

EXAMPLES FOR PRACTICE.

1. The height of the barometer being 30.4 in., find the boiling point of water having a saturation of $\frac{5}{32}$. Ans. 218.54° F.

2. A sample of water boils at 216.7° F., the height of the barometer being 30.3 in. Find the saturation. Ans. $\frac{3.52}{32}$.

3. A salinometer immersed in water having a temperature of 208° F. indicates 3 pounds of salt. Correct the reading. Ans. 3.1 lb.

4. How much water at $\frac{3}{32}$ saturation will have to be blown off to keep the saturation constant, the saturation of the feed-water being $\frac{1}{32}$, and 5,000 pounds entering the boiler every hour? Ans. 1,666.67 lb.

5. How much water at $\frac{1}{32}$ is evaporated for every 120 pounds of water blown off at $\frac{3}{32}$? Ans. 240 lb.

6. Find the percentage of heat lost in blowing off water at $\frac{4}{32}$, the steam pressure being 100 pounds, absolute, the temperature of the feed-water 190° F., and its saturation $\frac{1}{32}$. Ans. 4.29%, nearly.

COMBUSTION AND FUELS.

LAWS OF CHEMICAL COMBINATIONS.

823. Elements and Compounds.—Every body, every mass of matter, is either an **element, a compound,** or a **mixture.** Iron, silver, sulphur, and oxygen are elements; water, wood, lime, and carbonic acid are compounds.

824. A compound may be decomposed or divided into separate substances. For example, if an electric current is passed through water, the water slowly disappears and two gases are formed. These gases are entirely unlike, and neither resembles the water from which it was produced. Likewise, lime can be divided into two other substances, calcium and oxygen. Any substance which can thus be decomposed or divided into other substances is called a **compound.**

825. There are substances, however, which have never been decomposed into other substances. By no known process can sulphur be separated into other substances; neither can iron, gold, arsenic, and many other substances. Substances which have never been decomposed are called **elements.**

826. The elements which will be considered in this Course are the following:

Element.	Symbol.
Hydrogen	H
Oxygen	O
Nitrogen	N
Carbon	C
Sulphur	S

In referring to an element it is customary to simply use the symbol, which is usually the first letter of the name. Thus, H stands for hydrogen, C for carbon, etc.

827. Chemical Combination.—When two or more elements are brought into contact under favorable circumstances, they will combine and form a new substance which is unlike either of the elements. The nature of the compound thus formed will generally be very different from that of the elements, so far at least as it affects our senses.

For example, hydrogen, an exceedingly light gas, will burn in the air with a light blue flame. This burning is a form of chemical combination, in which the hydrogen unites with another gas, namely the oxygen in the air, and forms water, a substance familiar to us all. Chemical combination produces heat, whereas chemical separation or decomposition absorbs heat. Thus, if carbon and oxygen are brought together at high temperature, they will combine and form carbon dioxide; hydrogen and oxygen combine to form water; hydrogen, nitrogen, and oxygen, when combined in certain proportions, form nitric acid; a given volume of nitrogen and three times that volume of hydrogen combine and form ammonia, a gas which differs greatly from both nitrogen and hydrogen. In each of these combinations a certain amount of heat is produced, and an exactly equal amount of heat will be absorbed in separating them again.

828. It is supposed that each molecule of an element, such as hydrogen or oxygen, is composed of two atoms. It is further supposed by chemists that equal volumes of all *gases*, whether simple or compound, contain the same number of molecules. Thus, a cubic foot of hydrogen, a cubic foot of air, a cubic foot of steam, all contain the same number of molecules.

Suppose, now, that a cubic foot of hydrogen gas is allowed to come in contact with a cubic foot of *chlorine* gas (symbol Cl). The mixture is exposed to heat or light, and the gases combine. The process of combination is explained as follows: There is a certain attraction or affinity between the hydrogen atoms and the chlorine atoms. Under the influence of heat or light, this attraction becomes so strong that the two atoms composing the molecule of hydrogen are torn apart. Likewise, the atoms composing a molecule of chlorine separate. Each atom of chlorine seizes upon an atom of hydrogen and forms a molecule of an entirely new gas, viz., hydrochloric acid gas. Since each atom of chlorine takes *one* atom of hydrogen, it must be plain that the number of molecules of each gas must be the same. In other words, a cubic foot of chlorine requires a cubic foot of hydrogen to combine with it; these gases can not be made to combine in any other proportion. For example, if three cubic feet of chlorine were placed in contact with two cubic feet of hydrogen, four cubic feet of hydrochloric acid gas would be formed and the extra cubic foot of chlorine would still remain chlorine. The symbol for hydrochloric acid gas is $H\ Cl$.

829. Suppose, now, that hydrogen and oxygen are placed in contact and heated; they will combine and form steam (or water). But it will be found that each atom of oxygen seizes two atoms of hydrogen to form a molecule of water, and, therefore, the volume of hydrogen must be double the volume of the oxygen with which it combines. This is shown by the symbol for water, which is $H_2\ O$; that is, two parts of hydrogen to one of oxygen. Similarly, the symbol for ammonia is $N\ H_3$; that is, three parts of hydrogen to one of nitrogen. Again, hydrogen and carbon form a compound; each atom of carbon seizes four atoms of hydrogen and forms a molecule of marsh gas. The symbol for marsh gas is, therefore, $C\ H_4$. The small figures 2, 3, and 4 always indicate the number of atoms of the element represented by the symbol preceding them.

The symbol of any compound indicates how the atoms of the elements combine to form the compound. Thus, the symbol for water, $H_2\ O$, shows that two atoms of hydrogen and one of oxygen unite to form a molecule of water. The symbol $H_2\ S\ O_4$ (sulphuric acid) shows that a molecule of the sulphuric acid contains two atoms of hydrogen, one of sulphur, and four of oxygen.

830. Combination by Weight.—One cubic foot of hydrogen combines with just one cubic foot of chlorine. But, upon weighing each gas, it is found that the cubic foot of chlorine weighs 35.5 times as much as the cubic foot of hydrogen. A cubic foot of oxygen weighs 16 times as much as a cubic foot of hydrogen.

It has been stated that equal volumes of gases contain the same number of molecules. Therefore, a cubic foot of oxygen must contain the same number of atoms as a cubic foot of hydrogen. Now, since the former weighs 16 times as much as the latter, it follows that an atom of oxygen weighs 16 times as much as an atom of hydrogen. Similarly, an atom of chlorine weighs 35.5 times as much as an atom of hydrogen. This ratio between the weight of an atom of any element and the weight of an atom of hydrogen is called the **atomic weight,** or the **chemical equivalent,** of the element. The atomic weight of any element (or compound) may be found by dividing the weight of a given volume, say a cubic foot, of the element, when in a gaseous state, by the weight of a cubic foot of hydrogen. The atomic weight is, therefore, about the same thing as specific gravity, hydrogen being used as a base of comparison instead of water or air.

There are more than 60 elements, each having a certain fixed proportion for combining with other elements; this proportion is the atomic weight of the element, or some simple multiple of it.

831. The atomic weights of the elements named above are as follows:

Hydrogen (H)	1
Oxygen (O)	16
Nitrogen (N)	14
Carbon (C)	12
Sulphur (S)	32

By the aid of these atomic weights, the composition of any substance by weight can be found when its symbol is known. Take water whose symbol is H_2O; that is, there are two atoms of H to one of O. Multiply the number of atoms of each by the atomic weight of the atom. Thus,

$2 \times 1 = 2$ parts by weight of hydrogen.
$1 \times 16 = \underline{16}$ parts by weight of oxygen.
18 parts by weight of water.

Water, then, is composed of $\frac{2}{18} = 11.11\%$ of hydrogen, and $\frac{16}{18} = 88.89\%$ of oxygen.

As another example, take carbon dioxide, $C\,O_2$. We have

1 atom of C × atomic weight, $12 = 12$ parts by weight of C.
2 atoms of O × atomic weight, $16 = \underline{32}$ parts by weight of O.
44 parts by weight of CO_2.

Hence, $C\,O_2$ contains $\frac{12}{44} = 27.27\%$ carbon and $\frac{32}{44} = 72.73\%$ oxygen. From these examples it is plain that the atomic weight of water is 18, and of carbon dioxide 44.

832. Mixtures.—Two or more elementary substances may be mixed together, and yet not combine to form a new substance. They are then said to form a **mixture.** The mixture has the properties of the elements composing it. The most familiar example of a mixture is ordinary air. It is composed of oxygen and nitrogen, 23 parts by weight of the former to 77 parts by weight of the latter. The two gases are not chemically combined; they are simply mixed.

COMBUSTION.

833. Combustion *is very rapid chemical combination.* The atoms of some of the elements have a very great affinity or attraction for those of other elements, and when they combine they rush together with such rapidity and force that heat and light are produced. Oxygen, for example, has a great attraction for nearly all the other elements. An atom of oxygen is ready to combine with almost any substance with which it comes in contact. For carbon, oxygen has a particular liking, and whenever these two elements come in contact at a sufficiently high temperature they combine with great rapidity. The combustion of coal in the furnace of a boiler is of this nature. The temperature of the furnace is raised by kindling a fire, and then the carbon of the coal begins to combine with oxygen taken from the air. The combination is so rapid and violent that a great quantity of heat is given out.

834. The elements which enter into combustion are oxygen and usually either carbon or hydrogen. Coal, wood, and other fuels are composed almost entirely of these three elements. *Combustion is, therefore, the rapid chemical combination of oxygen with either carbon or hydrogen or both.*

We have seen that when carbon and oxygen combine they form CO_2, or carbon dioxide; when hydrogen and oxygen combine they form water, H_2O. These are called the **products of combustion.** When, as is ordinarily the case, the oxygen is obtained from the air, the nitrogen of the air passes into the furnace along with the oxygen. It takes no part in the combustion, and passes off up the chimney with the CO_2. Hence, nitrogen is also a product of combustion in air.

835. It has been shown that CO_2 is composed of 12 parts by weight of carbon to 32 of oxygen. Hence, to burn a pound of carbon requires $\frac{32}{12} = 2\frac{2}{3}$ pounds of oxygen. If the oxygen is taken from the air, it will take $2\frac{2}{3} \div .23 = 11.6$ pounds of air to supply the $2\frac{2}{3}$ pounds of oxygen. This is because only 23% of air is oxygen. The combustion of a pound of carbon may be represented as follows:

	Elements.	Products.
1.0 lb. carbon	1.00 lb. carbon	3.67 lb. CO_2
11.6 lb. air	2.67 lb. oxygen	
	8.93 lb. nitrogen	8.93 lb. nitrogen
12.6	12.6	12.6

That is, 1 pound of carbon requires 11.6 pounds of air for complete combustion. Of this air, 2.67 pounds is oxygen, and this combines with the pound of carbon, forming 3.67 pounds of carbon dioxide. The 8.93 pounds of nitrogen contained in the air pass off with the CO_2 as a product of combustion.

836. Next consider the complete combustion of a pound of hydrogen. The product of the combustion is water, H_2O. It has been shown that H_2O is composed of 2 parts by weight of hydrogen to 16 parts oxygen. Hence, 1 pound of H requires $\frac{16}{2} = 8$ pounds of O to unite with it. The air required to furnish 8 pounds of O is $8 \div .23 = 34.8$ lb. The process of combustion is, therefore, as follows:

	Elements.	Products.
1.0 lb. hydrogen......	1.0 lb. hydrogen	9.0 lb. water (H_2O)
34.8 lb. air	8.0 lb. oxygen	
	26.8 lb. nitrogen	26.8 lb. nitrogen
35.8	35.8	35.8

837. There is one other case that may occur; the combustion of carbon may not be complete. If an insufficient amount of air or oxygen is supplied to the burning carbon, it is possible for the carbon and oxygen to form another gas, carbon monoxide, or CO, instead of carbon dioxide CO_2.

The combustion of a pound of carbon to form CO requires, of course, only half of the oxygen that would be necessary to form CO_2. This is because in the gas CO 1 atom of carbon seizes 1 atom of oxygen instead of 2. To burn a pound of carbon to CO_2 requires 11.6 pounds of air. To burn it to CO would, therefore, require but $11.6 \div 2 = 5.8$ pounds of air.

The quantities of air required for combustion are shown in the following scheme:

One Pound.	Air at 62°.	Products of Combustion.
Hydrogen.....	34.8 lb., or 457 cu. ft.	Water. Nitrogen.
Carbon burned to CO_2......	11.6 lb., or 152 cu. ft.	Carbon dioxide. Nitrogen.
Carbon burned to CO.......	5.8 lb., or 76 cu. ft.	Carbon monoxide. Nitrogen.

838. The fuels in common use are composed chiefly of carbon, with sometimes a small percentage of hydrogen, oxygen, and incombustible matter called ash. When the percentages of carbon and hydrogen are known, the air required for the combustion of a pound of the fuel is easily found. For example, suppose a certain coal is 90% carbon and 10% hydrogen. To burn the carbon requires $152 \times \frac{90}{100} = 136.8$ cu. ft. of air; to burn the hydrogen requires $457 \times \frac{10}{100} = 45.7$ cu. ft. of air. Hence, to burn a pound of the fuel requires $136.8 + 45.7 = 182.5$ cu. ft. of air.

Let C = the number of parts of carbon contained in 100 parts of a given fuel;
H = the number of parts of hydrogen contained in 100 parts of the same fuel;
A = the number of cubic feet of air required for the combustion of a pound of the given fuel;
B = the heat of combustion of a pound of the fuel, in B. T. U.;
W = number of pounds of water at 212° evaporated by a pound of the above fuel.

Then,
$$A = 152 \times \frac{C}{100} + 457 \times \frac{H}{100};$$

or, $A = 1.52\,(C + 3\,H)$, very nearly.

Whence we have the following rule:

Rule 158.—*To find the air required to burn a given fuel: To the carbon contained in the fuel add three times the hydrogen, multiply the sum by 1.52, and the result will be the air required in cubic feet at a temperature of 62° F.*

Example.—The composition of a certain kind of coal is as follows:

Carbon	84 parts.
Hydrogen	5 parts.
Oxygen	7 parts.
Ash	4 parts.
	100

Find the quantity of air required to completely burn a pound of this fuel.

SOLUTION.—According to the above rule,

$$A = 1.52(C + 8H) = 1.52 \times (84 + 8 \times 5) = 150.48 \text{ cu. ft.} \quad \text{Ans.}$$

When the fuel already contains oxygen a little less air is required to burn it; if it contains sulphur, a little more air will be required than given by the above rule. In either case the difference is very slight. It will be found that a pound of coal requires practically the same amount of air whether it be anthracite or bituminous. Roughly speaking, it requires about 12 pounds, or 160 cubic feet, of air to burn a pound of carbon or coal. If less air is supplied, the combustion will be imperfect; that is, the carbon burns to CO instead of CO_2.

839. Heat of Combustion.—The quantities of heat produced by the complete combustion of the elements composing the fuels have been found by experiment. They are as follows:

Hydrogen	62,000 B. T. U. per pound.
Carbon burned to CO_2.....	14,500 B. T. U. per pound.
Carbon burned to CO	4,400 B. T. U. per pound.

Using the symbols given in Art. **838,** we have

$$B = 14{,}500 \times \frac{C}{100} + 62{,}000 \times \frac{H}{100},$$

$$\text{or,} \quad B = 145\,C + 620\,H.$$

giving us the following rule:

Rule 159.—*To find the heat of combustion of a pound of fuel, multiply the percentage of carbon by 145 and the percentage of hydrogen by 620. Add the products, and the sum will be the required heat of combustion in B. T. U.*

EXAMPLE.—A certain kind of coal has the following composition:

Carbon	76.5 parts.
Hydrogen	4.4 parts.
Oxygen	3.0 parts.
Nitrogen	1.1 parts.
Ash	15.0 parts.
	100.0

How many B. T. U. will be produced by the combustion of a pound of this coal?

SOLUTION.—According to the rule just given,

$B = 145\,C + 620\,H = 145 \times 76.5 + 620 \times 4.4 = 13{,}820.5$ B. T. U. Ans.

From the steam table it is found that to change a pound of water at 212° into steam at 212° requires about 966 B. T. U., hence, the following rule:

Rule 160.—*To find the number of pounds of water at 212° evaporated by a pound of fuel, divide the heat of combustion of the fuel by 966.*

$$\text{or,} \quad W = \frac{B}{966}.$$

EXAMPLE.—How many pounds of water at 212° can be evaporated by a pound of the coal of the last example?

SOLUTION.—The heat of combustion is 13,820.5 B. T. U.

$$13{,}820.5 \div 966 = 14.3 \text{ lb.} \quad \text{Ans.}$$

840. Temperature of Combustion.—The theoretical temperature of the combustion of a given fuel can be easily calculated. Making no allowance for losses of heat, and supposing just enough air is furnished for the combustion, burning carbon should have a temperature of about 4,940° above zero; burning hydrogen should have a temperature of about 5,800° above zero. In practice these temperatures are never attained, on account of the losses of heat. Usually, the quantity of air admitted to the furnace is from 50 to 100 per cent. more than is theoretically necessary for the combustion. This extra quantity of air enters at a temperature of 60° to 70°, and escapes up the smokestack at a temperature of from 400° to 600°. A large quantity of heat is thus wasted, and the temperature of the fire is greatly lowered. Where the fire is outside of the boiler, and the furnace is surrounded by brickwork, the furnace temperature may be 2,500° to 3,000°; but when the furnace is inside the boiler and surrounded on all sides by water, the temperature rarely rises above 2,000°, and is usually less. A high temperature is, of course, desirable, since the water of the boiler will take up heat much faster at a high temperature than at a low one.

841.—Conditions Required for Economical Combustion.—The chief consideration is the air supply. It is not possible to obtain perfect combustion with the theoretical quantity of air; viz., 12 pounds, or 160 cubic feet, per pound of coal. This is because the air has to pass through the thick bed of coal, and it does not, therefore, come in close enough contact with the burning surface to insure perfect combustion. It is usually estimated that about 24 pounds of air are required with ordinary natural draft, and 18 pounds with forced draft for each pound of coal burned. If more air is used than is actually necessary, the extra amount simply carries heat from the furnace to the smokestack. On the other hand, if too little air is admitted to the furnace, part of the carbon burns to CO instead of CO_2. It has been stated that the combustion of carbon to CO gives out but 4,400 B. T. U. per pound. Here, then, is a loss of $14,500 - 4,400 = 10,100$ B. T. U. for each pound of carbon burned. Hence, the admission of either too much or too little air results in loss of efficiency.

842. It is a general law that the excess of air required to produce the most economical results is inversely as the rate of combustion. In boilers that burn from 2 to 4 pounds of coal per square foot of grate surface per hour, the excess of air is 100%. When the rate of combustion is from 10 to 16 pounds of coal per square foot of grate surface per hour, the excess is only 25 to 50%; finally, when the rate of combustion is over 20 pounds per square foot of grate surface per hour, the excess of air need only be about 10%.

A common and serious source of loss is the escape of combustible gases from the surface of the coal. Most varieties of bituminous coal contain *hydrocarbons*. These are gases composed of carbon and hydrogen, such as marsh gas, common lighting gases, olefiant gas, and others. The carbon contained in these gases may be one-third or more of all the carbon contained in the coal.

When coal of this nature is thrown in the furnace, it becomes heated, but before it really begins to burn these gases are driven off, and, unless some means are provided to burn them, they escape up the smokestack. To burn these hydrocarbons it is necessary to admit air into the furnace *above* the fire, either through the furnace front or through the bridge wall.

The most economical rate of combustion, and the proper quantity of air to be admitted to the furnace, can only be determined by experiment. The conditions of highest economy will, in general, be different for the different types of boilers and furnaces.

The subject of combustion will be referred to later under the head "*firing.*"

EXAMPLES FOR PRACTICE.

1. How many pounds of air will be required for the perfect combustion of 7 lb. of carbon? Ans. 81.2 lb.

2. A fuel is 88% carbon and 12% hydrogen. How many cubic feet of air are required for complete combustion of a pound?

Ans. 188.48 cu. ft.

3. (*a*) How many B. T. U. would the combustion of the pound of coal of example 2 give out? (*b*) How many pounds of water at 212° would a pound of this fuel evaporate?

Ans. { (*a*) 20,200 B. T. U.
(*b*) 20.9 lb.

4. The chemical symbol of the product of combustion of sulphur with oxygen is SO_2 (sulphurous oxide). What is the composition of this gas by weight?

Ans. { Sulphur 50%.
Oxygen 50%.

5. Assume that with natural draft, double the theoretical quantity of air is used to burn a fuel. Under these circumstances, how many cubic feet of air would be required to burn 115 lb. of coal, the chemical composition being as follows:

Hydrogen	5 parts.
Carbon	90 parts.
Oxygen	3 parts.
Ash	2 parts.
	100

Ans. 36,708 cu. ft.

CIRCULATION OF WATER IN BOILERS.

843. The *transfer of heat* from the furnace to the water in the boiler is accomplished by radiation, conduction, and convection. It is estimated that when the fire is burning brightly, about one-half of the heat received from the furnace by the boiler is radiated. The transfer of heat through the water is due to convection, since liquids are poor conductors of heat. The particles of water next to the shell or plate become heated, and immediately rise into the main body of water, giving place to fresh particles of cold water. The rapidity with which heat will be absorbed by convection, therefore, depends upon the effectiveness of the water circulation in the boiler, and upon the extent and conductivity of the heating surfaces. The transfer of heat through the shell and furnace plates takes place by conduction. It has been proved, experimentally, that the quality or thickness of the material has little influence, thick iron tubes working practically as well as thin brass ones. Very thick plates are, however, liable to be injured by burning when exposed to the direct action of the fire.

844. Water circulation is essential to the efficient operation of a boiler. It has just been shown that the rapidity of the transfer of heat by convection depends upon the rapidity of circulation. Besides this, the circulation is useful in preventing, in some degree, the deposit of sediment which accumulates from the feed-water. Again, a rapid circulation keeps the parts of the boiler at a uniform temperature and is a safeguard against overheating.

845. Fig. 231 shows the circulation of the water in a plain cylindrical boiler. The heated currents rise from the hottest part of the shell directly over the furnace, and carry the bubbles of steam to the surface. The cooler water rushes in to take their place over the furnace, and thus the circulation is maintained. As shown in the figure, there are two currents, one carrying the cold water from rear to front, and the other carrying it down the outside of the shell and up through the center. It will also be noticed that the circulation is in a direction contrary to that of the furnace gases. Since in all cylindrical boilers the water is contained

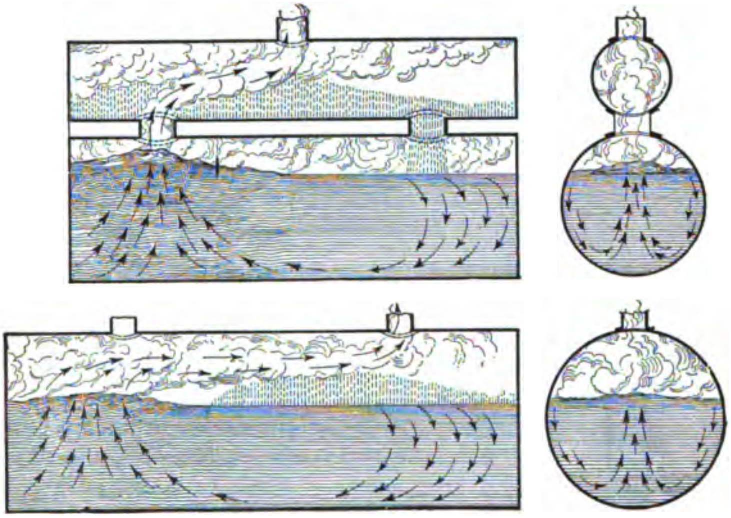


FIG. 231.

in a solid mass, broken only by flues or tubes, the circulation is more or less interfered with by opposing currents.

846. Fig. 232 illustrates the circulation of the water in a Scotch boiler. The water directly over the top of the

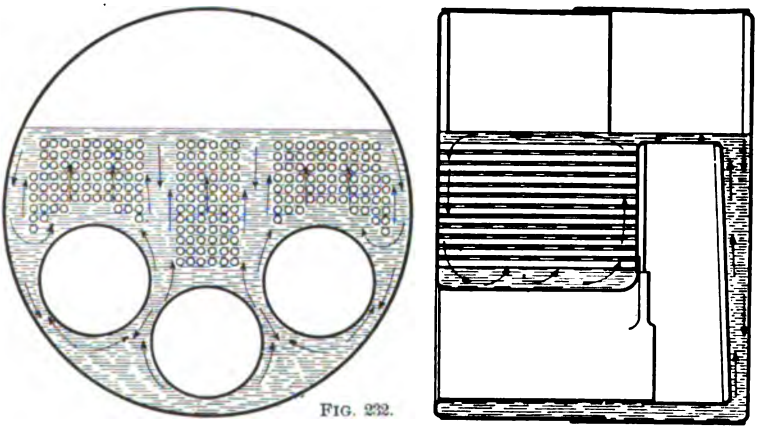


FIG. 232.

furnaces is heated first; it rises between the tubes and alongside each nest of tubes, cooler water coming down between the nests and alongside the shell. The circulation in the lower half of the boiler is very feeble, some of the water passing upwards alongside the furnaces, and the downward currents meeting it and almost neutralizing its motion. It will thus be seen that the circulation is greatly interfered with by opposing currents, and as a consequence the lower half of the boiler is considerably cooler than the upper half; the difference in temperature between different parts of the sheets sets up severe strains in the material, thus tending to shorten the life of the boiler. The circulation is more rapid and effective if the water is constrained to follow a particular path.

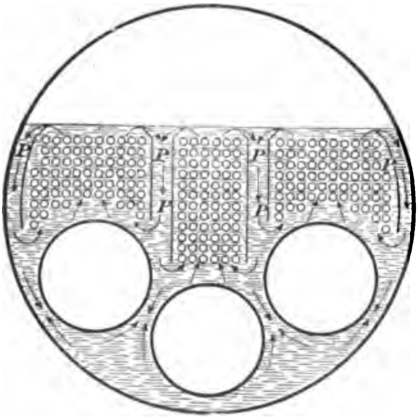


FIG. 233.

847. To accomplish the above object, thin sheet-iron plates are sometimes fitted in Scotch boilers in such a position that the upward and downward currents can not interfere with each other. The arrangement of the plates is shown in Fig. 233. The thin iron plates *P* enclose each separate nest of tubes, extending very nearly the whole length, and downwards to the lowest tubes of each nest. The upper ends of the plates enclosing each nest are inclined towards each other, and are carried up a short distance above the water level.

The water is lifted through the opening between the two plates by the ebullition on the surface of the water, and, after parting with the particles of steam suspended within it, flows down the upper inclined surfaces of the plates and augments the downward current. These plates will accelerate the circulation in a direction at right angles to the axis of the boiler, but do not influence any current in an axial direction. The arrows clearly show the direction of the currents. This arrangement helps the circulation somewhat, but still leaves much to be desired. It is one of the strong points of the water-tube boilers that the water must pass in one direction through a series of tubes; hence, the circulation is strong and uninterrupted. Of course, the above statement refers to properly designed water-tube boilers. In the earlier designs, the circulation of the water had either not been provided for at all, or but indifferently. For this reason the first water-tube boilers placed in a steamship were a failure; but since then, the importance of providing for a rapid circulation has been recognized, and the boilers designed accordingly, so that at the present day the above statement holds good for nearly all water-tube boilers used in steam vessels. The difference between the cylindrical and water-tube boilers in this respect may be illustrated as follows: The cylindrical boiler with its contained mass of water may be compared to an ordinary kettle in the process of boiling; see Fig. 234. The water rises rapidly around the outer edges and flows downwards in the center. If, however, the fire is quickened, the upward and downward currents interfere with each other, and the kettle boils over.

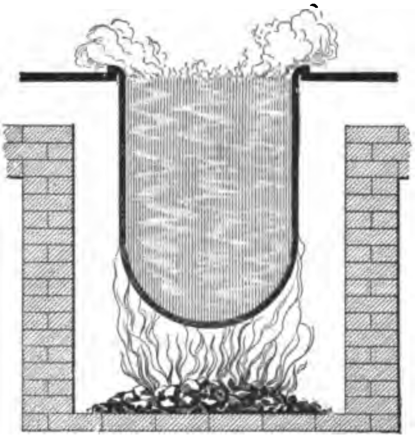


FIG. 234.

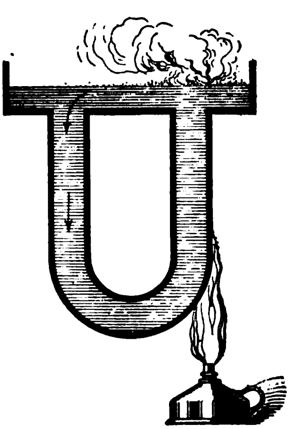


FIG. 235.

848. The water-tube boiler should be and is identical in principle with a **U** tube depending from a vessel filled with water, and with the heat applied to one leg; see Fig. 235. The circulation is set up immediately, and proceeds quietly, no matter how fierce the fire may be.

About the only way of improving the circulation of Scotch boilers is to do so by mechanical means. This may be done by the use of a small pump connected to the bottom of the boiler, drawing the cold water from the lower half, and discharging it through a perforated pipe between the nests of tubes. Lately an apparatus has been designed to circulate and heat the water, and has been adopted in most large steamships.

849. The **Craig heating and circulating apparatus** is shown in Figs. 236 and 237. In general design and principle of operation it greatly resembles an injector. In Fig. 236 the feed-water enters through the pipe *L*, passing

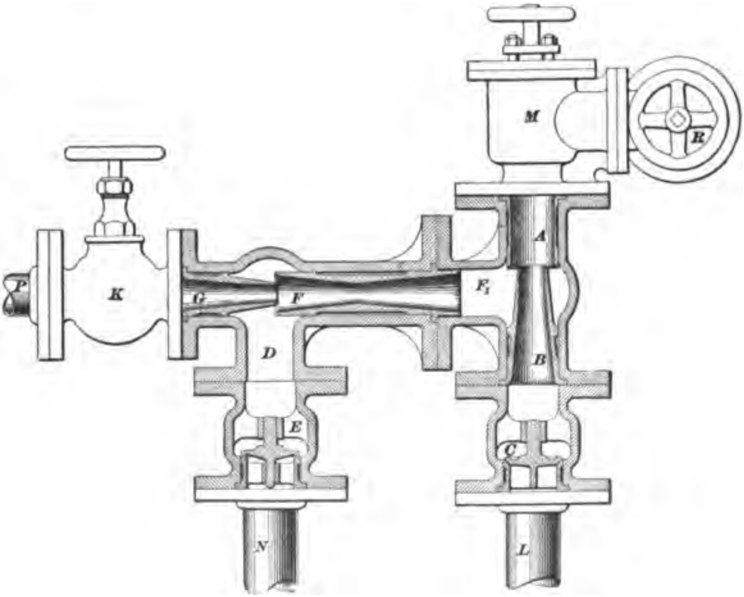


FIG. 236.

through the check-valve *C* into the nozzle *B*; thence through *A* into the feed check-valve *M*, and through the stop-valve *R* into the boiler. The water passing through *B* at a high velocity induces a current of water to pass through the induction pipe *N* connected to the bottom of the boiler, the water passing through the check-valve *E* into *D*, thence through the annular opening between the nozzles *F* and *G* into F_1, whence it passes into *A*, mingling with the feed-water. To heat the water a jet of live steam is admitted by the valve *K* and nozzle *G*. The steam pipe *P*, bolted to the valve *K*, leads to the donkey boiler. When it is desired to equalize the temperature and circulate the water while getting up steam in a boiler fitted with the apparatus, steam from the donkey boiler is admitted by the valve *K*. The steam flowing through *G* into *F* at a high velocity acts the same as an injector, inducing a flow of water through *N*, *E*, *D*, *F*, *A*, *M*, and *R* into the boiler, and delivering the water at a high temperature. The check-valve *C* prevents any of the water from entering the feed-pipe *L*. When the boiler is under steam, the apparatus is used to circulate and heat the feed-water, the circulation being induced by the feed-water flowing through *B* with a high velocity and inducing a current of water to pass up through *N*, *E*, *D*, *F*, and F_1 into *A*, and thence into the boiler. The feed-water is heated by mingling with the hot water coming from the bottom of the boiler.

850. In Fig. 237 the apparatus is shown applied to a Scotch boiler. At N_1 the internal induction pipe is shown. This is a perforated pipe running in the direction of the length of the boiler, and connected to the external induction pipe *N*, provided with a stop-valve *n*. The feed-pipe *L* is provided with a stop-valve *l*. At *M* the feed check-valve is shown, whence the water passes through the stop-valve *R* into the internal feed-pipe M_1, thence into the perforated distributing pipe M_2, discharging the water downwards between the nests of tubes. *P* is the steam pipe leading to the donkey boiler. This pipe is provided with a stop-valve *K*. To circulate the water while running, the valve *n* is opened, when the feed-water itself will maintain the circulation. To heat and circulate the water before fires are lighted, the valve *l* is closed, the valves *M* and *n* are opened, and steam from the donkey boiler admitted to the apparatus, the steam in the manner previously explained heating and circulating the water, thus bringing all the parts of the boiler to a uniform temperature, and thereby greatly reducing the local strains in the material of the boiler due to starting a fire in the furnaces. The apparatus may be connected by suitable piping to serve for the main feed as well as for the donkey feed.

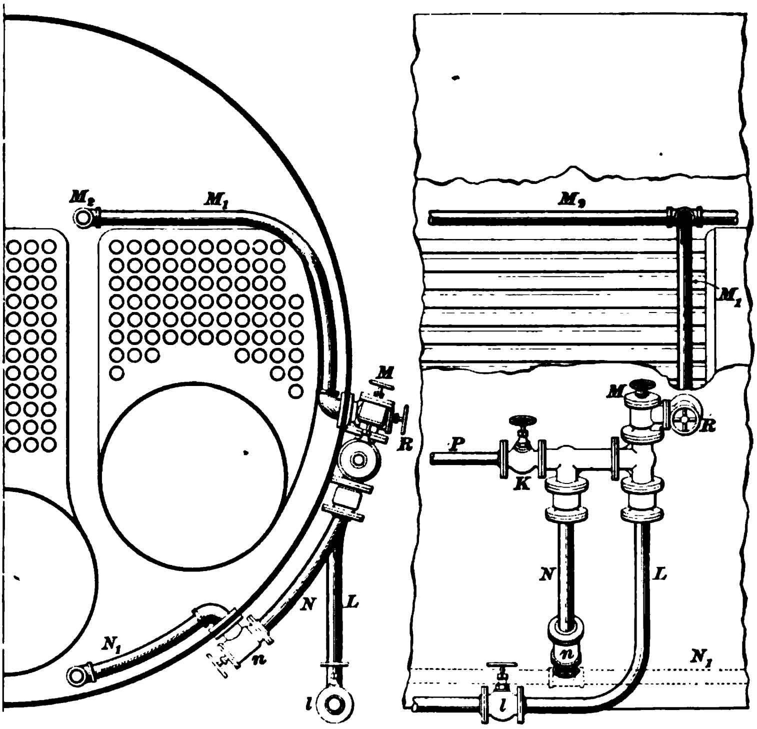


Fig. 287.

NATURAL AND FORCED DRAFT.

851. The necessary quantity of air is supplied to a furnace by a draft, either artificial or natural.

852. **Forced draft,** or **artificial draft,** is produced mechanically by fans or blowers; **natural draft** is produced by the smokestack, the impelling force being the difference between the weight of the hot gases in the stack and the weight of an equal column of cold air.

The smokestack serves the double purpose of creating a draft and carrying away obnoxious gases. The production of the draft depends upon the fact that the furnace gases (the products of combustion) passing up the stack have a high temperature, and are, consequently, lighter than an equal volume of outside air at ordinary temperature; that is, the pressure within the smokestack is slightly less than the pressure of the outside air. Consequently, the air will flow from the place of higher pressure to the place of lower pressure; that is, into the smokestack through the furnace.

The area of the smokestack must be such that the gases of combustion may be discharged freely. Experience has shown that an area of 1 square foot of smokestack to every 8 square feet of grate surface is a fair ratio, representing the practice of some of the best builders. The intensity of the draft depends upon the height of the smokestack; hence, the amount of coal that may be burned per square foot of grate surface per hour may be found from the height of the smokestack, provided the area of the stack is of the required size.

NOTE.—The height of a smokestack is the perpendicular distance between the top of the stack and the top of the grate.

853. The maximum rates of combustion attainable under natural draft are given by the following rules, which have been deduced from the experiments of Isherwood, U. S. N.:

Let W = weight (in pounds) of coal burned per square foot of grate area per hour;
H = height of smokestack in feet.

Rule 161.—*To find the amount of anthracite coal that may be burned per square foot of grate surface per hour under the most favorable conditions, subtract 1 from twice the square root of the height of the smokestack.*

That is, $W = 2\sqrt{H} - 1.$

Rule 162.—*For anthracite burning under ordinary conditions, subtract one from one and one-half times the square root of the height of the smokestack.*

That is, $W = 1.5\sqrt{H} - 1.$

Rule 163.—*For best semi-anthracite and bituminous coals, multiply the square root of the height of the smokestack by two and one-quarter.*

That is, $W = 2.25\sqrt{H}.$

Rule 164.—*For ordinary soft coals, multiply the square root of the height of the smokestack by three.*

That is, $W = 3\sqrt{H}.$

854. The maximum rate of combustion is thus fixed by the height of the smokestack; the minimum rate may be anything less.

Example.—What is the maximum coal consumption per hour of a vessel fitted with six boilers with two furnaces each, the length of the grate being 6 feet, the width 3 feet 6 inches, and the height of the smokestack 65 feet? Ordinary soft coal is being used.

Solution.—Using rule **164,** we get

$$W = 3\sqrt{65} = 24.187 \text{ lb. per sq. ft. of grate area per hour.}$$

The total grate area $= 6 \times 2 \times 6 \times 3.5 = 252$ sq. ft. Hence, the maximum coal consumption $= 252 \times 24.187 = 6{,}095.12$ lb. per hour. Ans.

855. As previously explained, natural draft is caused by the upward flow of heated air.

The force of the draft depends a great deal upon various conditions, such as the direction and the force of the wind, the temperature of the air, the height of the smokestack, the thickness of the fire, etc. To make the draft independent of any of these conditions, various mechanical arrangements are used.

856. Forced draft is practically applied in one of three ways:

1. The air is forced by a blower into an air-tight fire room.
2. The air is forced by a blower into air-tight ash pits.
3. The blower is placed in the uptake, and by driving out the waste gases creates a partial vacuum, which draws the air in through the grates.

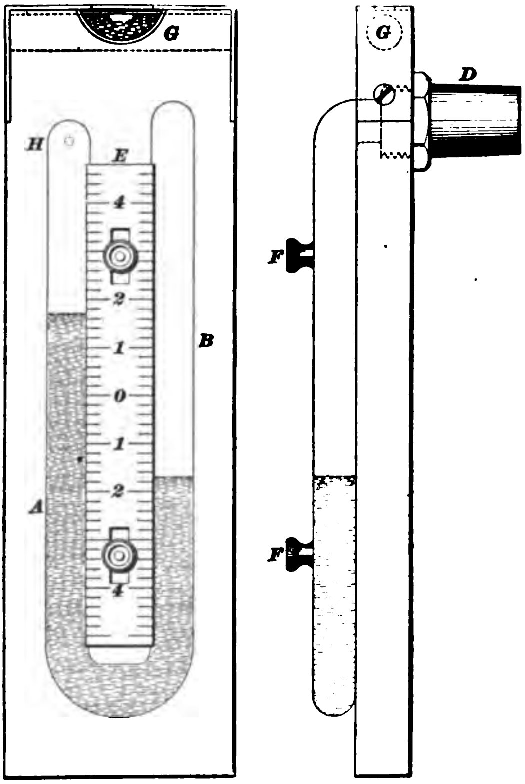


FIG. 238.

The pressure of the draft is measured in inches of water; thus, in the above cases the air is forced into the fire room

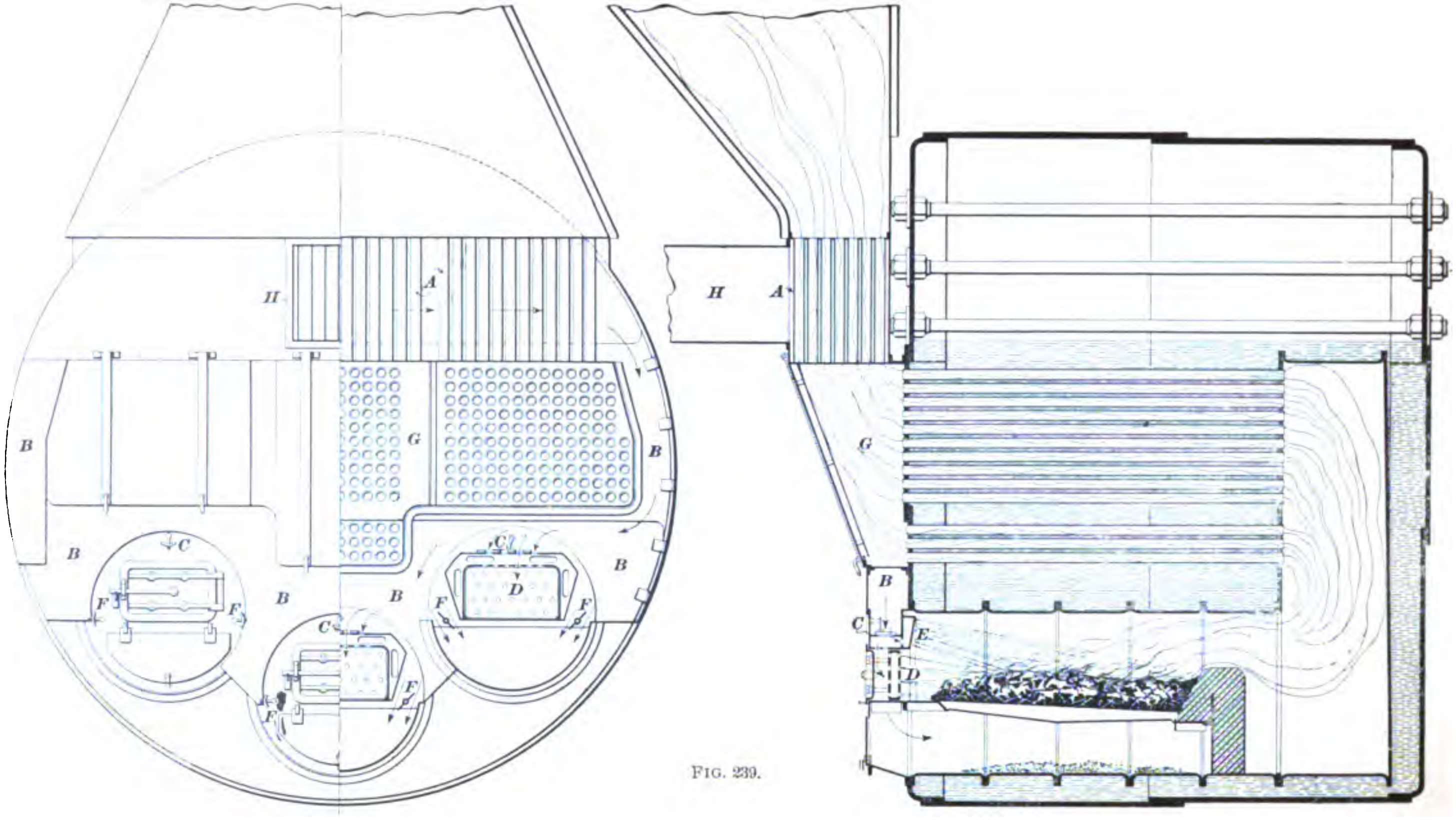


FIG. 239.

or ash pit under a pressure of from 1 in. to 3 in. of water, an inch of water corresponding to a pressure of .036 lb. per square inch, or about 5.2 pounds per square foot.

A water gauge is shown in Fig. 238. As will be seen, it is a glass tube open at both ends, bent to the shape of the letter **U**; the end *D* of leg *B* communicates with the vessel or compartment containing the air under pressure. The pressure forces the water contained in the tube upwards in the leg *A*, and downwards in the leg *B*, the difference in level between the water in the two legs representing the air pressure in inches of water. *F*, *F* are the set-screws by which the scale *E* is adjusted, *H* is a small air-hole, and *G* is a spirit level by means of which the gauge can be set horizontally.

In naval vessels, the closed fire-room system of forced draft is extensively used. All openings from the fire room are closed air-tight, and air forced in until the desired pressure has been reached. This system has not found much favor in the mercantile service, and the closed ash-pit system has been adopted instead.

857. Howden's closed ash-pit system, in which hot air under pressure is delivered into the ash pits and furnaces, is shown in Fig. 239. By means of a blower located in a suitable place, cold air is forced through the pipe *H* into a closed air chamber located in the uptake immediately above the doors of the front connection. The gases of combustion pass through numerous vertical tubes, shown at *A*, contained in the air chamber, thus heating the air surrounding the tubes. The heated air passes into an air-tight reservoir *B*, attached to the front end of the boiler and surrounding the furnaces, as well as the front connection *G*. This reservoir, projecting about 10 inches from the front of the boiler, receives the air under pressure. The air admitted above the fire enters through the valve *C* into a space between the outer and inner furnace doors, both of them swinging on one hinge. The inner door is perforated, and is provided with a perforated air-distributing box *D*, the outer door serving to retain the air pressure. The air passing through the valve *C*, besides filling the spaces between the doors, also fills the space around the furnace door, whence it passes into a perforated air-distributing box *E*, covering the whole surface of the furnace front inside the furnace. Part of the air passes through the perforated dead plate into the ash pit, which is closed by hinged doors. The rate of admission of air to the ash pit is regulated by means of the valves *F*.

858. In operation, the rate of combustion of the fuel is governed by the valves *F* regulating the air pressure. The valves *C* are adjusted at the beginning of the trip to suit the character of the fuel used. The jets of highly-heated air admitted by these valves above the fire tend to promote complete combustion. In some closed ash-pit systems, *cold air* is delivered under pressure into the ash pit.

859. An **induced** or **exhaust** draft system is shown in Fig. 240. It consists of a pair of fans *A*, *A* located one on each side of the base of the smokestack. Both fans are mounted on the same shaft, which passes through the smokestack. The shaft is belted to a steam engine, not shown in the figure. In operation, the damper *B* is closed while using the exhaust draft. The rapidly revolving fans exhaust the air in the uptake, thus reducing the pressure in the tubes

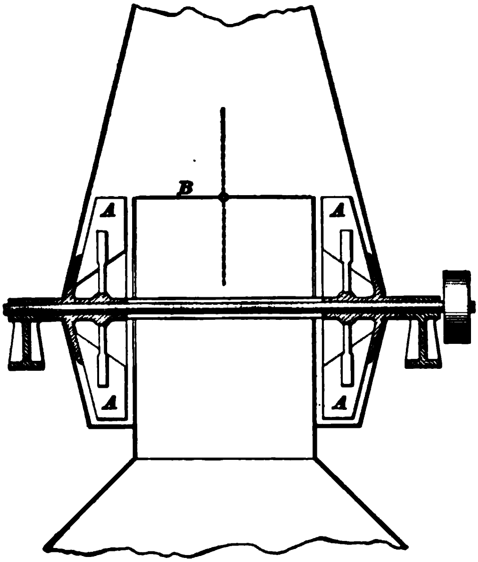


FIG. 240.

and furnaces, and thereby producing a more rapid current of air through the furnaces both below and above the grate. No alterations of the arrangement of the furnace fittings are necessary. This system can be applied very cheaply to steamers already built.

860. Sometimes the air in the chimney is exhausted by means of a **steam jet.** It consists of a wrought-iron pipe provided with a conical nozzle pointing upwards, and is placed in the center of the base of the smokestack. A steam pipe provided with a stop-valve connects the nozzle with the boiler. The steam issuing from the nozzle with a high velocity carries particles of the air with it, thus creating a partial vacuum, and, consequently, increasing the inflow of air into the furnace.

LOSS OF HEAT.

861. A portion of the heat generated in the furnace is usefully expended in evaporating the water. There is, however, more or less loss of heat due to the following causes:

1. A certain amount of heat is carried up the smokestack by the waste gases, the temperature of which is between 500° and 700°.
2. In some cases heat is lost by incomplete combustion.
3. Heat is required to vaporize the water formed by the combustion of hydrogen.
4. The escape of free carbon in the form of smoke is a loss.
5. Heat is lost by radiation.

862. The loss due to the first of the above causes can not be avoided where natural draft is used, since the temperature of the ascending gases must be 300° to 600° F. to insure a good draft. The loss may, however, be aggravated by using an excess of air.

The loss due to the second cause may be avoided by using a sufficient quantity of air to insure complete combustion.

The loss of the heat required to vaporize the water is unavoidable; it is, of course, much greater with wet fuels.

The formation of smoke is a very common and very fruitful source of waste. Bituminous coals, which are rich in hydrocarbon gases, produce the most smoke. The volatile hydrocarbons are driven off by the high temperature, and come in contact with the air; the oxygen of the air unites with the more inflammable hydrogen, leaving the carbon in a finely divided condition; upon being cooled down, the fine particles of carbon appear as smoke.

Smoke may be avoided by bringing a supply of fresh air in contact with the carbon while it is red hot in the flame; that is, before it cools down into smoke. After being formed, smoke is hard to burn, and the skill of the fireman should be exerted in preventing its formation.

The loss of heat by radiation may be greatly reduced by covering the exposed parts of the boiler with non-heat-conducting materials. The radiation from an internally fired boiler is less than from one externally fired, the excess of the latter over the former being due to the brickwork furnace walls.

FOAMING AND PRIMING.

863. **Foaming** is the violent ebullition of the surface of the water in a boiler. Foaming is sometimes caused, when using sea-water in the boiler, by changing off from sea-water to fresh water, or, when using fresh water, changing off to sea-water. Owing to the violent ebullition of the water, the true water level can not be ascertained without checking somewhat the outflow of steam from the boiler. Sometimes it is necessary to shut off the outflow of the steam entirely before the water level can be found. Further causes of foaming are the presence of oil or grease on the surface of the water, carried in by the feed-water, and insufficient steam space in the boiler, or too large a water space.

864. Foaming is always indicated by the violent agitation, the rapid rising and falling, and at the same time the muddy appearance of the water in the water gauge glass. Foaming is always attended by the danger of overheating the tubes and the crown sheet of the combustion chamber, and, therefore, as soon as a boiler begins to foam it is necessary to find the true water level by the means indicated above.

Violent foaming may be checked by covering the fire with fresh fuel or ashes, opening the furnace door and closing the damper. Foaming due to the presence of grease or oil may be prevented by using the surface blow-off frequently.

865. Priming means the passage of water from the boiler through the steam pipe in the form of spray instead of vapor. It is caused by insufficient steam space, either due to the design or to the water being carried too high. It is often due to the boiler being too small for the work required, and as the fires have to be urged to the utmost to generate sufficient steam, water is evaporated very rapidly and particles of the water are carried along with the steam. This is analogous to the boiling over of a tea-kettle when placed directly over a hot fire. Insufficient circulation of the water usually causes priming. Priming generally manifests itself by a clicking noise in the cylinder of the steam engine. This noise is caused by the piston dashing the water carried over with the steam into the cylinder against the cylinder head. Severe priming may break the piston or the cylinder. It may be prevented by lowering the water level, reducing the speed of the engine, which reduces the amount of steam taken from the boiler, and also by uniform firing.

RATE OF COMBUSTION.

866. The **rate of combustion** of the fuel in the furnace is usually stated in pounds per hour burned on each square foot of grate. For coal the usual rates of combustion in marine boilers are as follows:

Ordinary rates under natural draft...	15 to 25 lb.
Ordinary rates under forced draft....	25 to 60 lb.
In torpedo boats and high speed yachts under forced draft..	up to 125 lb.

867. The **fuels** used in the generation of steam for marine purposes are chiefly coal and wood, although liquid fuel (such as petroleum) is coming into use, having already met with undoubted success in locomotive practice; it has an evaporative power about 25% greater than that of the best coals. These fuels are composed of carbon in combination with hydrogen, oxygen, and non-combustible substances.

There are five leading varieties of **coal**:

1. Anthracite, or hard coal, consisting almost entirely of pure carbon.
2. Dry bituminous coal, containing 70 to 80% carbon.
3. Bituminous coking coal, containing 50 to 60% carbon.
4. Cannel coal, containing 70 to 85% carbon.
5. Lignite, or brown coal, containing 55 to 75% carbon.

868. Anthracite coal is difficult to burn, and requires a strong draft, high temperature, and much attention. It burns without flame or smoke, which gives it a peculiar value for some purposes.

The bituminous coals burn freely and rapidly, and give off flame and smoke. The best bituminous coals have a higher heating value than anthracite, and are more highly regarded as steam coals.

Lignite is a sort of incomplete coal, and is not a very valuable fuel.

Wood is used considerably on the Western rivers, where its supply is abundant and the price of coal is high. The chemical composition of ordinary seasoned firewood is as follows:

Carbon	37.50 parts.
Hydrogen	4.50 parts.
Oxygen	30.75 parts.
Nitrogen	.75 parts.
Ash	1.50 parts.
Water	25.00 parts.
	100.00

The effective heating value of all kinds of wood is the same, and is usually estimated at $\frac{2}{5}$ of the heating value of the same weight of coal.

FIRING.

869. As coal is almost universally used, either anthracite (hard coal) or bituminous (soft coal), only the firing of coal will be treated of here.

The thickness of the fire in a furnace depends to a large extent upon the draft, the nature and the quality of the fuel, the size of the grate, and the rate of combustion required. As a general rule, the stronger the draft, the thicker the fire can be. Where a high rate of combustion is required with ordinary natural draft, it may often be attained by a thin fire and frequent light charges of fresh coal. It is claimed that this is not the most economical way of firing, because the cold air rushing into the furnace whenever the fire door is open for charging the fire, reduces the temperature of the furnace and of the gases in combustion, and, consequently, reduces the evaporation. For instance, let 10 pounds of coal be burned per square foot of grate surface per hour, evaporating 100 pounds of water. Then, if 20 pounds of coal be burned on the same area in the same time, instead of evaporating twice the amount of water, that is, 200 pounds, the actual evaporation will probably be about 170 or 180 pounds. The average thickness of the fire varies from 8 to 14 inches.

870. There are three methods of firing in use at the present day:

1. Coking firing. This is especially adapted to coal rich in hydrocarbons, such as bituminous coal. The fresh charge of coal is piled upon the dead plate, the heat of the furnace driving off the volatile hydrocarbons, which mix with the air entering through the holes in the fire door, and are ignited by the heat of the fire and completely consumed. Then the coked coal is spread evenly over the fire, and a fresh charge is piled upon the dead plate.

2. Alternate firing. This is adapted to either anthracite or bituminous coal. First, one side of the furnace is supplied with fresh fuel, and then the other side of the furnace

is charged, the time between these charges being so regulated that one side of the fire will be burning brightly before the other side is supplied.

3. Spreading firing. This is the system most generally employed. The fresh charge of coal is spread evenly all over the fire.

871. The tools used in working the fires are shown in Fig. 241. The slice bar, shown at *A*, is made of 1 in. or 1⅛ in. round iron, about 8 feet long, one end of it being flattened and a handle formed at the other end. This is employed for breaking up the crust formed at the surface of the fire when bituminous coal is used. The heat of the fire fuses the fresh charge of coal in a short time, thus keeping a sufficient supply of air from passing through the grate. About 10 minutes after charging the fire, the slice bar is run into the fire on the surface of the grate, and, by depressing the end of the bar outside the furnace, the fire is broken up. This operation is called **slicing the fire.**

The T bar, shown at *B*, is a slice bar with a broad, flat point, and is used, when firing anthracite coal, for breaking up the clinkers and cinders lying on the grate. It is merely run along the top of the grate the whole length, this operation facilitating the admission of air to the fire.

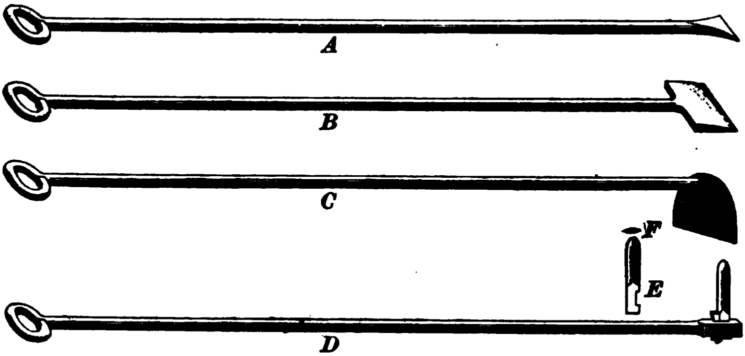


FIG. 241.

The hoe, made of $\frac{5}{8}$ in. bar iron, is shown at *C*. Generally two are used—a heavy hoe with a broad end and a lighter one. The lighter one is used for leveling the fire; the heavier one in cleaning the fire and ash pit.

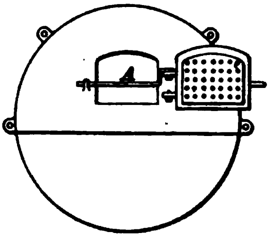


FIG. 242.

In connection with hoes, a bearing bar *A*, of the shape shown in Fig. 242, is often used, the figure clearly showing the way in which it is attached. This greatly facilitates the cleaning of the fire, as it provides a rest for the hoe.

Fig. 241 *D* illustrates the usual construction of the poker. A handle is formed at one end; the other end is enlarged and is rectangular in cross-section. It is provided with a slot for the insertion of the blade, which is shown enlarged at *E*. These blades are usually $\frac{5}{16}$ inch thick by $1\frac{1}{4}$ inches wide, and about 8 inches long, and are forged to the shape shown at *F*. A number of these blades are carried in stock, and are inserted when required by driving out the taper key, thus allowing the old blade to be removed and a new one to be inserted; the key is then driven home, locking the blade in position. This tool is used for cleaning the space between the grate bars from below, should it become choked.

The shovel is so well known as to need no description.

872. Cleaning Fires.—Two methods of cleaning a fire are in use. In the first method, the burning fuel or the **live coal,** as it is termed, is pushed to one side of the furnace while the other is being cleaned. The live coal is then distributed evenly all over the grate and covered with a light charge of fresh fuel.

In the second method, the live coal is pushed back against the bridge, and the cinders and clinkers covering the front of the grate are pulled out. The live coal is then pulled to the front of the furnace, and the cinders, etc., on the back part of the grate are pulled over the top of the live coal.

This is spread evenly over the grate, after cleaning, and covered with a light charge of fresh coal.

It is well to allow the fire to burn down somewhat before cleaning, as it will then be easier to clean. The damper should be partially closed, to prevent the cold air from rushing into the furnace and cooling the furnace plates above the fire. For this reason, and also to prevent the loss of steam pressure, the cleaning should be done quickly.

873. Banking Fires.—Should it be desirable to have a boiler lying idle, with steam up and the fires ready to generate steam to the full capacity of the boiler at short notice, the fires are banked. This is done by first cleaning the fire, and then either pushing the live coal against the bridge or pulling it to the front and covering it with fresh fuel. The fire will lie smouldering, the air supply being regulated by the damper, the fire door, and the ash-pit damper just enough to keep the fire burning sufficiently to keep up the steam pressure. When it is desired to start the fires again, some of the fuel covering the live coal is skimmed off, and the live coal is spread over the grate. The fire door is then closed, the ash-pit and smokestack dampers are opened and the fire allowed to burn up, for two or three minutes, perhaps, when it is covered with a light charge of fresh fuel.

874. Hauling Fires.—When the demand for steam suddenly ceases, as, for instance, in case of a breakage necessitating stoppage of the engine for a considerable period, the burning fuel has to be drawn from the furnace. The hard labor entailed in *hauling the fire*, as it is termed, may be reduced considerably by allowing the fire to burn low, if the circumstances permit it. When it becomes necessary to haul the fire, in case of the water getting low in the boiler, it is considered the best practice to deaden the fire before hauling it by spreading a heavy charge of fresh fuel or ashes over it.

875. Lighting Fires.—To start a fire the grate is lightly covered with coal. Wood is piled evenly on top of it and covered with some coal. Greasy waste is put in front of the wood. This, when ignited, will soon ignite the wood, the fire gradually working towards the bridge; the layer of coal below the burning wood protects the grate bars from the heat, and thus prevents warping of the bars. As soon as the wood is burning freely more coal is put on. The fire door should be kept open while the wood is burning and the ash-pit damper shut. The fire should burn very slowly at first, so as not to injure the boiler by the unequal expansion of its various parts. The rate at which the fire burns may be regulated by the smokestack damper.

876. Practical Hints on Firing.—As it is usually desirable to keep the steam at an even pressure, the following points ought to be observed: Two fires in the same boiler should never be cleaned at once. When putting a fresh charge of fuel into the furnace, do it as rapidly as possible to avoid the inrush of cold air, which will injure the furnace plates and also reduce the temperature of the gases of combustion. Two furnaces in the same boiler should not be charged at once, as the fresh charge deadens the fire and reduces the generation of steam. The usual method of managing the fire is as follows: Let *A* and *B* be two single-ended Scotch boilers with two furnaces each, numbered as in Fig. 243, and suppose that bituminous coal is used. Furnace No. *1* is charged with fresh fuel; the fire in furnace No. *2* is leveled off with the hoe; the fire in furnace No. *3* sliced, and, next, the grate of furnace No. *4* is cleaned from below; furnace No. *2* is now charged with fuel, and No. *3* is leveled, No. *4* sliced, and the grate of No. *1* cleaned; furnace No. *3* is charged, No. *4* leveled, No. *1* sliced, and the grate of No. *2* cleaned; furnace No. *4* is charged, No. *1* leveled, No. *2* sliced, and the grate of furnace No. *3* cleaned.

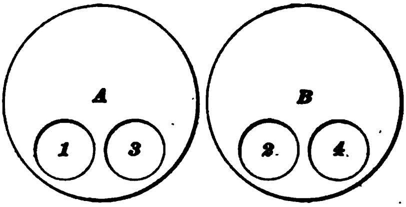


FIG. 243.

It will thus be seen that the same operation is not repeated at the same time in one boiler, and, furthermore, that each operation is performed in the numerical order of the furnaces. Thus, if the first fire sliced were in furnace No. *3*, the next one to be sliced would be in furnace No. *4*, the next one in furnace No. *1*, and the next one in No. *2*.

It will be observed that the furnaces numbered with the even numbers are in one boiler; those with the odd numbers in the second boiler.

This system of firing is in vogue in sea-going steamers, or wherever the engineer in charge believes in doing work systematically. Of course, the system may be varied to suit varying conditions, the arrangement of the operations as described being intended to furnish an idea as to a systematic way of firing. The same system may be and is followed in cleaning fires. The length of time a fire will burn without needing cleaning depends on the amount of coal burned and the quality of the coal. When consuming about 15 pounds of coal per square foot of grate surface per hour, with average coal, the fire will burn about 12 hours before it must be cleaned.

877. Lumps of coal should not be thrown into the fire; it should be broken up into pieces about the size of a man's fist. The fresh charge of fuel should be spread evenly over the fire. With irregular-sized coal, leveling with the hoe has to be resorted to some time before a fresh charge of fuel is put into the furnace. Should it be noticed while charging that the fire has burned out in one spot, owing to clinkers having formed on the grate and prevented admission of air to the fuel above it, or to the burning away of the fuel, the grate should be cleaned there, and the open spot filled with live coal before a fresh supply of fuel is put into the furnace. The back of the grate should not be allowed to become bare, as the inrushing cold air will cool the hot gases of combustion and greatly reduce the amount of steam generated. The ash pit should be kept clean, as ashes accumulating in it will prevent the free access of air to the furnace. The length of time that air is admitted above the fire, and the amount of it, depend upon the quantity and quality of the coal. For coal rich in hydrocarbons a longer time will be required than for other coal. But in no case should air be admitted after the hydrocarbons are expelled from the coal, which, with anthracite, will be in two or three minutes after charging. By watching the steam gauge the condition of the fires can be told, and a look at the burning fuel will show whether fresh fuel is needed, or the fires require slicing, or the air supply is insufficient, either by reason of the damper being closed or the grate clogged up.

878. Before cleaning fires it is often advisable to increase the feed and to work up the steam pressure, and to reduce the feed while cleaning. A certain quantity of heat is stored up in the extra water fed into the boiler, which is liberated when the feed is reduced, and tends to keep up the steam pressure.

A hard-coal fire does not need to be sliced, as no crust is formed on the surface of the fire. Slicing will merely mix the cinders and clinkers with the live coal and deaden the fire. The **T** bar should be run in under the fire to break up the cinders.

When burning small-size anthracite coal, it is advisable to resort to alternate firing. If a thick layer of anthracite is spread evenly all over the grate, as in spreading firing, a gas explosion is likely to occur. This may thus be accounted for: The gases driven off from the coal by the heat, when mixed with a sufficient quantity of oxygen, form an explosive mixture. Upon opening the furnace door, air rushes into the furnace, mixes with the gases and forms the mixture spoken of, which needs but a spark to ignite it. To prevent all danger of a gas explosion, either alternate firing should be resorted to, or a spot of say 1 square foot in area should be left uncovered when charging the fire. The gases will then be ignited and consumed as soon as they are formed.

MANAGEMENT AND CARE OF BOILERS.

INCRUSTATION.

879. The solid matter contained in sea-water consists chiefly of chloride of sodium (salt), sulphate of lime, and carbonate of lime; various other substances are usually present in minute quantities.

All these substances are held in solution at ordinary temperatures, but, with the exception of the chloride of sodium, are precipitated when the temperature of the water is raised. Chloride of sodium is not precipitated at ordinary temperatures until the water has become saturated with it. The point at which the different substances become insoluble has been found in various experiments to be about 290° F. On becoming insoluble, the agitation of the boiling water keeps the various substances suspended for some time, the rapidity with which they settle varying with the density of the water, the rapidity of the circulation, and the intensity of the ebullition.

When the circulation of the water is rapid, the various substances are carried away to a comparatively quiet place, where they are precipitated and deposited on the tubes or plates of the boiler.

880. The sulphate of lime depositing on the plates and tubes forms a crust varying in hardness according to the other substances with which it is combined. This crust is known as **scale.** When a considerable deposit of carbonate of lime is present in the boiler, it will remain soft for a certain time, and it deposits as a fine, loose powder if the boiler is cooled down very slowly before emptying it. This deposit can be washed out by a stream of water, a hose being used. Scale deposited on the plates exposed to the heat of the fire may lead to overheating, as scale is a bad conductor of heat. It has been found, however, that a coating of scale even as thick as $\frac{3}{16}$ inch is not so liable to lead to overheating as the fine deposit due to carbonate of lime, especially when mixed with organic matter, such as grease or oil. The grease seems to combine mechanically with the carbonate of lime, and when deposited on the plates forms a loose, spongy mass, sticking to the plates. This mass, by preventing contact of the water with the plates, and also by its great resistance to the transmission of heat, leads to overheating of the plates. The following simple experiment will prove that a loose, spongy mass is more likely to lead to overheating than a hard, solid layer of thick scale:

Take an ordinary saucepan and coat the bottom of it with plaster of paris, which acts practically the same as sulphate of lime scale, to the depth of about ¼ inch. It will be found that clean water can be boiled with but a small risk of overheating the bottom of the pan, unless the fire is very fierce. Next, take a clean saucepan and attempt to boil some water thickened with oatmeal to the consistency of gruel. In consequence of the resistance to the convection of heat, unless circulation be promoted by stirring the compound, the bottom of the pan will soon become overheated and the compound will be *burned.* Carbonate of lime may cause overheating in still another way. Carbonate of lime is held in suspension by the agitation of the water for some time before precipitating, and covers the surface of the water as scum. In the course of time the water becomes saturated with the substance, and as it offers a great resistance to the free escape of the steam bubbles, as well as preventing to some extent the convection of heat, the consequence is that the water is lifted off the plates by the steam, accumulating on the surface of the plates, and their overheating is likely to follow. To prevent overheating due to this cause, the surface blow-off cock should be used while the carbonate of lime is on the surface of the water in the form of scum, and before the water becomes saturated. Using the surface blow-off cock once an hour, keeping it open 10 or 15 seconds at a time, is considered good practice.

881. Sulphate of lime is not held in suspension for any length of time, but is deposited as soon as it reaches a quiet place in the boiler. Hence, the formation of sulphate of lime scale can not be prevented by the use of the surface or bottom blow-off cocks. It may, however, be prevented and partially removed by carbonate of soda, the common washing soda of commerce.

The carbonate of soda and sulphate of lime combine, when mixed in the boiler, to form sulphate of soda, which is very soluble. If carbonate of lime is present, it is precipitated by the soda. As is well known, common soda dissolves and removes grease, and, therefore, it is often used in boilers fed with greasy water. But, as its use will lead to violent foaming, the surface blowing-off cock must be used frequently. The soda is generally introduced by fitting a small tank, containing the soda dissolved in water, near the condenser and connecting the tank to it by a small pipe provided with a stop-cock.

This pipe enters from the top of the tank, reaching to within two or three inches of the bottom. When the stop-cock is opened, the pressure of the atmosphere forces the soda water into the condenser, whence it passes with the feed-water into the boiler. The amount of soda to be used is found by experience. It may vary from 1 to 10 pounds or more a day, according to the size and number of boilers.

882. Zinc plates have been found by experience to prevent and remove sulphate of lime scale to some extent. The zinc causes sulphate of lime to deposit in the form of powder instead of scale, chiefly upon the zinc. Furthermore, any grease present in the boiler, on decomposing, will form fatty acids, which, instead of corroding the iron or steel of the boiler, will attack the zinc. It has been found that the action of the zinc is best when making a metallic contact with the plates of the boiler. Iron or steel studs are screwed into the sides of the combustion chamber, and the zinc in the form of a plate about 10″ × 6″ × 1″ is bolted to these, taking care to have the contact surfaces clean and bright.

The usual proportions are 1 square foot of zinc to every 50 square feet of heating surface in new boilers, which are diminished after a time to 1 to 75, or even 1 to 100 square feet.

883. Such scale as is formed and not removed by use of either soda, or zinc, or both, must be removed mechanically. This is done by the scaling hammer shown in Fig. 244 at *A*, and the scaling bar shown at *B*. These tools are made of tool steel, and ground to an edge similar to that of a cold chisel.

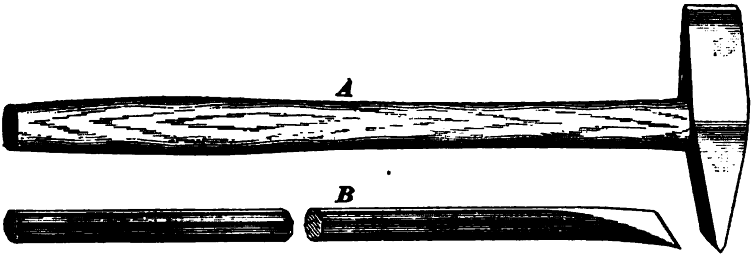


FIG. 244.

The scaling bar is made of various sizes and shapes to suit various conditions.

Besides leading to overheating, the presence of scale causes an enormous waste of fuel; it has been found in certain actual cases that a thickness of scale of about $\frac{1}{16}$ inch has required an extra expenditure of fuel of about 15%. With a thickness of scale of $\frac{1}{2}$ inch, the extra fuel required to evaporate the same amount of water was about 60%. Obviously, the best way to prevent the formation of scale is not to use any water containing the substance forming it. The chief value of the surface condenser lies in the fact that it supplies distilled water to the boiler.

WEAR AND TEAR OF BOILERS.

884. As is well known, water will cause iron or steel to rust. This rusting, known as **corrosion,** either internal or external, tends to shorten the life of the boiler. Internal corrosion is of three kinds.

INTERNAL CORROSION.

885. 1. **Uniform Corrosion, or Wasting.**—By this is meant the more or less uniform decrease in the thickness of the plates, extending over a considerable area. This is due to the chemical action of the water, and seems to be rusting, pure and simple.

Owing to the absence of any well-defined limit, it is often hard to detect. It is frequently necessary to drill a hole through the plate and measure the thickness to find the depth of corrosion. Sometimes the presence of a red streak in a crack of the scale will indicate corrosion below it. It is usually found under a sulphate of lime scale. In some cases the corroded part of the plate clings to the iron, and can only be detached by a hard blow with a hammer, when a big patch will come off. The usual places in which uniform corrosion may be looked for, are the sides of the furnaces about on a line with the grate bars, the bottom of the ash pits, the backs of the combustion chamber, the shell at the water-line, and the front and bottom of the boiler.

886. 2. **Pitting.**—This consists of a series of small holes, close together, eaten into the plates or tubes, honeycombing the plate as it were. The appearance of a pitted plate is shown in Fig. 245. It is due to chemical action of the water and may be looked for anywhere. It has been found in the water space as well as in the steam space, and even in the steam drums. Pitting is especially met with in boilers using distilled water from surface condensers.

Both pitting and uniform corrosion may be prevented to some extent, when distilled water is used, by allowing a thin scale, say about $\frac{1}{32}$ inch thick, to form on the plates of the boiler, when starting out after cleaning it. This is done by feeding salt water for a short time.

887. 3. **Grooving** is generally caused by the buckling action of the plates when under pressure. Thus, the ordinary lap joint of a boiler distorts the shell slightly from a truly cylindrical form, and the steam pressure tends to bend the plates at the joint. This bending action is liable to start a small crack along the lap, which, being acted upon by corrosive agents in the water, soon deepens into a groove, as shown in Fig. 246. The mark made along the seam by a sharp calking tool, when used by careless workmen, is almost certain to lead to grooving.

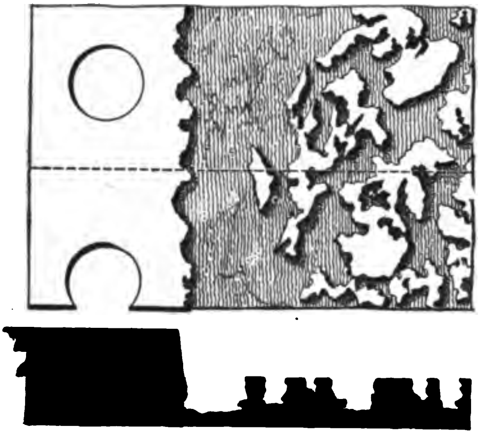


FIG. 245.

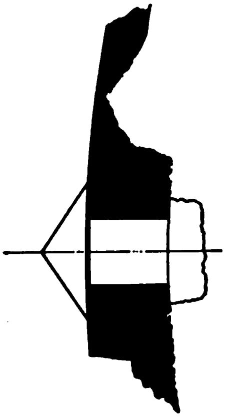


FIG. 246.

In the best practice, the inside butt straps are never calked.

As a rapid circulation of the water will equalize to some extent the strains due to the difference in temperature between different parts of the plates, a promotion of the circulation, when found to be sluggish, will have a good effect. This may be done in the manner described in the section on circulation.

If the grooving is caused by the cooling action of the feed-water, which is very likely to be the case when the feed-water is delivered in a body on the shell or furnace plates, the remedy would be to employ either a perforated pipe to distribute the water, or to heat the feed-water, or both.

EXTERNAL CORROSION.

888. External corrosion in marine boilers is due to water coming in contact with the shell and rusting it. The water may either come from leaky seams or rivets, or it may be the water in the bilges splashing against the plates when the vessel is rolling. Damp air will also rapidly cause corrosion. If corrosion is due to leaks, the best way of preventing it is to stop the leaks; and a thin coat of red lead paint applied when the plates are dry, and frequently renewed, will stop corrosion due to the other two causes.

889. Blistering.—This is a defect sometimes met with in wrought-iron plates exposed to the intense heat of the fire, such as the crown sheets. Through imperfection in the manufacture of the plate, instead of being solid, it is laminated in spots; that is, there are layers of iron with impurities between them. As these impurities are poor conductors of heat, the layer of iron exposed to the fire gets overheated and buckles up, forming what is termed a **blister.**

890. Overheating may cause the formation of a pocket, as shown in the accompanying figure, which represents the crown sheet of a flue boiler. In this instance some greasy waste had been left on the sheet when repairing the boiler. This, in the manner described in the section on incrustation, led to local overheating of the sheet, the steam pressure on the upper side of it causing it to assume the shape shown at *A*. Sometimes overheating causes the complete collapse of the furnace flues, especially when greasy

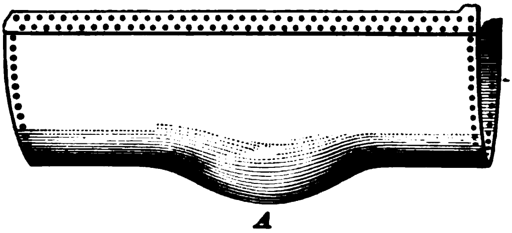


FIG. 247.

feed-water is used. Obviously, the best way to prevent a collapse is to keep the boiler clean, so that the water may always be in contact with the plates composing the furnaces. Overheating of the tops of the combustion chambers, due to low water in the boiler, will generally cause leakage of the tubes expanded into the overheated part of the tube sheet.

891. Fractured Plates.—Sometimes the metal between the rivet holes, or the metal between the rivet holes and the edge of the plate, is found to be fractured. This is due to local strains set up by expansion and contraction of the plates, either induced by cold feed-water being brought in contact with them, or by the plates being so intensely heated as to drive the water away from the plates. Then, upon cooling down, the water coming in contact with the plates again, will cause them to contract suddenly and fracture.

INSPECTION OF BOILERS.

892. All boilers in steam vessels coming under the jurisdiction of the Board of Supervising Inspectors of Steam Vessels, must be tested by the inspector of boilers once a year. The test applied is known as the *hydrostatic test*. The boiler is filled with water, all outlets being shut, and by means of a small force pump the pressure of the water is gradually increased until the required pressure has been reached, which is ascertained by means of a test pressure gauge supplied by the inspector. The test pressure is by law one and one-half times the working pressure of the boilers. The object of the hydrostatic test is to find whether the boiler is capable of withstanding safely the working pressure. At the same time it will reveal the presence of leaks, a sudden drop in the pressure denoting a serious leak.

The hydrostatic test is usually supplemented by an internal and external inspection of the boiler, it being the duty of the inspector to look for any defects, such as corrosion, either internal or external, pitting, grooving, evidence of overheating, bagging, blistering, fractures, etc.; in fact, to ascertain the condition of the boiler.

A hammer test is often employed to find the condition of the plates. A blow is struck with a light hammer, and the condition of the plate is judged by the sound. Considerable experience is necessary to judge the condition of the plate in this manner, and the student is recommended to test plates known to have certain defects, such as cracks, corrosion, grooves, pitting, and blisters, and compare the sounds produced. The position of the plate in the boiler has a great influence on the sound, and, therefore, no rule can be given which would apply in all cases.

CARE OF STEAM BOILERS.

893. When a vessel has reached port, the fire remaining in the furnaces is hauled, the boilers allowed to cool down, if possible, and emptied, and the manhole and handhole covers taken off. A thorough inspection of the boiler is made, both internally and externally, and all necessary repairs made. The mountings are next examined, first of all the pipes leading to the water column, which may be partially closed with sediment or scale and must be cleaned; then, the gauge-cocks are put in order, if required. With the high pressures used at present, these need frequent regrinding. The steam valves may need packing or repairing, likewise, the main and donkey feed check-valves, the safety valves, and all other attachments, which should be overhauled as far as possible.

It is the usual practice on ocean steamers to have a set time for overhauling each fitting liable to wear and tear. For instance, say once a month each check-valve is taken apart and examined, whether the working of it has revealed any trouble or not. This practice is considered better than to wait until a breakdown has occurred.

When the ship is ready to leave port, the lower manhole and handhole covers are put in position, and the boiler filled with water to the required depth, generally by means of a hose introduced into the top manhole and connected to a hydrant on the dock. The top manhole cover is next put in place and the safety valve is raised off its seat. The blow-off connections should be examined to see if they are properly shut. It is good practice to ease all steam stop-valves before raising steam. By **easing** a valve is meant the opening of it very slightly, just enough to remove it off its seat. If a steam valve be screwed down hard while cold, it will often be found next to impossible to open it when hot. This is due to the friction between the threads of the stem and nut induced by the elongation of the stem when heated. After all these precautions are taken, the fires are started and allowed to burn very slowly at first, so as not to strain the boiler. After steam is raised and all the air driven out of the boiler, the safety valve is lowered. The steam pressure should be raised very gradually. When there are several boilers connected to the same steam main, the pressures in all should be equal within a small limit of variation, say two pounds, before connecting them, so as to prevent a rush of steam from one boiler to another. The steam stop-valve of each boiler should be opened but a little at first, so as to warm the steam pipes gradually. The drain from the main steam pipes should be opened so as to allow all water collected in the pipe to escape. If the steam valves were suddenly opened wide, it might lead to an explosion of the boiler, as has been shown by some experiments made by Prof. Thurston with marine boilers. If there happens to be water collected in the pipe, the steam rushing in at a high velocity will drive the water ahead of it, and the momentum of the water dashing against the closed end of the pipe may cause it to break; this is technically known as the water hammer.

After the main steam pipe has been warmed thoroughly, the steam stop-valves may be gradually opened wide, shutting off the drain after a few minutes. The main steam stop-valve is next opened, the engine warmed up and started, taking care to open the main feed check-valves, if attached plunger feed-pumps are used. The damper is then opened, and the fires allowed to burn up as the demand for steam increases. The management of the fires has been treated of previously. The water level while running should be kept as nearly constant as possible. Should the demand for steam cease suddenly, as in case of stoppage of the engine for a short period, the damper must be shut and the furnace doors opened. The injector or donkey feed-pump should also be started, both to keep up the circulation and to keep down the steam, as with a surface condenser all steam blown off through the safety valves represents fresh water lost. If stopped for a considerable period, it may be necessary to bank or to haul the fires.

894. Low Water.—Should the water get low in the boiler, that is, fail to issue from the lowest gauge-cock when this is opened, or is out of sight in the gauge glass, or should the fusible plug give warning, the fires in the boiler should be covered with ashes or fresh coal, the damper closed and the steam worked off with the engine, and, if feasible, all auxiliary engines started up. *Under no circumstances should the safety valve be disturbed or feed-water be admitted to the boiler, as it may lead to an explosion.* The boiler should be allowed to cool down gradually, then be emptied and the manhole and handhole covers taken off, after which a thorough inspection should be made, looking for evidence of overheating. Before using the boiler again, it should be put in a good condition. A report must be made to the boiler inspector of any defects or accidents to the boiler.

895. The following are the rules and directions generally given for the guidance of firemen and attendants in the performance of their duties:

1. **Firing.**—Keep the fire of uniform thickness, and allow no airholes in the bed of fuel. Fire evenly and regularly, and not too much at a time. Keep the fire free from ashes and clinkers, and as clean at the corners and edges as at the center. Keep the ashpit clear. Do not clean the fires oftener than necessary.

2. **Water Level.**—The first duty of the fireman upon going to work should be to examine the water level. The

gauge-cocks should be tried; the gauge glass is not always reliable. In a battery of boilers the gauge-cocks on *each* boiler should be tried. Some serious explosions have resulted from the fact that the fireman only consulted the water level in the first boiler and took it for granted that the level in the other boilers was the same.

3. **Foaming or Priming.**—In case of foaming, close the throttle valve of the engine or the stop-valve of the supply pipe, and keep it closed long enough to show the true water level. The foaming can generally be stopped by blowing off and feeding fresh water. In cases of violent foaming, due to dirty water or to a change from fresh to salt water, check the draft and cover the fire with fresh coal.

4. **Leaks.**—When leaks are discovered they should be repaired at once.

5. **Filling Up.**—Allow the boiler to become cool before pumping in cold water; the practice of filling a hot boiler with cold water causes leaks and fractures, and sometimes explosions.

6. **Safety Valves.**—Raise the valves from their seats cautiously and frequently. Do not allow the valve to be overloaded.

7. **Pressure Gauge.**—The pressure gauge should stand at 0 when the steam pressure is off, and it should indicate the blowing-off pressure when the safety valve is in action. If the gauge does not do this, it should be compared with a standard gauge, and, if wrong, corrected.

8. **Gauge-Cocks and Gauge Glasses** should be kept clean and in constant use. The water gauge should be blown out frequently, and the glasses and passages to gauges kept clean. An obstructed gauge sometimes shows a false water level. The Manchester Boiler Association attributes more accidents to this cause than to all others combined.

9. **Removal of Sediment and Incrustation.**—Scale and sediment should be frequently removed. In tubular boilers, particularly, the handholes or manholes should be frequently opened and the sediment removed from the

portions of the plate over the furnace or the crown of the frame. Care should be taken to keep the boiler as free as possible from incrustation.

10. **Cleaning.**—All heating surfaces should be kept free from soot and dirt. Tubes should be often cleaned out.

11. **Exterior of Boiler.**—Care should be taken that no water comes in contact with the exterior of the boiler, either from leaky joints or other sources. Avoid dampness in the setting or in the covering of the boiler. Dampness leads to external corrosion.

12. **Blisters and Cracks.**—A blister should be at once examined and trimmed or patched. If a plate is badly cracked, it must be renewed.

13. **Fusible Plugs** should be examined when the boiler is cleaned, and scraped clean on both sides; otherwise, they are liable to prove worthless.

14. **Air Leaks.**—See that the furnace, combustion chamber, and smoke flue are tight. The admission of air through the brickwork of the setting is sometimes a source of considerable loss.

15. **Galvanic Action.**—Examine the parts of the boiler where brass or copper and iron come in contact in the presence of water. Galvanic action may produce corrosion under such circumstances; if such be the case, the corrosion may be prevented by placing pieces of zinc in the boiler.

16. **Rapid Firing.**—Steam should be raised slowly in boilers with thick seams or seams exposed to the fire. Otherwise, overheating may occur.

17. **Cleanliness.**—The boiler room, boiler, and mountings should be kept clean and in good order.

18. **Unused Boilers** may be kept in good condition by filling them full of water in which a quantity of common washing soda has been placed. Another method is to empty the boiler, dry it thoroughly, place trays of quicklime in the bottom, and seal it as nearly air-tight as possible. The latter method is often used for marine boilers.

REPAIRS OF BOILERS.

896. The repairs that can be made by the engineer are confined to patching plates, expanding leaky tubes, putting in a new staybolt, and other minor repairs.

897. By the term **hard patch** is meant a patch riveted to the worn part of the plate to be repaired. It is made of a piece of plate considerably larger than the worn part, and fitted on the inside of the boiler, so that the pressure may not force it off.

898. A **soft patch** is a patch put on with studs or bolts and nuts.

Leaky tubes are made tight by expanding them in the manner previously explained. If the tube ends are much worn, it may be necessary to drive them slightly into the combustion chamber, and expand and rebead them. Sometimes the tube sheet will be found cracked between two tubes. This may be repaired temporarily by fitting a plate over the cracked part in the manner shown in Fig. 248, and bolting it to the tube sheet.

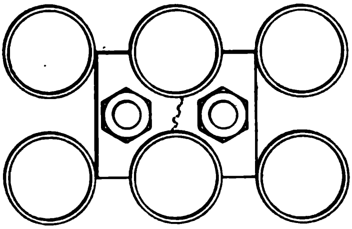


FIG. 248.

899. Split tubes may be repaired temporarily by using the **tube stopper** shown in Fig. 249. Washers *A* and *B*, having a rubber ring *f* about ½ inch thick between them, are placed on each end of a tie bolt *C*, provided with nuts and washers on each end; a sleeve *D* made of gas pipe is

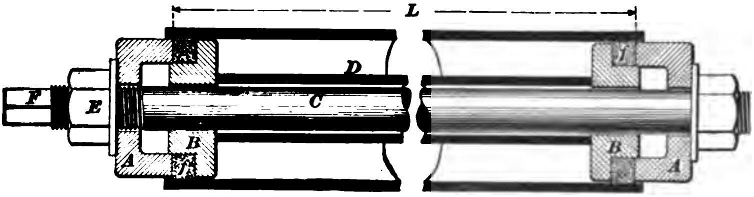


FIG. 249.

placed between the two pairs of washers. The distance L is made about 1 inch less than the length of the tube.

To stop a leaky tube, the stopper is inserted from the front, and the nut E is tightened, the bolt being held by a wrench put on the square end F of the bolt, to prevent it from turning. Tightening the nut E draws the washers A and B together, compressing the rubber and making a water-tight joint.

GRATE AREA AND HEATING SURFACE.

900. The **grate area,** or **grate surface,** depends upon the rate of combustion, the evaporation per pound of fuel, and the total weight of steam evaporated per hour.

For example, a Scotch boiler generates 8,000 pounds of steam per hour. Supposing the rate of combustion to be 16 pounds of coal per square foot of grate area per hour, and the evaporation to be 8 pounds of water per pound of coal, the necessary grate area will be $\frac{8,000}{16 \times 8} = 62.5$ sq. ft. This grate area would necessarily be divided between several furnaces, as a grate longer than 6 feet can not be easily fired, while the width is limited to four or six feet by practical considerations. Assuming the dimensions of the grate to 3 feet 6 inches by 6 feet, the above grate area would require $\frac{62.5}{6 \times 3.5} = 3$ furnaces.

From the above method of calculating the grate area, the following rule may be deduced:

Let G = area of grate in square feet ;
H = area of heating surface in square feet ;
F = rate of combustion in pounds per square foot of grate area per hour;
W = weight of steam generated per hour by boiler or boilers;
E = evaporation in pounds of water per pound of coal.
R = ratio of heating surface to grate area.

Rule 165.—*To find the grate area of a boiler, divide the weight of steam generated per hour by the product of the rate of combustion and the evaporation.*

That is, $$G = \frac{W}{FE}.$$

In rule **165** no account has been taken of the difference in the number of heat units required to evaporate steam from different temperatures of feed-water into steam at different pressures. Hence, the rule is but an approximation, but close enough for most practical work.

The evaporation per pound of coal for Scotch and fire-box boilers is approximately as given in the following table:

TABLE 30.

Coal per hour per sq. ft. of grate area. From	6–10	10–14	14–18	18–20
Pounds of water evaporated per pound of coal	10	9	8.25	7.5

For water-tube boilers add ½ pound to the evaporation.

Example.—A Scotch boiler is to generate 7,500 lb. of steam per hour, burning best bituminous coal. What should be the grate area, provided the height of the smokestack is 66 ft.?

Solution.—The amount of bituminous coal which may be burned per square foot of grate area is, from rule **163**, 18.28 pounds. From the table above, the evaporation per pound of coal may be taken at 7.5 pounds of water.

Substituting the values found, we have

$$G = \frac{7{,}500}{18.28 \times 7.5} = 54.7 \text{ sq. ft. Ans.}$$

901. Heating Surface.—To generate sufficient steam, the heating surface of the boiler must bear a certain ratio to the grate surface. This ratio R in Scotch and fire-box boilers varies from 30 to 35. That is, there are from 30 to 35 square feet of heating surface to every square foot of grate surface.

For water-tube boilers, the ratio varies between 35 and 40.

From the foregoing we may deduce the following approximate rule:

Rule 166.—*To find the necessary heating surface of a boiler, multiply the grate surface by the number of square feet of heating surface to be allowed for every square foot of grate surface;*

$$\text{or, } H = R\,G.$$

EXAMPLE.—Find the heating surface for the boiler mentioned in the last example, using a ratio of heating surface to grate surface of 35 : 1; that is, take $R = 35$.

SOLUTION.—$H = 35 \times 54.7 = 1,914.5$ sq. ft. Ans.

902. What constitutes the heating surface has been treated of previously in Art. **680.** Since a great part of the heating surface of tubular boilers is furnished by the tubes, particular attention must be paid to their arrangement. The tubes are from 2 to 3½ inches in diameter and are arranged in horizontal and vertical rows; if staggered, they hinder the circulation of the water. The tubes are usually arranged in nests, the space between the nests being large enough to admit a man's body. The pitch of the tubes, that is, the distance from center to center, should be from 1⅓ to 1½ times the diameter of the tube. The distance between the shell and outer row of tubes should not be less than 5 inches, preferably more. Neither should the lowest row of tubes (in a Scotch boiler) be placed too close to the furnace. By leaving a space of about 12 inches or more between the furnace and the lowest row of tubes, a large body of water rests directly over the fire, and good circulation is insured. Furthermore, the space greatly facilitates examination and repairs.

The steam space is usually made from ⅓ to ¼ the cubical contents of the boiler, the larger steam space being preferable, as there is more likelihood of the steam being dry.

The actual amount of steam space largely depends, however, on the speed of engines and the pressure carried. A set of engines in a screw steamer running at 150 revs. and carrying 180 lb. of steam would not require so much steam space as a paddle boat making 60 revs. and using steam at 80 lb.

BOILER EXPLOSIONS.

903. Boiler explosions are, in nearly all cases, due to one cause only—overpressure of steam. Either the boiler is not strong enough to safely carry its working pressure, or else the pressure has been allowed to rise above the usual point by the sticking or overloading of a safety valve or some similar cause.

A boiler may be unfit to bear its working pressure for any of the following reasons: 1. Defective design. 2. Defects in workmanship or material. 3. Corrosion and wear and tear in general. 4. Mismanagement in operation.

904. Defective design consists chiefly of designing boilers in such a manner that access to the places liable to wear and tear can not be had. The writer has seen several Scotch boilers in which it was a practical impossibility for the engineer to examine the top of the furnaces, the space between the nests and the tubes being but 7 inches. Defective water circulation in a boiler may lead to excessive incrustation and consequent collapse of the furnaces; a poorly designed feed apparatus may also cause a disastrous explosion.

905. Defects of workmanship and material may include the choice of faulty material, containing blisters, laminations, etc.; burned and broken rivets; plates, burned or otherwise injured in flanging, bending, or welding; scoring of the plates along the joints by sharp calking tools; injury of the plates by the reckless use of the drift pin. Old plates may be injured in patching them with new plates by the operation of removing the old rivets and putting in the new ones, and also by the greater expansion and contraction of the new plates when exposed to fire.

906. The strength of the shell may be weakened by corrosion, pitting, and grooving. In some boiler explosions the plates have been found wasted to little more than the thickness of paper.

907. Fractures, which ultimately end in explosions, may be produced by letting the cold feed-water come directly in contact with the hot plates. (It has already been remarked that the feed should be introduced into the coolest part of the boiler.)

908. External corrosion is a fruitful source of boiler explosions, especially in water-tube boilers. In one case wet ashes were allowed to accumulate in the ash pit, and the subsequent corrosion of the tubes led to a rupture causing the loss of several lives.

909. It has been previously explained how incrustation or the presence of g[illegible]ase may lead to the formation of pockets. If the pocket is not removed and replaced by a patch, rupture and explosion are liable to result.

910. The pressure of the steam may be allowed to rise above the normal blowing-off pressure by neglect or mismanagement. The safety valve may be neglected and allowed to stick fast to its seat. In one case a safety valve recovered from a boiler explosion was found to be corroded to such an extent that a pressure of 1½ tons was required to start it from its seat. Again, the safety valve may be temporarily shut off from the boiler by a stop-valve. Numerous very disastrous explosions have been due to this cause. It can not be too strongly emphasized that a stop-valve should never be placed between the safety valve and the boiler. In fact, this is forbidden by law. Overpressure and consequent explosion may be caused, also, by the practice of overweighting the safety valve.

911. Low water is the cause ascribed to many explosions. It was formerly the custom to consider nearly every boiler explosion as due to shortness of water. It is now known, however, that externally fired boilers rarely explode on account of low water, though internally fired boilers may do so.

The tubes of a return-tubular boiler may stand a large amount of overheating before giving way, and it is next to impossible to burst a plain cylindrical boiler by low water, as long as it contains any water at all. Low water may lead to explosion, however, in the following manner: The uncovered plates become red hot, and upon being suddenly covered with fresh cold feed-water may suddenly contract to such an extent as to produce rupture.

912. The superheating of the water is supposed to have produced explosions in rare instances. It has been observed that water from which the air has been expelled may be heated 10° or 20° above its normal boiling point without any signs of boiling. Upon being agitated, as by the introduction of fresh feed-water, the volume of superheated water, flashing suddenly into steam might, under favorable conditions, produce a pressure great enough to cause an explosion.

913. A weak boiler may possibly be exploded by a sudden opening or closing of the throttle valve. That such an explosion is possible is shown by the following experiment made by the United States Inspectors: A cylindrical boiler was tested and withstood a steam pressure of 300 pounds without injury. It was then filled again and the steam pressure was run up gradually, the discharge valve being opened at intervals and the fall in pressure noted. When the valve was suddenly opened at a pressure of 235 pounds, the boiler was torn into fragments, the iron being twisted and torn and thrown in all directions. The sudden rush of steam from the boiler into the discharge pipe reduces the pressure in the boiler very rapidly; the reduction of pressure causes a sudden formation of a great quantity of steam within the water, and the heavy mass of water, thrown towards the opening with great violence, strikes the portions of the boiler near the opening and bursts it open.

Explosions from this cause are probably rare; still, it is well to use caution in opening or closing stop-valves or safety valves.

The destructive nature of a steam-boiler explosion is due to the enormous amount of energy stored in the steam and heated water. Prof. Thurston calculates that a cubic foot of water heated at constant volume until its pressure is 60 to 70 pounds per square inch, has about the same energy as a pound of gunpowder, and that the energy stored in a plain cylindrical boiler at 100 pounds pressure is sufficient to project it to a height of over 3½ miles.

914. If a boiler fractures while undergoing the hydrostatic test, the water escapes through the rent in the plate and no explosion takes place, because the cold water has little or no stored energy. But when a boiler filled with steam and water at high temperature fractures, a violent explosion generally follows. The steam escaping through the opening diminishes the pressure, and, consequently, a new body of steam is formed from the water, which, by escaping, lowers the pressure still more, allowing the formation of another new body of steam at a lower pressure, and this operation is continued until the pressure reaches that of the atmosphere. This formation of successive large bodies of steam, occurring, as it does, almost instantly, can not fail to produce a disastrous explosion. Generally speaking, the larger the body of the contained water, the more disastrous is the result. The safety of water-tube boilers is thus accounted for by the fact that they consist of numerous parts or sections, each containing a relatively small body of water. The bursting of one of these sections is unusual on account of its small diameter, and should it occur, no disastrous explosion would be likely to follow, on account of the small quantity of contained water.

Boiler explosions may be prevented by the use of a properly designed and well-made boiler, and ensuring for it careful management. The boiler must be regularly inspected, repaired when necessary, and removed before becoming so worn out as to be dangerous.

BOILER TRIALS.

915. **Boiler trials** are made to determine the efficiency of the boiler; that is, to determine what percentage of the heat of the coal is usefully expended in heating the water. Sometimes the object of the trial of the marine boiler is to determine the quantity of water evaporated into steam at a given pressure for 1 pound of the fuel, and sometimes to determine the quantity of coal or steam consumed by a given engine.

Boiler trials will usually have to be made while the vessel is tied to a dock, unless it is navigating in still waters, as the rolling or pitching of the vessel would greatly interfere with the accuracy of the observations made with the different instruments used. The method of making a boiler trial is, briefly, as follows: The water fed into the boiler during the trial is weighed, the temperature of the feed-water is noted, and also the average pressure of the steam throughout the trial. From the data thus obtained the number of B. T. U. usefully expended in evaporating the water may be calculated by the aid of the steam table. The coal used during the test is also weighed. Knowing the B. T. U. developed by a pound of coal, the total heat supplied to the furnace is readily calculated.

A convenient plan for weighing the coal is to have a box with one side open placed on a platform scale. A weight is placed on the scale beam just heavy enough to balance the empty box. The scale is then set at 400, 500, or 600 pounds, as desired. The coal is shoveled into the box until the beam rises. By this means, the box contains just 400, 500, or 600 pounds, as desired. The coal is then fed from the box into the furnace.

After the test is over, the ashes and clinkers are raked from the ash pit and weighed. This weight is deducted from the weight of the coal and gives the weight of the coal actually burned, or the weight of the *combustible*, as it is called.

A convenient method of measuring the feed-water delivered to the boiler is to have two tanks *A* and *B*, Fig. 250, one above the other. The supply of water is fed through the pipe *C* into the upper tank *A*, which rests on a platform scale *D*. After balancing the tank, the scale may be set to weigh 500 or 600 pounds of water, the water being run in until the beam rises, and then shut off. The upper tank is provided with a pipe *E* and valve *F*, by means of which the water may be discharged into the lower tank, from which it is fed to the boiler through the pipe *G*.

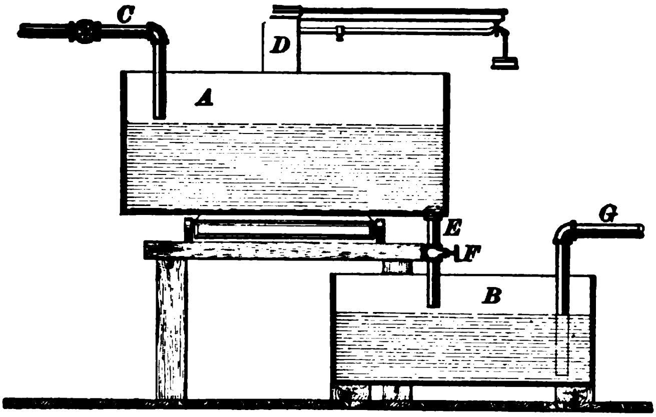


FIG. 250.

The attendant who keeps the record of coal supply or water supply should become accustomed to making the tally on his blank just before or after some regular operation. For example, the person who weighs the feed-water should record each tank full, say, immediately after closing the valve in the supply pipe, or, perhaps, after emptying the upper tank into the lower. If this precaution is not observed, the attendant is liable to become uncertain as to whether he has recorded the previous data, and a mistake is almost certain to result. A boiler trial may consist simply of an ordinary run of 10 or 12 hours, the only things observed being the weight and temperature of feed-water, the weight of coal, and pressure of steam. On the other hand, the trial may be very elaborate, including the analysis of coal and flue gases, tests of quality of steam, and many other refinements.

A standard method of making boiler trials has been adopted by the American Society of Mechanical Engineers. The code of rules given should be followed whether the trial is simple or elaborate. The following is the standard method in full:

CODE OF RULES FOR BOILER TRIALS.

916. Preliminaries to a Test.—I. In preparing for and in conducting trials of steam boilers, the specific object of the proposed trial should be clearly defined and steadily kept in view.

II. Measure and record the dimensions, position, etc., of grate and heating surfaces, flues, and smokestack; proportion of air space in the grate surface, kind of draft, natural or forced.

III. Put the boilers in good condition. Have all the heating surfaces clean, inside and out, grate bars and sides of furnace free from clinkers, dust and ashes removed from back connections, leaks in uptakes stopped, and all obstruction to draft removed. See that the damper will open to full extent, and that it may be closed when desired. Test for leaks in uptakes by firing a little smoky fuel and immediately closing damper; the smoke will then escape through the leaks.

IV. Have an understanding with the parties in whose interest the test is to be made as to the character of the coal to be used. The coal must be dry, or, if wet, a sample must be dried carefully, and a determination of the amount of moisture in the coal made and a calculation of the results of the test corrected accordingly. Wherever possible, the test should be made with standard coal of a known quality. For that portion of the country east of the Alleghany Mountains, good anthracite egg coal or Cumberland semi-bituminous coal may be taken as the standard for making tests. West of the Alleghany Mountains and east of the Missouri River, Pittsburgh lump coal may be used.

V. In all important tests a sample of coal should be selected for chemical analysis.

VI. Establish the correctness of all apparatus used in the test for weighing and measuring. These are:

1. Scales for weighing coal, ashes, and water.
2. Tanks or water meters for measuring water. Water meters should be used, as a rule, as a check on other measurements. For accurate work, the water should be weighed or measured in a tank.

3. Thermometers and pyrometers for taking temperatures of air, steam, feed-water, waste gases, etc.

4. Pressure gauges, draft gauges, etc.

VII. Before beginning a test, the boiler and smokestack should be thoroughly heated to their usual working temperature.

VIII. Before beginning a test, the boiler and connections should be free from leaks, and all water connections, including blow-out and extra feed-pipes should be disconnected or stopped with blank flanges, except the particular pipe through which the water is to be fed to the boiler during the trial. To disconnect the donkey feed, it may be necessary to obtain the consent of the supervising inspector of the district. In case the feed-pipes can not be disconnected, such pipes may be drilled so as to leave openings in their lower sides, which should be kept open throughout the test as a means of detecting leaks or accidental or unauthorized opening of valves. During the test the blow-off pipe should remain exposed. If an injector is used, it must receive steam directly from the boiler being tested, and not from a steam pipe or any other boiler.

See that the steam pipe is so arranged that the water of condensation can not run back into the boiler. If the steam pipe has such an inclination that the water of condensation from any portion of the steam-pipe system may run back into the boiler, it must be trapped so as to prevent this water getting into the boiler without being measured.

917. IX. **Starting and Stopping a Test.**—A test should last at least ten hours of continuous running, and twenty-four hours whenever practicable. The conditions of the boiler and furnace should be, as nearly as possible, the same at the end as at the beginning of the test. The steam pressure should be the same, the water level the same, the fire upon the grates should be the same in quantity and condition, and the uptakes, flues, etc., should be of the same temperature. To secure as near an approximation as possible to exact uniformity in conditions of the fire and in the temperatures of the uptakes and flues, the following method of starting and stopping a test should be adopted:

918. X. **Standard Method.**—Steam being raised to the working pressure, remove rapidly all the fire from the grate, and as quickly as possible start a new fire with weighed wood and coal, noting the time of starting the test and the height of the water level while the water is in a quiescent state just before starting the fire.

At the end of the test, remove the whole fire, clean the grates and ash pit, and note the water level when the water is in a quiescent state; record the time of hauling the fire at the end of the test. The water level should be as nearly as possible the same as at the beginning of the test. If it is not the same, a correction should be made by computation and not by operating the pump after the test is completed. When fires are being hauled at the beginning or end of a test, it will generally be necessary, in order to maintain the steam pressure up to the average during the test, to regulate by means of a stop-valve the discharge of steam from the boiler.

919. XI. **Alternate Method.**—Instead of the standard method above described, the following may be employed where local conditions render it necessary:

At the regular time for slicing and cleaning fires, arrange so that they shall be burned rather low, as is usual before cleaning, and then have them thoroughly cleaned; note the amount of coal left on the grate, as nearly as it can be estimated; note the pressure of steam and the height of the water level—which should be at the medium height to be carried throughout the test—at the same time, and note this time as the time of starting the test. Fresh coal, which has previously been weighed, should now be fired. The ash pits should be thoroughly cleaned at once after starting. Before the end of the test the fires should be burned low, just as before the start, and the fires cleaned in such a manner as to leave the same amount of fire and in the same condition on the grates as at the start. The water level and steam pressure should be brought to the same point as at the start, and the time of the ending of the test should be noted just before fresh coal is fired.

920. XII. **During the Test.—Keep the Conditions Uniform.** The boiler should be run continuously without stopping for meal times or for rise or fall of steam due to change of demand for steam. The draft being adjusted to the rate of evaporation or combustion desired before the test is begun, it should be kept constant during the test by means of the damper.

If the boiler is not connected to the same steam pipe with other boilers, an extra outlet for steam, with valve in same, should be provided, so that in case the pressure should rise to that at which the safety valve is set it may be reduced to the desired point by opening the extra outlet without checking the fires.

If the boiler is connected to the main steam pipe with other boilers, the safety valve on the boiler being tested should be set a few pounds higher than those of the other boilers, so that in case of a rise in pressure the other boilers may blow off and the pressure be reduced by closing their dampers, allowing the damper of the boiler being tested to remain open, and firing to go on as usual.

To set the safety valve to a higher pressure, the consent of the supervising inspector of the district may have to be obtained.

All the conditions should be kept as nearly uniform as possible, such as the force of draft, pressure of steam, and height of water. The time of cleaning the fires will depend upon the character of the fuel, the rapidity of combustion, and the kind of grates. When very good coal is used and the combustion is not too rapid, a ten-hour test may be run without any cleaning of the grates other than just before the beginning and just before the end of the test. But, in case the grates have to be cleaned during the test, the intervals between one cleaning and another should be uniform.

921. XIII. **Keeping the Records.**—The coal should be weighed and delivered to the firemen in equal portions, each sufficient for about one hour's run, and a fresh portion should not be delivered until the previous one has been all fired. The time required to consume each portion should be noted, the time being recorded at the instant of firing each new portion. It is desirable that the amount of water fed into the boiler should be accurately noted at the same time and recorded, noting, also, the height of water in the boiler, and the average pressure of steam and temperature of feed during the time. By thus recording the amount of water evaporated by successive portions of coal, the record of the test may be divided into several divisions (if desired) at the end of the test to discover the degree of uniformity of combustion, evaporation, and economy at different stages of the test.

922. XIV. **Priming Tests.**—In all tests in which accuracy of results is important, calorimetric tests should be made of the percentage of moisture in the steam, or of the degree of superheating. At least ten such tests should be made during the trial of the boiler, or so many as to reduce the probable average error to less than one per cent., and the final records of the boiler test should be corrected according to the average results of the calorimeter test.

On account of the difficulty of securing accuracy in these tests, the greatest care should be taken in the measurement of weights and temperatures. The thermometer should be accurate to within a tenth of a degree, and the scales on which the water is weighed should be correct to $\frac{1}{100}$ of a pound.

923. XV. **Analysis of Gases.**—The analysis of gases and measurement of the air supply are not generally necessary in ordinary commercial tests; they are, however, valuable in determining the relative values of different methods of firing or of different kinds of furnaces. Care must be taken to procure average samples of the gas, and the analysis should be made by a competent chemist.

924. XVI. **Record of the Test.**—A "log" of the test should be kept on properly prepared blanks containing headings as follows:

Time.	Pressures.			Temperatures.					Fuel.		Feed-Water.	
	Barometer.	Steam Gauge.	Draft Gauge.	External Air.	Boiler Room.	Flue.	Feed-Water.	Steam.	Time.	Pounds.	Time.	Pounds or Cubic Feet

925. XVII. **Reporting the Trial.**—The final results should be recorded upon a properly prepared blank, and should include as many of the following items as are adapted for the specific object for which the trial is made. The items marked with a * may be omitted for ordinary trials, but are desirable for a comparison with similar data from other sources.

Results of the trial of ....
Boilers at ....
To determine...............................

	ITEMS.	
1.	Date of trial.	
2.	Duration of trial.	hours.
	DIMENSIONS AND PROPORTIONS.	
3.	Grate surface, wide..... long..... area	sq. ft.
4.	Water-heating surface.	sq. ft.
5.	Superheating surface.	sq. ft.
6.	Ratio of water-heating surface to grate surface.	
	AVERAGE PRESSURES.	
7.	Steam pressure in boiler by gauge.	lb.
*8.	Absolute steam pressure.	lb.
*9.	Atmospheric pressure by barometer.	in.
10.	Force of draft in inches of water.	in.
	AVERAGE TEMPERATURES, FAHR.	
*11.	Temperature of external air.	deg.
*12.	Temperature of fire room.	deg.
*13.	Temperature of steam.	deg.
14.	Temperature of escaping gases.	deg.
15.	Temperature of feed-water.	deg.
	FUEL.	
15½.	Cost of coal per 2,000 lb. at boilers.	
16.	Total amount of coal consumed (includes wood × 0.4).	lb.

17.	Moisture in coal.	per cent.
18.	Dry coal consumed.	lb.
19.	Total refuse dry . . . . .pounds =	per cent.
20.	Total combustible (Item 18 less Item 19).	lb.
*21.	Dry coal consumed per hour.	lb.
*22.	Combustible consumed per hour.	lb.

RESULTS OF CALORIMETRIC TESTS.

23.	Quality of steam (dry steam taken as unity).	per cent.
24.	Percentage of moisture in steam.	per cent.
25.	Number of degrees superheated.	deg.

WATER.

26.	Total weight of water pumped into boiler and apparently evaporated.	lb.
27.	Water actually evaporated, corrected for quality of steam.	lb.
28.	Equivalent water evaporated into dry steam from and at 212° F.	lb.
*29.	Equivalent total heat derived from fuel.	B. T. U.
30.	Equivalent water evaporated into dry steam from and at 212° F. per hour.	lb.

ECONOMIC EVAPORATION.

31.	Water actually evaporated per pound of dry coal from actual pressure and temperature.	lb.
31½.	Equivalent water evaporated for $1.00, from and at 212° F.	lb.
32.	Equivalent water evaporated per pound of dry coal, from and at 212° F.	lb.
33.	Equivalent water evaporated per pound of combustible, from and at 212° F.	lb.

COMMERCIAL EVAPORATION.

34.	Equivalent water evaporated per pound of dry coal with one-sixth refuse at 70 pounds gauge pressure, from temperature of 100° F. (= Item 33 × 0.7249).		lb.

RATE OF COMBUSTION.

35.	Dry coal actually burned per square foot of grate surface per hour.		lb.
*36.	Consumption of dry coal per hour, coal assumed with one-sixth refuse.	Per square foot of grate surface.	lb.
*37.		Per sq. ft. of water-heating surface.	lb.
*38.		Per sq. ft. of least area for draft.	lb.

RATE OF EVAPORATION.

39.	Water evaporated from and at 212° F. per sq. ft. of heating surface per hr.		lb.
*40.	Water evaporated per hour from temperature of 100° F. into steam of 70 lb. gauge pressure.	Per sq. ft. of grate surface.	lb.
*41.		Per sq. ft. of water-heating surface.	lb.
*42.		Per sq. ft. of least area for draft.	lb.

COMMERCIAL HORSEPOWER.

43.	On a basis of 30 pounds of water per hour, evaporated from a temperature of 100° F. into steam of 70 pounds gauge pressure (= 34½ pounds from and at 212° F.).		H. P.
43½.	Number of horsepower obtained for $1.		H. P.
44.	Horsepower, builders' rating at ...... square feet per H. P.		H. P.
45.	Per cent. developed above or below rating.		per cent.

REMARKS.

926. Some of the items of the above schedule are derived from the others as follows:

Item 27 = Item 26 × Item 23.
Item 29 = Item 28 × 966.1.
Item 31 = Item 27 ÷ Item 18.
Item 32 = Item 28 ÷ Item 18.
Item 33 = Item 28 ÷ Item 20.
Item 43 = Item 30 ÷ 34½.

Items 32 and 33 are used as a sort of a standard to measure the performance of the boiler. The best boilers usually evaporate 9½ to 11½ pounds of water from and at 212° per pound of combustible. With poor boilers the rate may not exceed 5 to 8 pounds of water per pound of combustible.

Many engineers prefer to make Item 32 (the evaporations per pound of coal) the test of a boiler's performance. In case a boiler does not make a good showing, it is easy to make the result appear better by deducting a considerable quantity of coal actually consumed on the assumption that it is ash when Item 33 is used as a standard.

Items 43, 43½, 44, and 45 refer to stationary boilers which are sold by the horsepower, the standard horsepower, as adopted by the American Society of Mechanical Engineers, being 30 pounds of water evaporated from a temperature of 100° F. into steam of 70 pounds gauge pressure. As marine boilers are not sold in the open market to any extent, but are built chiefly to order, the above items referring to commercial horsepower may be omitted.

Item 28 is found from rule **167.**

927. Different boilers generate steam at different pressures, and receive the feed-water at different temperatures. In order to compare the relative performances of boilers, it is necessary to reduce the actual evaporation to *an equivalent evaporation from and at 212° F. per pound of combustible;* that is, to find what the evaporation would have been if the fuel had been without ash, the feed-water at 212° F., and the steam delivered at 0 lb. gauge pressure (14.7 lb., absolute).

Let W = the actual evaporation;
H = total heat of steam above 32° at pressure of actual evaporation;
t = observed temperature of feed-water;
W_1 = equivalent evaporation from and at 212° F.

Rule 167.—*To reduce actual evaporation to equivalent evaporation, subtract the observed temperature of the feed-water from the total heat of 1 pound of steam above 32° at pressure of actual evaporation. Add 32 to the remainder and multiply the sum by the actual evaporation in pounds. Divide the product by 966.1.*

That is, $$W_1 = \frac{W(H - t + 32)}{966.1}.$$

Example.—A boiler generates 2,200 pounds of steam per hour at a pressure of 120 pounds; the temperature of the feed-water is 70° F. What is the equivalent evaporation?

Solution.—From the steam table, the total heat H, corresponding to a pressure of 120 lb., gauge, is 1,188.64 B. T. U. Hence, from rule **167**,

$$W_1 = \frac{2{,}200 \times (1{,}188.64 - 70 + 32)}{966.1} = 2{,}620 \text{ lb., nearly. Ans.}$$

928. Quality of Steam.—In making a boiler trial it is important to determine as closely as possible how much moisture, if any, the steam contains. Many boilers, especially when generating steam rapidly, furnish wet steam. The water is carried along with the steam in the form of spray or even in drops. Of course, this water is not evaporated, and if not taken account of, would show for the boiler a higher efficiency than it really possessed.

By the expression **quality** of steam is meant the percentage of the water *fed into* the boiler which is evaporated into pure dry steam. For example, suppose that for every 100 pounds of water fed to the boiler 98 pounds are changed to dry steam and 2 pounds are carried over in the form of water. Then, the quality of steam is $\frac{100 - 2}{100} = .98 = 98\%$, and the percentage of moisture is 2%.

929. It is a rather difficult matter to make a very exact determination of the moisture contained in steam. The apparatus or instrument used for this purpose is called a **calorimeter.** There are many more or less complicated calorimeters in use; about the simplest and most available one for general use is the so-called **barrel calorimeter.** (See Fig. 251.) A barrel or tank *T* holding 400 or 500 pounds of water is placed on a platform scale *S*, filled with water, and weighed. The temperature of the water is registered by a thermometer inserted in the side of the barrel. Steam from the boiler is led through a pipe or hose *U* into the barrel until the temperature of the water reaches 130° to 140° F. The steam is then turned off, and the barrel and contents are again weighed. The difference between this weight and the original weight is the weight of the steam led in from the boiler. The average steam pressure throughout the observation must be observed. It is well to have the tube bent as shown in the figure.

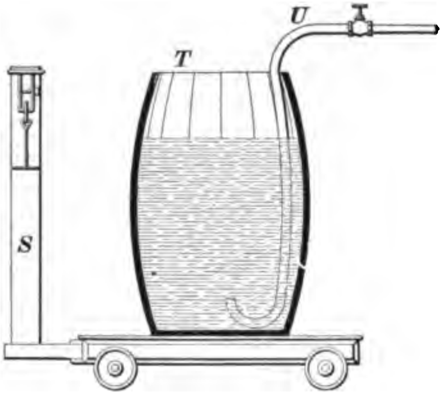


FIG. 251.

We know the weight of the cold water and the rise in temperature; we also know the weight of the steam or the mixed steam and water that is led in from the boiler. From the steam table, the temperature of the steam can be found, since the pressure is known.

Now, if dry steam comes through the pipe *U*, the condensation of this steam should raise the temperature of the cold water in the barrel a certain amount. If the temperture is not raised that much it must be because some of the mixture led into the barrel was water. Very rarely the temperature of the cold water might rise more than it would if the steam were dry, thus indicating that the steam is *superheated.*

Let W = weight of cold water in barrel;
w = weight of steam and water run into the barrel;
t = temperature of steam corresponding to the observed pressure;
t_1 = original temperature of the water W;
t_2 = temperature of the mixture after steam is condensed;
l = latent heat of a pound of steam at the observed pressure;
x = portion of weight w that is dry steam;
Q = quality of steam $= \frac{x}{w}$.

Rule 168.—*Multiply the rise in temperature of the cold water by the weight of cold water and divide the product by the weight of the mixture led in through the hose. Subtract the final temperature of the water from the temperature of the steam, and subtract the remainder thus obtained from the number first obtained. Divide this final remainder by the latent heat of a pound of steam at the observed pressure. The result will be the quality of the steam.*

That is, $Q = \frac{x}{w} = \frac{1}{l}\left[\frac{W}{w}(t_2 - t_1) - (t - t_2)\right]$.

EXAMPLE.—In a calorimetric test, the weight of cold water was 420 pounds; of steam condensed, 36 pounds. The initial temperature of cold water was 40° F., the final temperature was 130° F., and the steam pressure was 60 pounds. Find the quality of the steam.

SOLUTION.—Absolute pressure = 60 + 14.7 = 74.7 lb. per sq. in. Latent heat of steam at this pressure is (see steam table) 898.5. The temperature of steam at this pressure is 307.2°. Hence, by rule **168,**

$$Q = \frac{1}{l}\left[\frac{W}{w}(t_2 - t_1) - (t - t_2)\right] =$$

$$\frac{1}{898.5}\left[\frac{420}{36}(130 - 40) - (307.2 - 130)\right] = .9714 = 97.14\%;$$

that is, the boiler generates a mixture which is composed of 97.14 per cent. dry steam and 2.86 per cent. water. Ans.

The barrel calorimeter must be used very carefully in order to obtain accurate results. The operation should be repeated once or twice before the actual test is made, in order to warm up the barrel. The most important observation is that of the temperature. This should be taken by a thermometer graduated to fifths or tenths of a degree. The weights should be as accurate as possible. The chief merit of the barrel calorimeter is its availability. It can be rigged up in almost any situation.

930. To show the method of working up the data of a boiler trial, the following example is given:

EXAMPLE.—Given the following data observed during a boiler trial, it is required to make the necessary calculations for economic evaporation:

Duration of test	10 hr.
Average gauge pressure	72 lb.
Average temperature of feed-water	122° F.
Pounds of coal burned	15,232 lb.
Percentage of ash	4½%.
Water fed to boiler	124,600 lb.
Average quality of steam	97.2%.

CALCULATIONS.—Water evaporated = 124,600 × .972 = 121,111.2 lb.

Water evaporated per pound of coal — actual conditions = 121,111.2 ÷ 15,232 = 7.95 lb.

Water evaporated per pound of combustible =

$$121,111.2 \div \left[15,232 \times \frac{(100 - 4\tfrac{1}{2})}{100}\right] = 121,111.2 \div 14,546.6 = 8.33 \text{ lb.}$$

From rule **167,** the equivalent evaporation from and at 212° per pound of coal is

$$\frac{7.95 \times (1,178.765 - 122 + 32)}{966.1} = 8.96 \text{ lb.}$$

From the same rule, the equivalent evaporation from and at 212° per pound of combustible is

$$\frac{8.33 \times (1,178.765 - 122 + 32)}{966.1} = 9.38 \text{ lb.}$$

THE INJECTOR.

931. One form of the injector, shown in Fig. 127, Art. **606,** has already been described. Another form of injector largely used for feeding boilers is shown in Fig. 252. Steam enters from the boiler and passes through the connection *V* and nozzle *R* into the conical combining tube *S*. Here it meets the water, which enters through the pipe *P* (shown on the side in the cut, but really situated in the rear) and surrounds the nozzle *R*. At first the mingled steam and water flows through the valve *Q* and out of the overflow pipe *T*; but the condensation of the steam soon causes a partial vacuum in the upper chamber, and the atmospheric pressure on the top of the valve *Q* forces it to its seat and checks the overflow. The jet of water then flows steadily through the delivery tube *Y* and pipe *Z* into the boiler, as shown by the arrows. The injector is started by opening the globe valves in the steam and feed-water pipes. If the

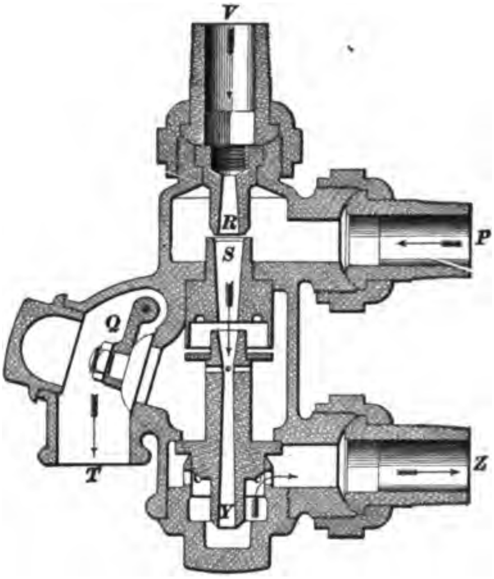


FIG. 252.

water supply is too great, water will appear at the overflow, and the feed valve must be closed a little or the valve in the steam pipe opened wider.

932. It may at first seem inconsistent that steam can flow from a boiler at say 80 pounds pressure and force water back into the boiler against the same pressure. It is, however, another example of heat changed into work. Each pound of steam entering the injector gives out about 900 B. T. U. (depending upon the pressure), and each of these B. T. U. is capable of developing 778 foot-pounds of work. Thus the steam jet possesses ample energy to force the water into the boiler. In fact, only a small per cent. of the energy is so expended, the larger part of it being utilized in heating the feed-water. Thus, while the injector is a wasteful machine, considered merely as a pump, it is a very economical boiler feeder, since the heating of the feed leaves so much less work for the boiler.

The capacity of an injector depends mainly upon the diameter of the delivery tube. Table 31 gives the capacities of the various sizes of the Penberthy injector just described, for pressures from 45 to 75 pounds, and lifts of from 1 to 6 feet.

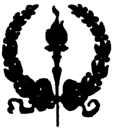
933. It is considered good practice to provide the boiler with an injector considerably larger than actually needed, for, by throttling the feed, the injector will deliver much less than its maximum capacity, but will heat the water nearly to boiling point. The injector thus acts as a feed-water heater, and effects an economy of fuel.

An injector will not work efficiently when the feed is at high temperature. For example, the Penberthy injector will not work without overflowing when the feed temperature is above about 120°, or when the temperature of the delivery is above about 200°.

Injectors are used almost exclusively on locomotives, and to a large extent on marine boilers, especially those of small dimensions.

TABLE 31.

Size.	Pipe Connection.			Capacity per Hour.	
	Steam.	Suction.	Delivery.	Maximum.	Minimum.
	In.	In.	In.	Gal.	Gal.
O O	3/8	3/8	3/8	80	55
A	1/2	1/2	1/2	120	70
A A	1/2	1/2	1/2	165	90
B	3/4	3/4	3/4	250	135
B B	3/4	3/4	3/4	340	165
C	1	1	1	475	300
C C	1	1	1	575	350
D	1 1/4	1 1/4	1 1/4	750	400
D D	1 1/4	1 1/4	1 1/4	920	500
E	1 1/2	1 1/2	1 1/2	1,300	700
E E	1 1/2	1 1/2	1 1/2	1,740	900
F	2	2	2	2,270	1,100
F F	2	2	2	2,800	1,400



A SERIES

OF

QUESTIONS AND EXAMPLES

RELATING TO THE SUBJECTS
TREATED OF IN THIS VOLUME.

It will be noticed that, although the various questions are numbered in sequence from **1** to **499,** inclusive, these questions are divided into seven different sections, corresponding to the seven sections of the preceding pages of this volume. No attempt should be made to answer any question or to solve any example occurring in one of the following sections until the corresponding Paper has been carefully studied.

ARITHMETIC.

EXAMINATION QUESTIONS.

(1) What is arithmetic?

(2) What is a number?

(3) What is the difference between a concrete number and an abstract number?

(4) Define notation and numeration.

(5) Write each of the following numbers in words: 980; 605; 28,284; 9,006,042; 850,317,002; 700,004.

(6) Represent in figures the following expressions: Seven thousand, six hundred. Eighty-one thousand, four hundred two. Five million, four thousand, seven. One hundred eight million, ten thousand, one. Eighteen million, six. Thirty thousand, ten.

(7) What is a fraction?

(8) What are the terms of a fraction?

(9) What does the denominator show?

(10) What does the numerator show?

(11) How do you find the value of a fraction?

(12) Is $\frac{13}{8}$ a proper or an improper fraction and why?

(13) Write three mixed numbers.

(14) Reduce the following fractions to their lowest terms: $\frac{4}{8}$, $\frac{4}{16}$, $\frac{8}{32}$, $\frac{32}{64}$. Ans. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{2}$.

(15) Reduce 6 to an improper fraction whose denominator is 4. Ans. $\frac{24}{4}$.

(16) Reduce $7\frac{7}{8}$, $13\frac{5}{16}$, and $10\frac{3}{4}$ to improper fractions. Ans. $\frac{63}{8}$, $\frac{213}{16}$, $\frac{43}{4}$.

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(17) What is the value of each of the following: $\frac{13}{2}$, $\frac{17}{4}$, $\frac{69}{16}$, $\frac{16}{8}$, $\frac{67}{64}$? Ans. $6\frac{1}{2}$, $4\frac{1}{4}$, $4\frac{5}{16}$, 2, $1\frac{3}{64}$.

(18) State the difference between a fraction and a decimal.

(19) State how to reduce a fraction to a decimal.

(20) Reduce the following fractions to equivalent decimals: $\frac{1}{2}$, $\frac{7}{8}$, $\frac{5}{32}$, $\frac{65}{100}$, and $\frac{125}{1000}$.

$$\text{Ans.}\begin{cases}.5.\\.875.\\.15625.\\.65.\\.125.\end{cases}$$

(21) Write out in words the following numbers: .08, .131, .0001, .000027, .0108, and 93.0101.

(22) How do you place decimals for addition and subtraction?

(23) Give a rule for multiplication of decimals.

(24) Give a rule for division of decimals.

(25) What is the sum of 3,290, 504, 865,403, 2,074, 81, and 7? Ans. 871,359.

(26) 709 + 8,304,725 + 391 + 100,302 + 300 + 909 = what? Ans. 8,407,336.

(27) $\frac{1}{8}+\frac{2}{8}+\frac{5}{8}=?$ Ans. 1.

(28) $\frac{1}{4}+\frac{3}{8}+\frac{5}{16}=?$ Ans. $\frac{15}{16}$.

(29) $42+31\frac{5}{8}+9\frac{7}{16}=?$ Ans. $83\frac{1}{16}$.

(30) An iron plate is divided into four sections; the first contains $29\frac{3}{4}$ square inches; the second, $50\frac{5}{8}$ square inches; the third, 41 square inches, and the fourth, $69\frac{3}{16}$ square inches. How many square inches are in the plate? Ans. $190\frac{9}{16}$ sq. in.

(31) What is the sum of .125, .7, .089, .4005, .9, and .000027? Ans. 2.214527.

(32) 927.416 + 8.274 + 372.6 + 62.07938 = ? Ans. 1,370.36938.

(33) Add 17 thousandths, 2 tenths, and 47 millionths. Ans. .217047.

(34) The weights of four lengths of shafting are 316.5, 402.3, $348\frac{3}{4}$, and 309.4 pounds, respectively. What is their total weight? Ans. 1,376.95 lb.

(35) The cost of the coal consumed under a nest of steam boilers during a week's run was as follows: On Monday, $15.83; Tuesday, $14.70; Wednesday, $14.28; Thursday, $13.87; Friday, $14.98; Saturday, $12.65. What was the cost of the week's supply of coal? Ans. $86.31.

(36) During a 12-hour test upon a steam engine the counter showed the number of revolutions per hour to have been as follows: 12,600, 12,444, 12,467, 12,528, 12,468, 12,590, 12,610, 12,589, 12,576, 12,558, 12,546, 12,532. How many revolutions were made during the test?

Ans. 150,508 revolutions.

(37) Find the difference between the following:

(*a*) 50,962 and 3,338; (*b*) 10,001 and 15,339.

$$\text{Ans.}\ \begin{cases} (a)\ 47{,}624. \\ (b)\ 5{,}338. \end{cases}$$

(38) (*a*) $70{,}968 - 32{,}975 = ?$ (*b*) $100{,}000 - 98{,}735 = ?$

$$\text{Ans.}\ \begin{cases} (a)\ 37{,}993. \\ (b)\ 1{,}265. \end{cases}$$

(39) What is the difference between $\frac{1}{2}$ and $\frac{7}{16}$, 13 and $7\frac{7}{16}$, $312\frac{9}{16}$ and $229\frac{5}{32}$? Ans. $\frac{1}{16}$, $5\frac{9}{16}$, $83\frac{13}{32}$.

(40) Work out the following examples:

(*a*) $709.63 - .8514$; (*b*) $81.963 - 1.7$; (*c*) $18 - .18$; (*d*) $1 - .001$; (*e*) $872.1 - (.8721 + .008)$; (*f*) $(5.028 + .0073) - (6.704 - 2.38)$.

$$\text{Ans.}\ \begin{cases} (a)\ 708.7786. \\ (b)\ 80.263. \\ (c)\ 17.82. \\ (d)\ .999. \\ (e)\ 871.2199. \\ (f)\ .7113. \end{cases}$$

(41) Work out the following:

(*a*) $\frac{7}{8} - .807$; (*b*) $.875 - \frac{3}{8}$; (*c*) $(\frac{5}{32} + .435) - (\frac{21}{100} - .07)$. (*d*) What is the difference between the sum of 33 millionths and 17 thousandths, and the sum of 53 hundredths and 274 thousandths?

Ans. (*a*) .068.
(*b*) .5.
(*c*) .45125.
(*d*) .786967.

(42) It is desired to increase the capacity of an electric-light plant to 1,500 horsepower by adding a new engine. If the indicated horsepower of the engines already in use is 482⅘, 316⅓, and 390¾, what power must the new engine develop? Ans. 310.11⅔ H. P.

(43) On a certain morning 7,240 gallons of water were drawn from an engine-room tank, and 4,780 gallons were pumped in. In the afternoon 7,633 gallons were drawn out, and 8,675 gallons pumped in. How many gallons remained in the tank at night, if it contained 3,040 gallons at the beginning of the day? Ans. 1,622 gal.

(44) A metal stack 45 feet high is made up of 7 plates, 6 of which are 7 feet long. Allowing 15 inches for laps, what is the length of the seventh plate? Ans. 4 ft. 3 in.

(45) The inside diameter of a 6-inch steam pipe is 6.06 inches, and the outside diameter is 6.62 inches. How thick is the pipe? Ans. .28 in.

(46) Find the products of the following:

(*a*) 526,387 × 7; (*b*) 700,298 × 17; (*c*) 217 × 103 × 67.

Ans. (*a*) 3,684,709.
(*b*) 11,905,066.
(*c*) 1,497,517.

(47) If your watch ticks once every second, how many times will it tick in one week? Ans. 604,800.

(48) Find the products of the following expressions:

(*a*) .013 × .107; (*b*) 203 × 2.03 × .203; (*c*) (2.7 × 31.85) × (3.16 − .316); (*d*) (107.8 + 6.541 − 31.96) × 1.742.

Ans. (*a*) .001391.
(*b*) 83.65427.
(*c*) 244.56978.
(*d*) 143.507702.

(49) An engine and boiler in a manufactory are worth \$3,246. The building is worth three times as much plus \$1,200, and the tools are worth twice as much as the building plus \$1,875; (*a*) what is the value of the building and tools? (*b*) What is the value of the whole plant?

Ans. { (*a*) \$34,689.
(*b*) \$37,935.

(50) How many square feet of heating surface are in the tubes of a boiler having 60 3-inch tubes, each 15½ feet long, if the heating surface of each tube per foot in length is .728 square foot? Ans. 677.04 sq. ft.

(51) Suppose that in one hour 10 pounds of coal are burned per square foot of grate area in a certain boiler, and that 9 pounds of water are evaporated per pound of coal burned. If the grate area is 30 square feet, how many pounds of water would be evaporated in a day of 10 hours?

Ans. 27,000 lb.

(52) How many feet does the piston of a steam engine pass over in a week of 6 days, running 8½ hours a day, if the length of the stroke of the engine is 1½ feet, and the number of revolutions per minute 160? Ans. 1,468,800 ft.

(53) A number of boilers are constantly fed by 3 pumps; the first delivers 1¼ gallons per stroke, and runs at 75 strokes per minute; the second delivers ⅞ of a gallon per stroke, and runs at 115 strokes per minute; the third delivers 1⅝ gallons per stroke, and runs at 96 strokes per minute. How many gallons of water are fed to the boilers per hour?

Ans. 21,022.5 gal.

NOTE.—If in the following examples there be a remainder, carry the quotient to four decimal places.

(54) Divide the following:

(*a*) 962,842 by 84; (*b*) 39,728 by 63; (*c*) 29,714 by 108; (*d*) 406,089 by 135.

Ans. { (*a*) 11,462.4048.
(*b*) 630.6032.
(*c*) 275.1296.
(*d*) 3,008.0667.

(55) Solve the following:

(*a*) $35 \div \frac{5}{16}$; (*b*) $\frac{9}{16} \div 3$; (*c*) $\frac{17}{2} \div 9$; (*d*) $\frac{113}{64} \div \frac{7}{16}$; (*e*) $15\frac{3}{4} \div 4\frac{3}{8}$.

Ans. (*a*) 112.
(*b*) $\frac{3}{16}$.
(*c*) $\frac{17}{18}$.
(*d*) $4\frac{1}{28}$.
(*e*) $3\frac{3}{5}$.

(56) Solve the following:

(*a*) $.875 \div \frac{1}{2}$; (*b*) $\frac{7}{8} \div .5$; (*c*) $\frac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125}$.

Ans. (*a*) 1.75.
(*b*) 1.75
(*c*) 5.

(57) Solve the following by cancelation:

(*a*) $(72 \times 48 \times 28 \times 5) \div (84 \times 15 \times 7 \times 6)$.

(*b*) $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20)$.

Ans. (*a*) $9\frac{1}{7}$.
(*b*) 32

(58) Find the values of the following expressions:

(*a*) $\frac{7}{\frac{3}{16}}$; (*b*) $\frac{\frac{15}{32}}{\frac{5}{8}}$; (*c*) $\frac{1.25 \times 20 \times 3}{\frac{87 + 88}{459 + 32}}$.

Ans. (*a*) $37\frac{1}{3}$.
(*b*) .75.
(*c*) $210\frac{3}{7}$.

(59) The distance around a cylindrical boiler is 166.85 inches. If there are 72 rivets in one of the circular seams, find what the pitch (distance between the centers of any two rivets) of the rivets is. Ans. 2.317 + in.

(60) A keg of $\frac{7}{8} \times 2\frac{3}{4}$ inches boiler rivets weighs 100 pounds, and contains 133 rivets. What is the weight of each rivet? Ans. .75 + lb.

(61) The distance around a wheel equals 3.1416 times its diameter, or the distance across it. If the distance around a fly-wheel is 56.5488 feet, what is the diameter of a wheel half as large? Ans. 9 ft.

(62) If 980,000 bricks are required to build an engine house, how many days will it take for 6 teams to draw them, each team drawing 8 loads a day, and there being 1,600 bricks to a load? Ans. 12.76 + days.

(63) If a mechanic earns $1,500 a year for his labor, and his expenses are $968 per year, in what time can he save enough to buy 28 acres of land at $133 an acre?
Ans. 7 years.

(64) The numerator of a fraction is 28, and the value of the fraction $\frac{7}{8}$; what is the denominator? Ans. 32.

(65) From 1 plus .001 take .01 plus .000001.
Ans. .990999.

(66) A freight train ran 365 miles in one week, and 3 times as far, lacking 246 miles, the next week; how far did it run the second week? Ans. 849 miles.

(67) If the driving wheel of a locomotive is 16 ft. in circumference, how many revolutions will it make in going from Philadelphia to Pittsburg, the distance of which is 354 miles, there being 5,280 feet in one mile?
Ans. 116,820 rev.

(68) How many inches in .875 of a foot? Ans. $10\frac{1}{2}$ in.

(69) What decimal part of a foot is $\frac{3}{16}$ of an inch?
Ans. .015625 ft.

(70) If water be conducted from a tank by 7 lengths of gas pipe coupled together, each length being 12 feet 6 inches long, $\frac{3}{4}$ of an inch being added at each of the joints for coupling together, how far from the tank is the water discharged? The first length screws into the tank $\frac{3}{4}$ of an inch. Ans. 87.8125 ft.

(71) If by selling a carload of coal for $82.50, at a profit of $1.65 per ton, I make enough to pay for 72.6 ft. of fencing at $.50 a foot, how many tons of coal were there in the car? Ans. 22 tons.

(72) The connection between an engine and boiler is made up of 6 lengths of pipe, three of which are 14 feet 5 inches long, two 12 feet 6 inches long, and one 8 feet 10 inches long. If the pipe weighs $10\frac{1}{2}$ pounds per foot, what is the total weight of the pipe used? Ans. 809.375 lb.

(73) Four bolts are required, $2\frac{1}{2}$, $6\frac{7}{8}$, $3\frac{1}{16}$, and 4 inches long. How long a piece of iron will be required from

which to cut them, allowing $\frac{7}{16}$ of an inch to each bolt for cutting off and finishing? Ans. $18\frac{3}{16}$.

(74) A double belt of a certain width can transmit $64\frac{1}{2}$ horsepower. How many horsepower can two single belts of the same width transmit, when running under the same conditions, supposing that the double belt is capable of transmitting $\frac{10}{7}$ as much power as one of the single belts?

Ans. 90.3 H. P.

(75) The lengths of belting required to connect four countershafts with the main line shaft were 18 feet 6 inches, 16 feet $9\frac{1}{2}$ inches, 22 feet 2 inches, and 20 feet $8\frac{1}{2}$ inches. How many feet of belting were required? Ans. $78\frac{1}{6}$ ft.

(76) In a steam-engine test of an hour's duration, the horsepower developed was found to be as follows, at 10-minute intervals: 48.63, 45.7, 46.32, 47.9, 48.74, 48.38, 48.59. What was the average horsepower?

Ans. 47.75 + H. P.

ARITHMETIC.

(CONTINUED.)

EXAMINATION QUESTIONS.

(77) What is 25% of 8,428 lb.? Ans. 2,107 lb.

(78) What is 1% of $100? Ans. $1.

(79) What is ½% of $35,000? Ans. $175.

(80) What per cent. of 50 is 2? Ans. 4%.

(81) What per cent. of 10 is 10? Ans. 100%.

(82) Solve the following:

(*a*) Base = $2,522, and percentage = $176.54. What is the rate?

(*b*) Percentage = 16.96, and rate = 8%. What is the base?

(*c*) Amount = 216.7025, and base = 213.5. What is the rate?

(*d*) Difference = 201.825, and base = 207. What is the rate?

Ans. (*a*) 7%.
(*b*) 212.
(*c*) 1½%.
(*d*) 2½%.

(83) The coal consumption of a steam plant is 5,500 lb. per day when the condenser is not running, or an increase of 15% over the consumption when the condenser is used. How many pounds are used per day when the condenser is running? Ans. 4,782.61 lb.

(84) An engineer receives a salary of $950. He pays 24% of it for board, 12½% of it for clothing, and 17% of it for other expenses. How much does he save a year? Ans. $441.75.

(85) If 37½% of a number is 961.38, what is the number? Ans. 2,563.68.

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(86) A man owns ¾ of a manufacturing plant. 30% of his share is worth $1,125. What is the whole property worth?
Ans. $5,000.

(87) What number diminished by 35% of itself equals 4,810? Ans. 7,400.

(88) The volume of the clearance in a steam-engine cylinder is found to be 18.3 cu. in., and the volume of the cylinder, neglecting the clearance, 254.5 cu. in. What percentage of the cylinder volume is the clearance?
Ans. 7.2%, nearly.

(89) The distance between two stations on a certain railroad is 16.5 miles, which is 12½% of the entire length of the road. What is the length of the road? Ans. 132 miles.

(90) The speed of an engine running unloaded was 1½% greater than when running loaded. If it made 298 revolutions per minute with the load, what was its speed running unloaded? Ans. 302.47 revolutions per minute.

(91) Reduce 4 yd. 2 ft. 10 in. to inches. Ans. 178 in.

(92) Reduce 3,722 in. to higher denominations.
Ans. 103 yd. 1 ft. 2 in.

(93) How many seconds in 5 weeks and 3.5 days?
Ans. 3,326,400 sec.

(94) Reduce 764,325 cu. in. to cu. yd.
Ans. 16 cu. yd. 10 cu. ft. 549 cu. in.

(95) How many gallons of water can be put into a tank holding 4 bbl. 10 gal. 3 qt.? Ans. 136¾ gal.

(96) A carload of coal weighed 16 T. 8 cwt. 75 lb. How many pounds did this amount to? Ans. 32,875 lb.

(97) Reduce 25,396 lb. to higher denominations.
Ans. 12 T. 13 cwt. 96 lb.

(98) Reduce 25,396 pt. to higher denominations.
Ans. 100 bbl. 24 gal. 2 qt.

(99) What is the sum of 2 yd. 2 ft. 3 in.; 4 yd. 1 ft. 9 in.; 2 ft. 7 in.? Ans. 8 yd. 7 in.

(100) What is the sum of 3 gal. 3 qt. 1 pt.; 6 gal. 1 pt.; 4 gal. 8 qt. 5 pt.? Ans. 16 gal. 2 qt. 1 pt.

(101) From 52 yd. 2 ft. 9 in. take 115 ft.
Ans. 14 yd. 1 ft. 9 in.

(102) From a barrel of machine oil is sold at one time 10 gal. 2 qt. 1 pt., and at another time 16 gal. 3 qt. How much remained? Ans. 4 gal. 1 pt.

(103) If 1 iron rail is 17 ft. 3 in. long, how long would 51 such rails be, if placed end to end? Ans. 879 ft. 9 in.

(104) Multiply 3 qt. 1 pt. by 4.7. Ans. 32.9 pt.

(105) How many iron rails, each 30 ft. long, are required to lay a single railroad track 23 miles long?
Ans. 8,096 rails.

(106) A main line shaft is composed of four lengths each 15 ft. 5 in. long, of one piece 14 ft. 8 in. long, and one piece 8 ft. 10 in. long. There are six hangers spaced equally distant apart, one being placed at each extremity of the shaft 8 in. from the ends. What is the distance between the hangers? Ans. 16 ft. 9$\frac{1}{5}$ in.

(107) The distance around a wheel is approximately $\frac{22}{7}$ times the diameter. If the diameter of a fly-wheel is 9 ft. 6½ in., find the distance around it. Ans. 29 ft. 11$\frac{6}{7}$ in.

(108) If the length of a boiler shell is 18 ft. 11¼ in., how many rivets should there be in one of the longitudinal seams if it is a single-riveted seam, supposing the rivets to have a pitch of 1¼ in., and the two end rivets to be 1⅛ in. from each end of the boiler? Ans. 181 rivets.

(109)
(*a*) What is the second power of 108?
(*b*) What is the third power of 181.25?
(*c*) What is the fourth power of 27.61?

Ans. (*a*) 11,664.
(*b*) 5,954,345.703125.
(*c*) 581,119.73780641.

(110) Solve the following:

(*a*) 106^2; (*b*) $(182\frac{1}{8})^2$; (*c*) $.005^2$; (*d*) $.0063^2$; (*e*) 10.06^2. (*f*) 67.85^2; (*g*) $967,845^2$; (*h*) $(\frac{7}{16})^2$; (*i*) $(\frac{1}{4})^2$.

Ans.
(*a*) 11,236.
(*b*) 33,169.515625.
(*c*) .000025.
(*d*) .00003969.
(*e*) 101.2036.
(*f*) 4,603.6225.
(*g*) 936,723,944,025.
(*h*) $\frac{49}{256}$.
(*i*) $\frac{1}{16}$.

(111) Solve the following:

(*a*) 753^3; (*b*) 987.4^3; (*c*) $.005^3$; (*d*) $.4044^3$; (*e*) $.0133^3$; (*f*) 301.011^3; (*g*) $(\frac{1}{8})^3$; (*h*) $(3\frac{3}{4})^3$.

Ans.
(*a*) 426,957,777.
(*b*) 962,674,279.624.
(*c*) .000000125.
(*d*) .066135317184.
(*e*) .000002352637.
(*f*) 27,273,890.942264331.
(*g*) $\frac{1}{512}$.
(*h*) 52.734375

(112) What is the fifth power of 2? Ans. 32.

(113) What is the fourth power of 3? Ans. 81.

(114) What is the ninth power of 7? Ans. 40,353,607.

(115) Solve the following:

(*a*) 1.2^4; (*b*) 11^5; (*c*) 1^8; (*d*) $.01^4$; (*e*) $.1^5$.

Ans.
(*a*) 2.0736.
(*b*) 161,051.
(*c*) 1.
(*d*) .00000001.
(*e*) .00001.

(116) In what respect does evolution differ from involution?

(117) **Find the square root of the following:**

(*a*) 3,486,784.401; (*b*) 9,000,099.4009; (*c*) .001225, (*d*) 10,795.21; (*e*) 73,008.05; (*f*) 9; (*g*) .9.

Ans.
(*a*) 1,867.29 +.
(*b*) 3,000.016.
(*c*) .035.
(*d*) 103.9.
(*e*) 270.2.
(*f*) 3.
(*g*) .948 +.

(118) **What is the cube root of the following:**

(*a*) .32768? (*b*) 74,088? (*c*) 92,416? (*d*) .373248? (*e*) 1,758.416743? (*f*) 1,191,016? (*g*) $\frac{4}{32}$? (*h*) $\frac{27}{256}$?

Ans.
(*a*) .689 +.
(*b*) 42.
(*c*) 45.2115 +.
(*d*) .72.
(*e*) 12.07.
(*f*) 106.
(*g*) .5.
(*h*) .472 +.

(119) Find the cube root of 2 to four decimal places.
Ans. 1.2599 +.

(120) Find the cube root of 3 to three decimal places.
Ans. 1.442 +.

(121) Solve the following:

(*a*) $\sqrt{123.21}$; (*b*) $\sqrt{114.921}$;
(*c*) $\sqrt{502,681}$; (*d*) $\sqrt{.00041209}$.

Ans.
(*a*) 11.1.
(*b*) 10.72 +
(*c*) 709.
(*d*) .0203.

(122) Solve the following:

(*a*) $\sqrt[3]{.0065}$; (*b*) $\sqrt[3]{.021}$; (*c*) $\sqrt[3]{8,036,054,027}$; (*d*) $\sqrt[3]{.000004096}$; (*e*) $\sqrt[3]{17}$.

$$\text{Ans} \begin{cases} (a)\ .186+. \\ (b)\ .27+. \\ (c)\ 2,003. \\ (d)\ .016. \\ (e)\ 2.57+. \end{cases}$$

(123) Extract the square root of:

(*a*) $\frac{1225}{5476}$.
(*b*) .3364.
(*c*) .1.
(*d*) $25.0\frac{3}{4}$.
(*e*) $.000\frac{4}{9}$.

$$\text{Ans.} \begin{cases} (a)\ \frac{35}{74}. \\ (b)\ .58. \\ (c)\ .31622+. \\ (d)\ 5.00749. \\ (e)\ .02108. \end{cases}$$

(124) Find the value of x in the following:

$11.7 : 13 :: 20 : x$. Ans. 22.22 +.

(125) (*a*) $20 + 7 : 10 + 8 :: 3 : x$.
(*b*) $12^2 : 100^2 :: 4 : x$.

$$\text{Ans.} \begin{cases} (a)\ 2. \\ (b)\ 277.7+. \end{cases}$$

(126) (*a*) $\frac{4}{x} = \frac{7}{21}$; (*b*) $\frac{x}{24} = \frac{8}{16}$; (*c*) $\frac{2}{10} = \frac{x}{100}$; (*d*) $\frac{15}{45} = \frac{60}{x}$; (*e*) $\frac{10}{150} = \frac{x}{600}$.

$$\text{Ans.} \begin{cases} (a)\ x = 12. \\ (b)\ x = 12. \\ (c)\ x = 20. \\ (d)\ x = 180. \\ (e)\ x = 40. \end{cases}$$

(127) $x : 5 :: 27 : 12.5$. Ans. $10\frac{4}{5}$.

(128) $45 : 60 :: x : 24$. Ans. 18.

(129) $x : 35 :: 4 : 7$. Ans. 20.

(130) $9 : x :: 6 : 24$. Ans. 36.

(131) $\sqrt[3]{1,000} : \sqrt[3]{1,331} :: 27 : x$. Ans. 29.7.

(132) $64 : 81 :: 21^2 : x^2$. Ans. 23.625.

(133) $7 + 8 : 7 :: 30 : x$. Ans. 14.

(134) If a piece of 2-inch shafting 3½ ft. long weighs 37.45 lb., how much would a piece 6¾ ft. long weigh?

Ans. 72.225 lb.

(135) The intensity of heat from a burning body varies inversely as the square of the distance from it. If a thermometer held 6 ft. from a stove rises 24 degrees, how many degrees will it rise if held 12 ft. from the stove? Ans. 6°.

(136) If sound travels at the rate of 6,160 ft. in 5½ sec., how far does it travel in 1 min.? Ans. 67,200 ft.

(137) If a railway train runs 444 mi. in 8 hr. 40 min., in what time can it run 1,060 mi. at the same rate of speed? Ans. 20 hr. 41.44 min.

(138) If a pump discharging 135 gal. per minute fills a tank in 38 minutes, how long would it take a pump discharging 85 gal. per minute to fill it? Ans. $60\frac{6}{17}$ min.

(139) If a quantity of babbitt metal contains 8 lb. of copper, 8 lb. of antimony, and 80 lb. of tin, how much copper will 32 lb. of the same metal contain? Ans. $2\frac{2}{3}$ lb.

(140) The distances around the drive wheels of two locomotives are 12.56 ft. and 15.7 ft., respectively. How many times will the larger turn while the smaller turns 520 times? Ans. 416 times.

(141) If a cistern 28 ft. long, 12 ft. wide, 10 ft. deep holds 798 bbl. of water, how many barrels of water will a cistern hold that is 20 ft. long, 17 ft. wide, and 6 ft. deep? Ans. 484½ bbl.

MENSURATION AND USE OF LETTERS IN FORMULAS.

EXAMINATION QUESTIONS.

$A = 5$ $\qquad h = 200$
$B = 10$ $\qquad x = 12$
$i = 3.5$ $\qquad D = 120$

Work out the solutions to the following formulas, using the above values for the letters:

(142) $C = \frac{D - x}{B + i}$. Ans. $C = 8$.

(143) $Q = \frac{A\,h + D}{2\,x + 6} + D$. Ans. $Q = 157\frac{1}{3}$.

(144) $r = \frac{3.246\,B\,h}{\frac{A\,x + h}{A\,i - B}}$. Ans. $r = 187.269 +$.

(145) $v = \sqrt{\frac{A\,D}{i\,B + 1.5}}$. Ans. $v = 4.05 +$.

(146) $u = \sqrt[3]{\frac{B\,x}{.00018\,h\,(A^2 - x)}}$. Ans. $u = 6.35 +$.

(147) $f = \frac{10\,(h - D)^2}{\sqrt[3]{D + A}}$. Ans. $f = 12{,}800$.

(148) $g = \frac{(B - A)^2 - \sqrt[3]{D + A}}{A^2 - (1 + D)}$. Ans. $g = 5$.

(149) $k = \sqrt{\frac{A\,B^2}{\sqrt[3]{A\,h}}}$. Ans. $k = 7.071 +$.

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(150) $$T = \sqrt{\frac{A^2\left[490 + \frac{(h\,x)^2}{D^2}\right]}{h + \frac{x}{D}(A^2 - B)^2}}.$$ Ans. $T = 10$.

(151) If one of the angles formed by one straight line meeting another straight line equals 152° 3′, what is the other angle equal to? Ans. 27° 57′.

(152) How many seconds are there in 140° 17′ 10″? Ans. 505,030″.

(153) Write a definition for a degree, as applied to the measurement of angles.

(154) (*a*) How many degrees are there in 240 minutes? (*b*) How many seconds? Ans. $\begin{cases} (a)\ 4°. \\ (b)\ 14{,}400″. \end{cases}$

(155) Draw an obtuse angle, a right angle, and an acute angle. State the name of each angle by using letters to designate them.

(156) Draw a rhombus and then draw a rectangle having the same area.

(157) Can a quadrilateral be formed with lines whose lengths are 20 inches, 9 inches, 4 inches, and 7 inches? Give reasons.

(158) A sheet of zinc measures 11½ inches by 2½ feet. How many square inches does it contain? Ans. 345 sq. in.

(159) If the zinc in the last example weighs 5¼ pounds, what is its weight per square foot? Ans. 2.19 lb.

(160) How many boards 16 feet long and 5 inches wide would be required to lay a floor measuring 15 × 24 feet? Ans. 54 boards.

(161) A lot of land is in the shape of a trapezoid. It is 16 rods long, 9 rods wide at the front, and 6 rods wide at the back. The front and back being parallel, what part of an acre does the lot contain? Ans. ¾ of an acre.

(162) The accompanying figure shows the floor plan of an electric-light station. From the dimensions given, calculate the number of square feet of unoccupied floor space.
Ans. 2,059.08 sq. ft.

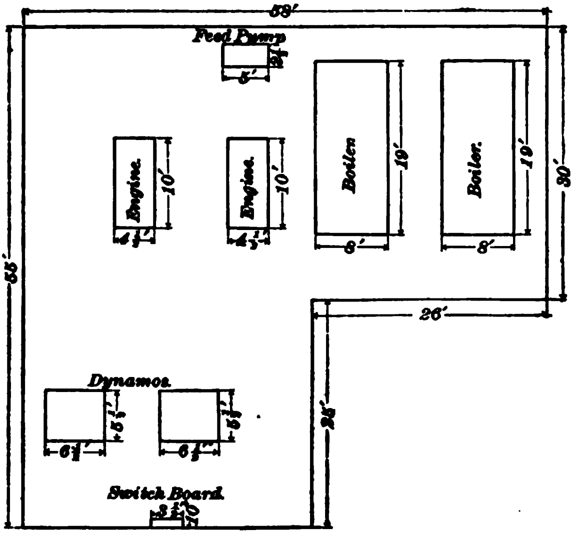

FIG. 1.

(163) A sidewalk 10 feet wide extends around a city block 528 feet long and 352 feet wide. Assuming that there is no space between the walk and the buildings, how many square yards does the walk contain? Ans. 2,000 sq. yd.

(164) How many square yards of plastering will be required for the four side walls of a hall 90 feet long, 50 feet wide, and 20 feet high, with four doors 5½ × 10 feet and fourteen windows 5 × 11 feet? There is a baseboard 9 inches high. Ans. 490.72 sq. yd.

(165) A triangle has three equal angles; what is it called?

(166) If a triangle has two equal angles, what kind of a triangle is it?

(167) Can a triangle be formed with three lines whose lengths are 12 inches, 7 inches, and 4 inches? Give reasons for your opinion.

(168) (*a*) What is the altitude of an equilateral triangle whose sides are each 6 feet? (*b*) What is its area?

Ans. { (*a*) 5.196 ft.
(*b*) 15.588 sq. ft.

(169) In a triangle ABC, angle $A = 23°$, and $B = 32° 32'$; what does angle C equal? Ans. $C = 124° 28'$.

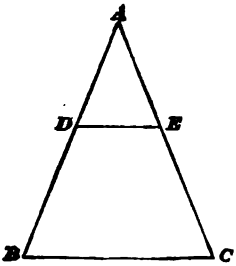

FIG. 2.

(170) In the figure, if $AD = 10$ inches, $AB = 24$ inches, and $BC = 13\frac{1}{2}$ inches, how long is DE, DE being parallel to BC? Ans. $DE = 5.625$ in.

(171) An engine room is 52 feet long and 39 feet wide. How many feet is it from one corner to a diagonally opposite one, measured in a straight line? Ans. 65 ft.

(172) A ladder 24 feet long rests against a house with its upper end 8 feet from the ground. How far on the ground is the lower end of the ladder from the house? Ans. 22 ft. 7½ + in.

(173) (*a*) Show why it is that the area of a triangle equals one-half the product of the base by the altitude. (*b*) Does it make any difference which side is taken as the base?

(174) (*a*) The area of an isosceles triangle is 200 square inches. If its altitude is 20 inches, how long is its base? (*b*) What is the length of one of its equal sides?

Ans. { (*a*) 20 in.
(*b*) 22.36 in.

(175) In an equilateral heptagon one of the sides equals 3 inches; what is the length of the perimeter? Ans. 21 in.

(176) The perimeter of a regular decagon is 40 inches; what is the length of the side? Ans. 4 in.

(177) What is one angle of a regular dodecagon equal to? Ans. 150°.

(178) The area of a regular pentagon is 43 square inches. If one side is 5 inches long, what is the perpendicular distance from the center to one side? Ans. 3.44 in.

(179) It is required to make a miter-box, in which to cut molding to fit around an octagon post. At what angle with the side of the box should the saw run? Ans. 67½°

(180) Calculate the area of the irregular polygon, Fig. 3. The dimensions are to be obtained by measuring. Ans. 1.78 + sq. in.

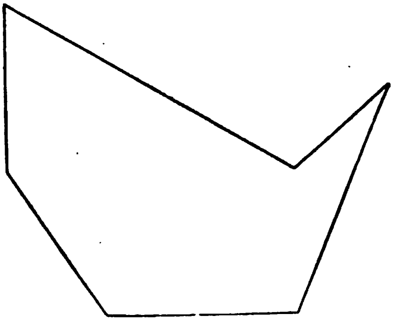
FIG. 3.

(181) An angle inscribed in a circle intercepts one-fourth the circumference. How many degrees are there in the angle? Ans. 45°.

(182) If the distance between two opposite corners of a hexagonal nut is two inches, what is the distance between two opposite sides? Ans. 1.732 + in.

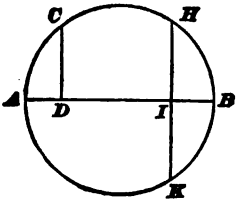

FIG. 4.

(183) In the accompanying figure, if the distance $B\,I$ is 6 inches and $H\,K$ 18 inches, what is the diameter of the circle? Ans. 19.5 in.

(184) In the same figure, if the diameter $A\ B = 32\frac{1}{2}$ feet, and the distance $I\ B = 8$ feet, what is the length of the chord $H\,K$? Ans. 28 ft.

(185) The trunk of a tree measures 7.854 feet around it; what is its diameter? Ans. 2½ ft.

(186) How many revolutions will a 72-inch locomotive driver make in going one mile? Ans. 280.112 revolutions.

(187) A pipe has an internal diameter of 6.06 inches; what is the area of a circle having this diameter? Ans. 28.8427 sq. in.

(188) The area of a circle having a diameter corresponding to the internal diameter of a certain pipe is 113.0976 square inches. What is the outside diameter, the pipe being ¾ inch thick? Ans. 13½ inches.

(189) How long must the arc of a circle be to contain 12°, supposing the radius of the circle to be 6 inches? Ans. 1.25664 in.

(190) Calculate the area of a flat circular ring whose outside diameter is 22 inches, and whose inside diameter is 21 inches. Ans. 33.7722 sq. in.

(191) Find the area of a segment of a circle whose diameter is 56 inches, the height of the segment being 5 inches. Ans. 108.5 sq. in

(192) What are the dimensions of the end of the largest square bar that can be planed from an iron bar 2 inches in diameter? Ans. 1.4142 in. square.

(193) What is the area of the sector of a circle 15 inches in diameter, the angle between the two radii forming the sector being 12½°? Ans. 6.1359 sq. in.

(194) (*a*) What would be the length of the side of a square metal plate having an area of 103.8691 square inches? (*b*) What would be the diameter of a round plate having this area? (*c*) How much shorter is the circumference of the round plate than the perimeter of the square plate?

$$\text{Ans.}\begin{cases}(a)\ 10.1916\text{ in.}\\(b)\ 11\tfrac{1}{2}\text{ in.}\\(c)\ 4.638\text{ in.}\end{cases}$$

(195) A plate 46 inches long is to be rolled into a shell; what will be the diameter of the shell if 5 inches are allowed for lap? Ans. 13.05 in.

(196) What is the convex area in square feet of a section of a smokestack 26 inches in diameter and 10½ feet long? Ans. 71.4714 sq. ft.

(197) Find the area in square feet of the entire surface of a hexagonal column 12 feet long, each edge of the ends of the column being 4 inches long Ans. 24.5774 sq. ft.

(198) Find the cubical contents of the above column in cubic inches. Ans. 5,985.9648 cu. in.

(199) Compute the weight per foot of an iron boiler tube 4 inches outside diameter and 3.73 inches inside diameter, the weight of the iron being taken at .28 pound per cubic inch. Ans. 5½ lb.

(200) The dimensions of a return-tubular boiler are as follows: Diameter, 60 inches; length between heads, 16 feet; outside diameter of tubes, 3½ inches; number of tubes, 64; distance of mean water-line from top of boiler, 18 inches. (*a*) Compute the steam space of the boiler in cubic feet. (*b*) Determine the number of gallons of water that will be required to fill the boiler up to the mean water level

Ans. { (*a*) 79.2 cu. ft.
(*b*) 1,246 gal., nearly.

(201) The cylinders of a compound engine are 19 and 31 inches in diameter, and the stroke is 24 inches; if the clearance at *each* end of the small cylinder is 14% of the *stroke*, and in the large cylinder 8% of the *stroke*, (*a*) what is the total volume in cubic feet of the steam in the small cylinder during one stroke? (*b*) In the large cylinder? (*c*) What is the ratio between the two?

Ans. { (*a*) 4.489 cu. ft.
(*b*) 11.321 cu. ft.
(*c*) Ratio = 2 522 : 1.

(202) Find the volume of a triangular pyramid 10 inches high; the base is an equilateral triangle, and one edge measures 10 inches. Ans. 144.336 cu. in.

(203) The slant height of a square pyramid is 25 inches, and one edge of its base is 16 inches. Find its altitude by the principle of the right-angled triangle. Ans. 23.6854 in.

(204) The length of the circumference of the base of a cone is 18.8496 inches, and its slant height is 10 inches. Find the area of the entire surface of the cone.

Ans. 122.5224 sq. in.

(205) If the altitude of the above cone were 9 inches, what would be its volume? Ans. 84.8232 cu. in.

(206) A square vat is 11 feet deep, 15 feet square at the top, and 12 feet square at the bottom. How many gallons will it hold? Ans. 15,058.29 gal.

(207) How many pails of water would be required to fill the vat, the pail having the following dimensions: Depth, 11 inches; diameter at the top, 12 inches; diameter at the bottom, 9 inches? Ans. 3,627.28.

(208) Required the area of the convex surface of the frustum of a hexagonal pyramid whose slant height is 32 inches, the perimeter of the lower base measuring 48 inches, and of the upper base 36 inches. Ans. 1,344 sq. in.

(209) It is desired to find the number of gallons that a certain tank will hold. By measuring with a tape measure, it is found to be 190 inches in circumference at the bottom and 170½ inches in circumference at the top. The depth of the tank is 7 feet, and the thickness of the sides 1½ inches. Make the calculation in round numbers. Ans. 861 gal.

NOTE.—To obtain the area of the upper and lower bases, first find the outside diameters and then deduct the thickness of the walls, or 2 × 1½ inches.

(210) Find (*a*) the area of the surface, and (*b*) the cubical contents of a ball 22½ inches in diameter.

Ans. { (*a*) 1,590.435 sq. in.
(*b*) 5,964.1313 cu. in.

(211) What is the volume of a ball whose surface has an area of 201.0624 square inches? Ans. 268.0832 cu. in.

(212) (*a*) What is the volume and area of a cylindrical ring whose outside diameter is 16 inches and inside diameter 13 inches? (*b*) If made of cast iron, what is its weight? Take the weight of 1 cubic inch of cast iron as .261 pound.
Ans. Weight = 21 lb.

(213) The volume of a certain cylindrical ring is 144.349 cu. in., its length before bending (see line *D* in Fig. 61, Art. **432**) was 20.42 inches; what is the area of its surface?
Ans. 192.45 sq. in.

MECHANICS.

EXAMINATION QUESTIONS.

(214) (*a*) What is a molecule? (*b*) An atom?

(215) If a body has an average velocity of 40 feet per second, how far will it travel in 14 minutes? Ans. $6\frac{4}{11}$ miles.

(216) Show how to represent a force by a line.

(217) In Fig. 78, Art. **491**, let the distance $F\,c$ be 21″, and $F\,b$ $3\frac{1}{2}$″; what weight will a force of 85 lb. applied at P raise? Ans. 510 lb.

(218) What must be the speed of the driver pulley in order that the driven may make 80 R. P. M. and be 28″ in diameter, the diameter of the driver being 21″?

Ans. $106\frac{2}{3}$ R. P. M.

(219) The number of teeth in a spur gear is 50 and the pitch is $1\frac{1}{2}$″; (*a*) what is the pitch diameter? (*b*) What is the outside diameter?

Ans. $\begin{cases} (a)\ 23.87''. \\ (b)\ 24.77''. \end{cases}$

(220) The driving gear has 45 teeth and the driven 180; if the driver makes 212 R. P. M., how many will the driven make? Ans. 53 R. P. M.

(221) What pressure can be exerted by a force of 24 lb. on a half-inch screw which has 13 threads per inch, the distance from the center of the screw to the point on the handle where the force is applied being 11″?

Ans. 21,563.94 lb.

(222) A ball weighing 5 lb. revolves in a circle whose radius is 32″ at the rate of 350 R. P. M.; what is the pull on the support caused by the ball? Ans. $555\frac{1}{2}$ lb.

(223) A body weighing 2 lb. has a velocity of 600 ft. per sec.; what is its kinetic energy? Ans. 11,194 ft.-lb.

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(224) What should be the width of a double leather belt to transmit 150 horsepower, when the belt has a velocity of 3,000 feet per minute, and has 7 feet of its length in contact with the smaller pulley, whose diameter is 63″? Give width to nearest ⅛″. Ans. 29.5″.

(225) (*a*) What are the three states of matter? (*b*) Name some of the general properties of matter; (*c*) some of the specific properties.

(226) If a man could run a mile at the average rate of 100 yards in 12 seconds, how long would it take him?
Ans. 3 min. 31.2 sec.

(227) What is meant by "center of gravity"?

(228) (*a*) Why is crowning usually given to the face of a pulley? (*b*) Why should high-speed pulleys be balanced?

(229) At what speed must the engine run when the diameter of the band-wheel is 13 feet and of the main pulley 91″, if the speed of the main shaft is to be 108 R. P. M.?
Ans. 63 R. P. M.

(230) The pitch of a gear is 2½″, and the number of teeth is 192; what is the pitch diameter? Ans. 152.79″.

(231) How many revolutions per minute must the driving gear make if it has 18 teeth, and the driven has 81 teeth and makes 80 R. P. M.? Ans. 360 R. P. M.

(232) The nuts on a cylinder head are tightened by means of a wrench 26″ long. The threads in the nuts are 8 to the inch, and the efficiency of the screw is 40%. What pressure will the nut exert against the head when a force of 60 lb. is applied to the end of the wrench? Ans. 31,365.7 lb.

(233) What do you understand by specific gravity?

(234) If the center of gravity of a section of an engine fly-wheel rim is 6 ft. 1¾ in. from the center of the shaft, and the weight of the rim is 13,000 pounds, what is its kinetic energy when making 150 R. P. M.? Ans. 1,883,661.7 ft.-lb.

(235) What should be the width of a single leather belt to transmit 2½ horsepower when the belt has a velocity of

2,000 feet per minute? The diameter of the smaller pulley is 14″, and the belt has 18″ of its length in contact with it.
Ans. 1⅛″.

(236) (*a*) What is meant by inertia? (*b*) By weight? (*c*) How is weight measured?

(237) The speed of a certain belt is 3,000 ft. per min.; if it drives a 48″ pulley, how long will it take the pulley to make 100 revolutions? Ans. 25.13 sec., nearly.

(238) Find the point of suspension of a rectangular cast-iron lever 4 ft. 6 in. long, 2 in. deep, and ¾ in. thick, having weights 47 and 71 pounds hung from each end, in order that there may be equilibrium. Take the weight of a cubic inch of cast iron as .261 lb.

Ans. $\left\{\begin{array}{l}\text{Short arm} = 22.342''.\\ \text{Long arm} = 31.658''.\end{array}\right.$

(239) When two pulleys are used to transmit power, which is called the driven and which the driver?

(240) The driver is 2 feet in diameter and the driven 32″; if the driven makes 63 R. P. M., how many must the driver make? Ans. 84 R. P. M.

(241) A certain gear has a pitch of 1⅛″, and its pitch diameter is 11.48″; how many teeth are in the gear?
Ans. 32 teeth.

(242) A fly-ball governor is designed to run at 88 R.P.M. The speed of the engine is 200 R. P. M. The diameter of the governor pulley is 8″; the number of teeth in the bevel gear which it turns, 44, and the number of teeth in the other bevel gear, 75; what must be the diameter of the pulley on the crank-shaft which drives the governor belt? Ans. 6″.

(243) A bookbinder has a press, the screw of which has 4 threads to the inch. It is worked by a lever 15″ long, to which is applied a force of 25 lb.; (*a*) what will be the pressure if the loss by friction is 5,000 lb.? (*b*) What would be the theoretical pressure?

Ans. $\left\{\begin{array}{l}(a)\ 4{,}424.8\text{ lb.}\\ (b)\ 9{,}424.8\text{ lb.}\end{array}\right.$

(244) A cubic foot of a certain kind of wood weighs 51 lb.; what is its specific gravity? Ans. .816.

(245) The piston of an engine weighs 325 lb., including the piston rod; what is its kinetic energy when moving at the rate of 660 ft. per min.? Ans. 611.4 ft.-lb., nearly.

(246) What horsepower can be safely transmitted by a gear whose pitch is 1″, a point on the pitch circle having a velocity of 1,200 ft. per min.? Ans. 12 H. P.

(247) (*a*) What is motion? (*b*) Velocity? (*c*) Rest? (*d*) Can a body be in motion with respect to one object and at rest with respect to another? Explain fully.

(248) (*a*) What is force? (*b*) Name several kinds of forces.

(249) Find by measurement the center of gravity of a triangle whose sides are 4″, 5″, and 6″ long.

Ans. $1\frac{3}{32}$″ from 6″ side.

(250) The driving pulley makes 40 R. P. M.; the driven makes 60 R. P. M., and is 36″ in diameter. What is the diameter of the driver? Ans. 54″.

(251) The diameter of the band-wheel of an engine is 12 ft.; of the main pulley, 8 ft.; of a driving pulley on the main shaft, 20″; of the driven pulley on the countershaft, 6″; of the driver on the countershaft, 6″, and of the driven on an emery-wheel spindle, 4″; if the engine makes 80 R. P. M., (*a*) what is the speed of the main shaft? (*b*) Of the countershaft? (*c*) Of the emery wheel?

Ans. { (*a*) 120 R. P. M.
(*b*) 400 R. P. M.
(*c*) 600 R. P. M.

(252) The pitch diameter of a gear is 34.15″; if the pitch is $1\frac{3}{8}$″, how many teeth has the gear? Ans. 78 teeth.

(253) In order to raise a weight, a combination of a fixed and movable pulley is used; if a force of 225 lb. be applied to the free end of the rope what load will it raise?

Ans. 450 lb.

(254) If the power moves through a distance of 5 ft. 6 in. while the weight is moving 6 in., (*a*) what is the velocity

ratio of the machine? (*b*) What weight would a force of 5 lb. applied to the power arm raise? Ans. $\begin{cases} (a)\ 11. \\ (b)\ 55\text{ lb.} \end{cases}$

(255) In the last example, if the efficiency were 65%, what weight could the machine raise? Ans. 35.75 lb.

(256) How many cubic inches of platinum will it take to weigh 10 lb.? Ans. 12.86 cu. in., nearly.

(257) (*a*) For what are belts used? (*b*) What is a single belt? (*c*) A double belt?

(258) What horsepower can be safely transmitted by a gear whose pitch is 1.57″, pitch diameter 30″, and which makes 100 revolutions per minute? Ans. 19.36 H. P.

(259) (*a*) What is uniform motion? (*b*) What is variable motion? (*c*) If a body moves 10 feet the first second, 12 feet the second second, 15 feet the third second, etc., is its motion uniform or variable, and why?

(260) What three conditions are required to be known in order to compare forces?

(261) A steel rod $\frac{7}{8}$″ in diameter has on one end a cast-iron spherical ball 5″ in diameter; if the length of the rod is 40″ between the ball and the end, where is the center of gravity?

SUGGESTION.—First calculate the weights of the ball and rod by the aid of the specific gravity tables.

Ans. 6.427″ from the center of the ball.

(262) The driving pulley makes 240 R. P. M., and the driven pulley 180 R. P. M.; if the diameter of the driven pulley is 30″, what is the diameter of the driver? Ans. $22\frac{1}{2}$″.

(263) In a train of gears used to raise a weight of 6,000 lb. in a manner similar to that shown in Fig. 83, Art. **500**, the diameters of the drivers and belt pulley are 18″, 12″, 15″, and 12″, and of the pinions and drum 6″, 5″, 8″, and 3″; what force must be applied to the belt to raise the weight, if 20% of the total force is lost through friction?

Ans. $138\frac{8}{9}$ lb.

(264) The pitch diameter of a gear is 24.16″, and the number of teeth is 38; what is the pitch? Ans. 1.997″.

(265) It is required to raise a load of 1,890 lb. by means of a block and tackle which has four fixed and four movable pulleys; what force is required to be applied to the free end of the rope? Ans. $236\frac{1}{4}$ lb.

(266) In a block and tackle, the theoretical power necessary to raise a weight of 1,000 lb. is 50 lb.; (*a*) what is the velocity ratio? (*b*) If the actual power necessary to raise the load is 95 lb., what is the efficiency?

$$\text{Ans.} \begin{cases} (a)\ 20. \\ (b)\ 52.63\%. \end{cases}$$

(267) A piece of lead is $\frac{1}{2}''$ in diameter and $10''$ long; how much does it weigh? Ans. 12.91 oz.

(268) If the distance between the centers of the crank-shaft and main shaft is 38 ft., and the diameters of the band-wheel and main pulley are 11 ft. and 7 ft., respectively, what must be the length of the main belt? Ans. 105 ft. $3''$.

(269) The entire solar system is moving through space at the rate of 18 miles per second; (*a*) what is its velocity in miles per hour? (*b*) How far will it go in one day?

$$\text{Ans.} \begin{cases} (a)\ 64{,}800 \text{ mi. per hr.} \\ (b)\ 1{,}555{,}200 \text{ mi.} \end{cases}$$

(270) (*a*) What is a path of a body? (*b*) What line do we measure when we wish to find the distance a body has traveled.

(271) (*a*) How are forces measured? (*b*) What kind of an effect do forces always tend to produce?

(272) It is required to raise a weight of 1,500 lb. by means of a lever like that shown in Fig. 71, Art. **484.** The length of the lever is 4 ft., and the distance from the fulcrum to the weight is $4''$; what force will it be necessary to apply? Ans. $136\frac{4}{11}$ lb.

(273) Had the lever in the above example been like that shown in Fig. 72, Art. **484,** what force would have been required? Ans. 125 lb.

(274) The band-wheel of an engine is 10 feet in diameter; what must be the diameter of the pulley on the main shaft in order to make 110 R. P. M., if the band-wheel makes 88 R. P. M? Ans. 8 ft.

(275) (*a*) What is a spur gear? (*b*) A miter gear? (*c*) A bevel gear?

(276) The pitch diameter of a gear is 36.56". and the number of teeth is 42; what is the pitch? Ans. 2.735".

(277) The length of an inclined plane is 400 feet, and the height is 45 feet; what force acting parallel to the plane will be required to pull up the plane a weight of 4,000 lb.?

Ans. 450 lb.

(278) (*a*) What is meant by the velocity ratio of a machine? (*b*) By the efficiency?

(279) If a coil of brass wire weighs 10 lb., and the diameter of the wire is $\frac{1}{16}$", how long is the wire?

Ans. 896 ft., nearly.

(280) The diameters of two pulleys are 14" and 18", and the distance between their centers is 14 feet; what must be the length of a belt to drive these pulleys? Ans. 32 ft. 4".

(281) A velocity of 30 miles per hour corresponds to how many feet per second? Ans. 44 ft. per sec.

(282) The stroke of a steam engine is 28", and it makes 1,500 strokes in 6 minutes; what is the velocity of the piston in feet per second? Ans. $9\frac{13}{18}$ ft. per sec.

(283) State the three relations between force and motion.

(284) A pulley on the main shaft is 40" in diameter, and makes 120 R. P. M.; what must be the diameter of a pulley on the countershaft that is to make 160 R. P. M.?

Ans. 30".

(285) (*a*) What is a rack? (*b*) A worm-wheel? (*c*) A worm?

(286) (*a*) What distinguishes the epicycloidal teeth from the involute teeth? (*b*) Name two advantages which the latter possess over the former.

(287) An inclined plane has a length of 1,200 feet and a height of 125 feet. It is required to pull a load of 50,000 lb. up this plane. A block and tackle having 6 fixed and 6 movable pulleys is stationed at the top of the plane, and the weight end of the rope is attached to the load. If the rope which connects the block to the load is parallel to the plane,

what force will it be necessary to exert on the free end of the rope to pull up the load, no allowance being made for friction? Ans. 434 lb.

(288) (*a*) What do you understand by centrifugal force? (*b*) By centripetal force?

(289) Define (*a*) work; (*b*) horsepower; (*c*) kinetic energy; (*d*) potential energy.

(290) Two pulleys have diameters of 8″ and 20″; the distance between their centers being 19 ft. 3″, what must be the length of a belt to drive them? Ans. 42 ft. 3½″.

(291) Two bodies starting from the same point, move in opposite directions, one at the rate of 11 feet per second, and the other, 15 miles per hour; (*a*) what will be the distance between them at the end of 8 minutes; (*b*) How long before they will be 825 feet apart?
Ans. { (*a*) 3 miles.
(*b*) 25 seconds.

(292) A railroad train runs 2 miles in 2 minutes and 10 seconds; what is its average velocity in feet per second? Ans. 81.23 ft. per sec.

(293) Why is it difficult to jump from a rowboat into the water?

(294) A compound lever, similar to the one shown in Fig. 77, Art. **490,** is required to lift a weight of 1,250 lb. The lengths of the power arms PF are 30″, 20″, 10″, and 15″, of the weight arms WF 6″, 5″, 4″, and 7″; what force will be required? Ans. 11⅔ lb.

(295) The driving pulley is 20″ in diameter and driven 16″; if the driver makes 150 R. P. M., how many will the driven make? Ans. 187½ R. P. M.

(296) How is the diameter of a gear measured?

(297) The driving gear makes 100 and the driven, 40 R. P. M.; if the driven has 60 teeth, how many has the driver? Ans. 24.

(298) The base of an inclined plane is 80 feet long, and the height of the plane is 50 feet; what force exerted parallel to the base will raise a load of 750 lb.? Ans. 468¾ lb.

(299) What is the centrifugal force of the counterweight of a steam engine, the counterweight weighing 128 lb., and its center of gravity being 8¾″ from the center of the shaft? The crank makes 180 R. P. M. Ans. 1,028.16 lb.

(300) How much work can be done by 20 cubic feet of water falling from a height of 50 feet? Ans. 62,500 ft.-lb.

(301) What horsepower can be transmitted by a single leather belt 5″ wide, which runs at the rate of 1,960 ft. per min.? The diameter of the smaller pulley is 15″ and the length of the arc of contact is 21″? Ans. 11.4 H. P., nearly.

(302) It is required to raise a weight of 18,000 lb. by means of a screw having 3 threads per inch; if the length of the handle is 15″, and there is a loss of 10,000 lb. due to friction, etc., what force will it be necessary to apply to the handle? Ans. 99 lb., nearly.

(303) The fly-wheel of an engine is 9 feet in diameter (outside); if the fly-wheel makes 100 R. P. M., how many miles will a point on the rim travel in 1½ hours? Ans. 40.16½ miles.

(304) Suppose that an air gun can throw a ball with a velocity of 100 feet per second, and that a man standing on a railroad train, which is moving at the rate of 100 feet per second, were to fire the gun in a direction exactly opposite to that in which the train is moving, what would become of the ball? Why?

(305) If the distance between the center line of the handle and the axis of the drum shown in Fig. 79, Art. **491**, is 14½″, and the diameter of the drum is 5″, what load will a force of 30 lb. exerted on the handle raise? Ans. 174 lb.

(306) A pulley on the main shaft is 42″ in diameter, and makes 108 R. P. M.; what will be the speed of the countershaft if the driven pulley is 36″ in diameter? Ans. 126 R. P. M.

(307) What is (*a*) the pitch circle? (*b*) The pitch of a gear?

(308) The driving gear makes 360 R. P. M., and the driven 170 R. P. M.; if the driver has 34 teeth, how many has the driven? Ans. 72 teeth.

(309) Name some particular use in the engine room or shop to which you have seen the inclined plane put.

(310) The mean diameter of the rim of an engine cast-iron fly-wheel is 9 ft. 10¾", its width is 22", and its thickness 2½"; what is the centrifugal force when running at 210 R. P. M.? Ans. 397,450 lb.

(311) Assuming the average pressure upon the piston of a steam engine to be 41.38 pounds per square inch, what is the horsepower? The diameter of the piston is 10"; the stroke 16", and the number of strokes per minute 450.

Ans. 59.091 H. P., nearly.

(312) What horsepower can be transmitted by a 20" double leather belt which has a velocity of 2,800 ft. per min.? The diameter of the smaller pulley is 4 ft., and the belt has 5 ft. 9 in. of its length in contact with it. Ans. 99.4 H. P.

MECHANICS.

(CONTINUED.)

EXAMINATION QUESTIONS.

(313) State Pascal's law.

(314) A cylinder fitted with a piston is used as a lifting cylinder by passing a rope over a pulley and fastening one end to the piston rod. The piston is moved by means of water obtained from the city reservoir, and a gauge attached to a pipe near the cylinder shows the pressure to be 90 lb. per sq. in. The diameter of the cylinder is 19 in. and of the pipe ½ in. If friction be neglected, how great a weight can be raised? Ans. 25,517.6 lb.

(315) Give Mariotte's law.

(316) What do you understand by (*a*) the tensile strength of a material? (*b*) The working stress?

(317) A close-link wrought-iron chain is made from ⅜" iron; what is the greatest safe load that it will carry? Ans. 1,687.5 lb.

(318) What is the allowable working load for a steel-wire rope 5¼" in circumference? Ans. 27,562.5 lb.

(319) What steady force is required to shear a steel crank-pin which is 6" in diameter? Ans. 1,696,464 lb.

(320) What must be the diameter of a wrought-iron shaft to transmit 40 horsepower when running at 116 revolutions per minute, no power being taken off between bearings. Ans. 2.89". A shaft 2 15/16" in diameter would be used, that is, 3" round iron turned down.

(321) In Fig. 104, Art. **562**, suppose that the area of the piston *c* is .6 sq. in.; of *d*, 3 sq. in.; of *e*, 6 sq. in.; of *f*, 2 sq. in.; of *a*, 14 sq. in., and of *b*, 9 sq. in.; if a force of

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24 lb. be applied to *c*, what must be the forces applied to the other pistons to counterbalance the force at *c*, neglecting the weight of the water and the weight of the pistons?

Ans.
At *d*, 120 lb.
At *e*, 240 lb.
At *f*, 80 lb.
At *a*, 560 lb.
At *b*, 360 lb.

(322) The upper base of a cylinder submerged in water is 40 ft. below the surface. The diameter of the cylinder is 20 inches, the altitude 36 inches, and the bases are parallel. If the bases are horizontal, what is (*a*) the upward pressure of the water on the cylinder? (*b*) The downward pressure?

Ans.
(*a*) 5,863.39 lb.
(*b*) 5,454.32 lb.

(323) (*a*) What is a barometer? (*b*) What is the essential difference between a mercurial and an aneroid barometer?

(324) The smallest section of a connecting-rod is 3.5 sq. in.; what is the unit stress when subjected to a tensile stress of 12,400 lb.? Ans. 3,543 lb. per sq. in., nearly.

(325) What load may be safely carried by a hemp rope 4″ in circumference? Ans. 1,600 lb.

(326) What load can be safely sustained by a round wooden pillar, 8″ in diameter and 10 ft. long, having both ends flat? Ans. 13⅓ tons.

(327) What force is required to punch a 1″ hole through a wrought-iron plate $\frac{7}{16}$″ thick? Ans. 54,978 lb.

(328) What horsepower will a 1⅞″ wrought-iron shaft transmit when running at 180 revolutions per minute, an average amount of power being taken off between the bearings? Ans. 12.49 H. P.

(329) Does the shape of the vessel have any effect in regard to the pressure exerted by a liquid upon its bottom?

(330) In Fig. 111, Art. **576**, suppose that the diameter of the piston *a* is 2″, and of *b* 7½″; if a weight of 400 lb. be

laid upon *b*, what weight must be applied to *a* to balance the weight on *b*? Ans. 28.44 + lb.

(331) A vessel contains 42 cu. ft. of coal gas having a tension of 20¼ lb. per sq. in.; what will be the new tension when allowed to communicate with a perfectly empty vessel whose volume is 14 cu. ft.? Ans. 15.19 lb. per sq. in., nearly.

(332) What safe steady load can be sustained by a 1½" round wrought-iron bar, the load producing a tensile stress? Ans. 21,205.2 lb.

(333) What load can a hemp rope 6" in circumference carry with safety? Ans. 3,600 lb.

(334) What load will a hollow cast-iron pillar support with safety if the pillar is 20 ft. long, outside diameter 14", inside diameter 11½", and both ends are fixed? Ans. 219.24 tons.

(335) What force is required to punch a hole 1½" in diameter through a ¾" steel plate? Ans. 212,058 lb.

(336) A vessel is filled with water to the depth of 18"; if the area of the bottom is 46 sq. in., what is the total pressure on the bottom? Ans. 29.95 lb.

(337) Why does water seek its level?

(338) The volume of steam in an engine cylinder is ⅝ cu. ft. at cut-off, and its pressure is 94.7 lb. per sq. in.; what will be its pressure when the volume has increased to 2½ cu. ft.? Ans. 23.675 lb. per sq. in.

(339) A bar of steel having a cross-section of 1¾" × 3" is subjected to a tensile stress; if the stress is suddenly applied, what is the greatest load that it will safely carry? Ans. 31,500 lb.

(340) A load of 2,400 lb. is to be raised by means of a hemp rope; what should be the circumference of the rope? Ans. 4.9".

(341) Regarding the piston rod of a steam engine as a pillar which has one end flat and the other round, what should be the greatest diameter of the piston if the rod is 4 ft. 8 in. long, 3¼ in. in diameter, and the greatest steam

pressure is not to exceed 100 lb. per sq. in.? The rod is made of wrought iron. Ans. 25″, nearly.

(342) (*a*) What is cold-rolled shafting? (*b*) Bright shafting? (*c*) Black shafting?

(343) A tank contains cylinder oil having a specific gravity of say .92. This tank is connected to a four-gallon can by means of a pipe whose internal diameter is ⅝″. If the vertical distance between the level of the oil in the tank and the end of the pipe is 8 ft. 7 in., what is the pressure per square inch at the end of the pipe?

Ans. 3.427 lb. per sq. in.

(344) In Fig. 113 (see examples following Art. **578**), suppose that the diameter of the plunger *A* is 12″, and of the plunger *C* of the pump *B* ⅝″; if a force of 160 lb. be applied to *C* by means of the lever *D*, how great a weight can the plunger *A* raise if the weight of *A* is 600 lb.?

Ans. 58,382.4 lb.

(345) The volume of steam in an engine cylinder at the beginning of compression is 1.11 cu. ft., and its pressure is 18 lb. per sq. in. At the end of compression the volume is .3 cu. ft.; what is the final pressure?

Ans. 66.6 lb. per sq. in.

(346) What should be the least area of one of the 14 wrought-iron cylinder head stud bolts if the diameter of the cylinder is 19″, and the greatest steam pressure is 180 lb. per sq. in.? Assume that the studs are subjected to shocks.

Ans. .729 sq. in.

(347) What should be the circumference of a hemp rope to safely sustain a load of 4,200 lb.? Ans. 6½″.

(348) Regarding the connecting-rod of a steam engine as a pillar with two round ends, what is the greatest force that may be exerted on the cross-head if the connecting-rod is made of wrought iron, is 10 ft. long, and has a rectangular cross-section 6″ by 2½″? Ans. 35,489 lb.

(349) If you were to order some 2½″ bright-turned shafting, what size would you expect to get?

(350) A tank having the shape of a frustum of a cone is filled with water. If the diameter of the large end is 8 ft., of the small end 6 ft., and the perpendicular distance between the two ends is 10″, (*a*) what is the pressure on the bottom when the large end is down? (*b*) When the small end is down?

Ans. { (*a*) 2,618 lb.
(*b*) 1,473 lb., nearly.

(351) What is the total pressure on all of the six sides of a cube which has been sunk in the water until its top is 50 ft. below the surface? One edge of the cube measures 2 ft., and its upper base is parallel with the water level.

Ans. 76,502 lb.

(352) A vessel contains 25 cu. ft. of air having a pressure of 45 lb. per sq. in. When allowed to communicate with a second vessel which is entirely empty, the pressure falls to 15 lb. per sq. in. What is the volume of the second vessel? Ans. 50 cu. ft.

(353) A steam cylinder is 44″ in diameter and sustains a steam pressure of 100 lb. per sq. in. The diameter of the cylinder head studs is $1\frac{3}{8}$″ and the area at the bottom of the thread is 1.057 sq. in. How many wrought-iron studs are required? Assume that the studs are subjected to shocks. Ans. 29 studs.

(354) An iron-wire rope 4″ in circumference is used on a crane for hoisting loads; what is the greatest load that the rope will sustain with safety? Ans. 9,600 lb.

(355) A cast-iron rectangular cantilever beam having a cross-section of $1\frac{1}{2}$″ wide by $2\frac{1}{2}$″ deep is 4 ft. 8″ long; how great a weight will the beam sustain at its end?

Ans. 201 lb., nearly.

(356) What horsepower will a $2\frac{7}{16}$″ steel shaft transmit, when running at 120 revolutions per minute, pulleys being carried between the bearings to distribute the power along the line? Ans. 20.445 H. P.

(357) A board 8″ × 20″ and $1\frac{1}{2}$″ thick is so placed in water that its flat sides are horizontal; if the distance from the

level of the water to the top of the board is 5 feet, what is the total upward pressure on the board? Ans. 355.91 + lb.

(358) Will any solid body whose specific gravity is greater than that of water, sink in it to any depth? Why?

(359) The volume of steam in an engine cylinder at cut-off is 1.6 cu. ft., and its pressure is 90 lb. per sq. in.; what must be its volume at release when the pressure has fallen to 21 lb.? Ans. 6$\frac{6}{7}$ cu. ft.

(360) What should be the least diameter of a wrought-iron bolt that is to resist a sudden pull of 12,000 lb.?
Ans. 1.74″ +.

(361) A steel-wire rope is 4$\frac{3}{4}$″ in circumference; what load will it safely sustain? Ans. 22,562.5 lb.

(362) A white-pine beam supported at both ends has a rectangular cross-section 8″ wide by 10″ deep; if the beam is 28 ft. long, what total uniform load will it support in safety? Ans. 6,857$\frac{1}{7}$ lb.

(363) What horsepower can a 10″ wrought-iron crank-shaft transmit, when running at 200 revolutions per minute?
Ans. 2,857$\frac{1}{7}$.

(364) A vertical cylinder having a diameter of 20″ and a length inside of 36″ is filled with water; a pipe having a diameter of $\frac{3}{8}$″ is screwed into the upper head and fitted with a piston weighing 10 oz., on which is laid a weight of 25 lb.; if the end of the pipe is 10 ft. above the level of the water in the cylinder, (*a*) what is the pressure per square inch on the top of the cylinder? (*b*) On the bottom? (*c*) What equivalent weight laid on the lower cylinder head would replace the pressure it sustains?

$$\text{Ans.} \begin{cases} (a)\ 236.45 \text{ lb. per sq. in.} \\ (b)\ 237.75 \text{ lb. per sq. in.} \\ (c)\ 74{,}691.54 \text{ lb.} \end{cases}$$

(365) If, in the last example, a hole one inch in diameter is drilled through the cylinder wall midway of its length, and covered by a flat plate in such a manner that the water can not leak out, what would be the pressure against the plate? Ans. 186.22 lb.

(366) The vacuum gauge of a condensing engine indicates 20″ of vacuum; what is the pressure in the condenser?
Ans. 4.9 lb. per sq. in.

(367) The volume of the receiver of an air compressor is 300 cu. ft., and the pressure of the air which is contained in it is 52 lb. per sq. in.; if 120 cu. ft. of air, having a pressure of 52 lb. per sq. in., are removed from the receiver, what will be the pressure of the air which remains?
Ans. 31.2 lb. per sq. in.

(368) What is the greatest safe load that may be applied to a stud-link wrought-iron chain if the diameter of the iron from which the link is made is ½″? Ans. 4,500 lb.

(369) It is desired to handle loads up to 14,000 lb. by means of an iron-wire rope; what should be its circumference? Ans. 4.83″, nearly.

(370) What is the greatest load that a bar of wrought iron 2″ in diameter and 6 ft. long can safely sustain in the middle? The bar is merely supported at its ends.
Ans. 480 lb.

(371) What must be the diameter of a cast-iron crankshaft to transmit 1,000 horsepower at 80 revolutions per minute? Ans. 10.4 in.

(372) (*a*) What must be the height of the mercury column to indicate a vacuum of 12″? (*b*) Of 18″?

(373) (*a*) What is a stress? (*b*) A strain? (*c*) A unit stress?

(374) The links in a stud-link wrought-iron chain are made from iron 13/16″ in diameter; what is the greatest safe load that the chain can handle? Ans. 11,883 lb.

(375) A steel-wire rope is used to haul loads up an inclined plane; the greatest stress in the rope is 8,000 lb.; what should be its circumference? Ans. 2.83″.

(376) What uniform load can be safely sustained by a steel beam 20 ft. long, 2″ wide, and 6″ deep? Ans. 4,608 lb.

(377) A 4″ steel shaft is to transmit 80 horsepower; how many revolutions per minute must it make if used for transmission only (that is, no power being taken off at intermediate points)? Ans. 81¼ R. P. M.

(378) Define tension as applied to gases.

(379) (*a*) What is elasticity? (*b*) Elastic limit? (*c*) What is meant by set?

(380) What safe load may be carried by a close-link wrought-iron chain whose links are made from ⅝″ iron?
Ans. 4,687.5 lb.

(381) What is the allowable working load for an iron-wire rope 6″ in circumference? Ans. 21,600 lb.

(382) What force is required to shear a wrought-iron strip 4 ft. long and ½″ thick? Ans. 960,000 lb.

(383) A 7″ wrought-iron crank-shaft is to transmit 200 horsepower; how many revolutions per minute must it make?
Ans. 40.8 rev., nearly.

STEAM AND STEAM BOILERS.

EXAMINATION QUESTIONS.

(384) (*a*) What is heat? (*b*) Suppose a closed vessel containing air is placed in a furnace; describe the effect of the heat upon the molecules in the air. (*c*) If the vessel is so arranged that the air can not escape or expand, will the pressure of the air increase as it is heated? (*d*) Why?

(385) (*a*) What is temperature? (*b*) Describe the thermometer. (*c*) Of what is temperature a measure?

(386) A bar of iron weighing 2½ lb. has a temperature of 460°. Does the bar contain more or less heat above 32° than 10 pounds of water at a temperature of 60°?

(387) (*a*) What are some of the effects of heat? (*b*) Give some practical illustrations of the expansion of bodies by heat.

(388) Define:—(*a*) B. T. U. (*b*) Latent heat. (*c*) Sensible heat. (*d*) What is meant by the specific heat of a substance?

(389) A pound of ice at 16° is heated until it finally is changed to steam at atmospheric pressure. (*a*) Describe the action of the heat upon the ice and water. (*b*) How many B. T. U. are required for the operation? (*c*) What part of the heat applied is sensible heat? (*d*) What part is latent heat.

(390) (*a*) What is the mechanical equivalent of heat? (*b*) Give examples of heat changed to work, and *vice versa.* (*c*) How many foot-pounds of work are equivalent to 30½ B. T. U.? Ans. (*c*) 23,729 ft.-lb.

(391) Assuming 20% of the heat to be utilized, how many heat units per hour are required to run an engine developing 35 horsepower? Ans. 445,372.5 B. T. U.

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(392) In a power plant, the engine extracts 8% of the heat produced by the combustion of coal. Assuming that the combustion of a pound of the coal produces 14,000 B. T. U., how many pounds of coal per horsepower per hour are used by the engine? Ans. 2.27 lb.

(393) How many B. T. U. are required to raise the temperature of 22½ pounds of sulphur from 44° to 68°?
Ans. 109.4 B. T. U.

(394) (*a*) What is the latent heat of fusion of ice? (*b*) How many B. T. U. are required to melt a cake of ice weighing 11 pounds and having a temperature of 17°?
Ans. (*b*) 1,667.16 B. T. U.

(395) How many B. T. U. are required to raise 6 pounds of superheated steam from 310° to 342°?
Ans. 92.256 B. T. U.

(396) A ball of copper at 305°, weighing 18 pounds, and an iron rod at 278°, weighing 13 pounds, are plunged into a bath of water at 56°. If the water weighs 32 pounds, what will be its final temperature? Ans. 77.45°.

(397) (*a*) How many B. T. U. are required to change a pound of water at 212° into steam at atmospheric pressure? (*b*) How many B. T. U. are required to change 8 pounds of water at 63° into steam at 212°?
Ans. (*b*) 8,924.8 B. T. U.

(398) How many B. T. U. are required to change 2.2 lb. of ice at 23° into steam at 212°? Ans. 2,849.3 B. T. U.

(399) What is (*a*) saturated steam? (*b*) superheated steam? (*c*) In what way does saturated steam differ from a perfect gas?

(400) (*a*) Define the following: total heat; heat of the liquid; latent heat of vaporization. (*b*) How may the heat of the liquid be found approximately?

(401) If the pressure of the steam in a boiler is 81 pounds per square inch (absolute), what is the volume in cubic feet of 6 pounds of the steam? Ans. 31.779 cu. ft.

(402) If a pound of steam occupies a volume of 7 cu. ft., what is its gauge pressure? Ans. 45.3 lb. per sq. in.

(403) How many pounds of steam at 78 lb. pressure (absolute) will raise the temperature of 250 pounds of water from 50° to 135°? Ans. 19.795 lb.

(404) What is the volume of $6\frac{1}{8}$ pounds of steam at a temperature of 350°? Ans. 20.041 cu. ft.

(405) (*a*) What is a steam boiler? (*b*) What are its essential parts?

(406) Describe in your own language the construction of a single-ended Scotch boiler, and state in what respect it differs from a double-ended Scotch boiler.

(407) What is the object of the diagonal braces attached to the top and bottom of a Scotch boiler?

(408) Describe in your own language the construction and the setting of a fire-box boiler.

(409) What do you understand by a (*a*) dry uptake? (*b*) wet uptake? (*c*) dry-bottomed boiler? (*d*) wet-bottomed boiler?

(410) What is the object of the feed-coils of a Roberts boiler?

(411) (*a*) In what respect do the Scotch boiler and the fire-box boilers chiefly differ from the water-tube boilers? (*b*) Give some points for and against water-tube boilers.

(412) What is the object of a donkey boiler?

(413) A strip of iron 1″ wide by $\frac{5}{8}$″ thick cut from a plate to be used in a marine boiler, when pulled apart in a testing machine, shows a tensile strength of 53,000 pounds per square inch, and is reduced to an area of .375 square inch at the point of fracture. Would you allow the plate to be used? Give your reasons.

(414) Explain why flat surfaces of steam boilers subjected to pressure must be braced.

(415) The sides of the combustion chamber of a Scotch boiler carrying 150 pounds gauge pressure are stayed by

steel staybolts having a pitch of $7\frac{1}{4}$ inches. Find (*a*) the load on each stay; (*b*) the diameter of each stay.

Ans. $\begin{cases} (a)\ 7{,}884.375 \text{ lb.} \\ (b)\ 1\frac{5}{16} \text{ in., nearly.} \end{cases}$

(416) The furnace sheets of a fire-box boiler are $\frac{1}{2}$ inch thick. What should be the pitch of wrought-iron staybolts, provided the boiler is to carry 90 pounds pressure, absolute ? Ans. 10 in., nearly.

(417) What working pressure should the boiler mentioned in the previous question be allowed to carry, supposing the pitch of the staybolts to be $8\frac{1}{2}$ inches ? Ans. 106.3 lb.

(418) Suppose on examination you found the staybolts of the boiler in question 415 wasted away to $\frac{7}{8}$ inch diameter. To what should you reduce the steam pressure?

Ans. 68.64 lb.

(419) The through stayrods of a boiler are made of steel $2\frac{3}{8}$ inches in diameter at the thinnest part, and are spaced $14\frac{1}{2}$ inches by 15 inches, center to center. Assuming the steam pressure to be 170 pounds, (*a*) what will be the stress per square inch of section ? (*b*) Would you allow the stays to be used ?

(420) In Fig. 5 is shown the front head of a boiler stayed by a palm stay. From the dimensions given, find the stress in the stay, assuming the steam pressure to be 80 pounds and the area supported by the stay 215 square inches.

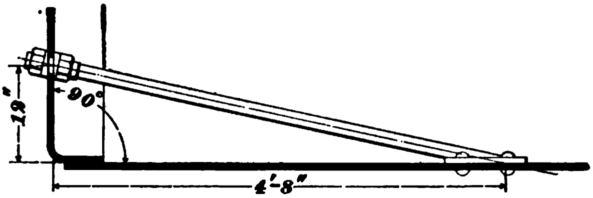

FIG. 5

Ans. 17,600 lb.

(421) What is the object of the tubes in a fire-tube boiler ?

(422) A furnace flue constructed of material ½ inch in thickness is made up of sections 3 feet 3 inches in length with flanged ends and strengthening rings. The diameter of the flue being 39 inches, what should be the working pressure allowed on this flue? Ans. 176.73 lb.

(423) A corrugated furnace flue constructed of material $\frac{7}{16}$ inch thick measures .38½ inches over the bottom of the corrugations. Find the working pressure allowed. Ans. 159.09 lb.

(424) What working pressure would you allow on a double-riveted bumped head bumped to a radius of 48 inches, assuming the tensile strength of the material to be 52,000 pounds and its thickness ⅝ inch? Ans. 270.83 lb.

(425) Find the safe working pressure on a boiler having single-riveted seams, the diameter of the boiler being 96 inches, the thickness of the shell .375 inch, and its tensile strength 65,000 pounds. Ans. 84.63 lb.

(426) At what pressure would a cast-iron steam pipe burst, the diameter of the pipe being 10 inches, its thickness ½ inch, and the ultimate tensile strength of the iron 17,800 pounds? Ans. 1,780 lb. per sq. in.

(427) (*a*) What is the principle of operation of a safety valve? (*b*) Name the different types of safety valves.

(428) How may the blow-off pressure be altered in the case of a pop safety valve?

(429) What should be the total weight on a dead-weight safety valve, the valve opening being 2½ inches in diameter and the blow-off pressure 60 lb. per square inch? Ans. 294.52 lb.

(430) The area of a lever safety valve is 10 square inches. The distance from the valve stem to the fulcrum is 4 inches. The distance from the fulcrum to the point of suspension of the weight is 40 inches. The weight of the valve and stem is 5.54 pounds. The lever is 42 inches long, made of a wrought-iron bar 1½ inches by ¾ inch. Take the weight of a cubic inch of wrought iron as .28 pound. Find the weight

necessary to balance at the following pressures : (*a*) 65 lb.; (*b*) 75 lb.; (*c*) 85 lb.; (*d*) 100 lb.

Ans. (*a*) 57.5 lb.
(*b*) 67.5 lb.
(*c*) 77.5 lb.
(*d*) 92.5 lb.

(431) (*a*) What is the principle of operation of a Bourdon steam gauge ? (*b*) What is the object of the siphon used in connection with a steam gauge ? (*c*) Is there any case in which a siphon may influence the indication of a steam gauge ? If so, state it and give explanation.

(432) (*a*) Will a gauge-cock attached directly to the boiler always indicate the true water level ? (*b*) Why are glass water gauges often fitted to a water column instead of attaching them directly to the boiler ? (*c*) How can you find out whether a glass water gauge is operating properly ?

(433) What is the object of a fusible plug ?

(434) (*a*) What is the bottom blow-off cock used for ? (*b*) What is the surface blow-off cock used for ? (*c*) What is a spanner guard ?

(435) Why is a manhole made elliptical ?

(436) What is the principle of operation of a check valve ?

(437) (*a*) How may the expansion of a steam pipe be provided for ? (*b*) What materials are used in the construction of steam pipes ?

(438) What is the bridge of a furnace ?

(439) Describe the construction and piping of a superheater.

(440) (*a*) What is the object of a steam drum ? (*b*) What is the object of a dry pipe ?

(441) What is the principle of operation of a separator ?

(442) What is the donkey valve used for ?

(443) Why is a baffle plate fitted to the furnace door ?

(444) Is air ever admitted behind the bridge ?

(444*a*) A concaved head bumped to a radius of 42 inches is attached to the shell of a steam drum by a single row of rivets. (*a*) If the plate of which the head is constructed is $\frac{9}{16}$ of an inch thick and has a tensile strength of 45,000 pounds, what pressure would be allowed on the head? (*b*) If the shell of the drum is 36 inches in diameter, $\frac{5}{16}$ of an inch thick, and 56,000 pounds tensile strength, single riveted, what pressure would be allowed on the drum with the head given above?

Ans. { (*a*) 120.5 lb. per sq. in.
(*b*) 120.5 lb. per sq. in.

(444*b*) Calculate the load on a palm stay 7 feet 8 inches long and attached to the shell at a point 6 feet 10 inches from the head. The steam pressure is 160 lb., and the area supported 196 sq. in. Ans. 35,184.4 lb.

(444*c*) In a double zigzag-riveted lap joint the pitch of the rivets is 3 inches, and the rivet diameter is $\frac{7}{8}$ of an inch. What should be the distance between the rows of rivets? Ans. 1.54 in.

(444*d*) In the last example, what should the diagonal pitch be? Ans. 2.15 in.

(444*e*) Would a double-riveted lap joint be allowed, in which the pitch of the rivets is 4 inches and the plate $\frac{5}{8}$ of an inch thick?

(444*f*) The joint of an iron plate $\frac{5}{8}$ of an inch thick is double chain-riveted. How far should the rows of rivets be apart? Ans. 2.375 in.

(444*g*) Given a double-riveted lap joint on a steel plate $\frac{7}{16}$ of an inch thick and having $\frac{13}{16}$ of an inch rivets. What should be (*a*) the proper pitch? (*b*) the maximum pitch allowable?

Ans. { (*a*) 2.759 in.
(*b*) 2.771 in.

STEAM AND STEAM BOILERS.

(CONTINUED.)

EXAMINATION QUESTIONS.

(445) Describe the passage of the feed-water to the boiler and back to the feed pumps, when a surface condenser is used.

(446) Why should a stop-valve be attached between the boiler and each feed check-valve?

(447) How is the amount of feed-water entering each boiler regulated?

(448) What is the object of a feed relief valve?

(449) Why is the internal feed-pipe sometimes made perforated?

(450) Why should the feed-water be filtered when using a surface condenser?

(451) Describe the operation of a Wass grease extractor.

(452) What is the object of heating feed-water, and how may this be done?

(453) What do you understand by the term "salt feed"?

(454) Describe the principle of operation of a Baird evaporator.

(455) Describe the principle of operation and the object of a steam trap.

(456) What do you understand by $\frac{5}{32}$ saturation?

(457) A sample of water boils at 219.4° F., the barometer standing at 29.8 in. Determine the saturation. Ans. $\frac{6.43}{32}$.

(458) For what is a hydrometer used, and how is it graduated?

(459) (*a*) State the influence of temperature of the water under test upon the indication of the hydrometer. (*b*) How may it be corrected?

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(460) 8,500 pounds of feed-water enter a boiler every hour at a saturation of $\frac{1}{32}$. How many gallons of water must be blown off every hour to keep the saturation at $\frac{2\frac{1}{2}}{32}$?

Ans. 404.76 gallons.

(461) How much water at $\frac{1}{32}$ is evaporated into steam for every pound of water blown off at $\frac{5\frac{1}{8}}{32}$? Ans. $4\frac{1}{8}$ lb.

(462) What influence has the circulation of the water upon the efficiency of the boiler?

(463) Why is the lower part of a Scotch boiler much cooler than the upper part?

(464) (*a*) Describe the operation of a Craig circulating apparatus. (*b*) State some other ways of improving the circulation of Scotch boilers.

(465) State how (*a*) natural draft and (*b*) forced draft are produced.

(466) Find the maximum coal consumption, under natural draft, of a vessel making a trip of 2,282 miles at a speed of 14 miles an hour and burning ordinary soft coal. The height of stack is 81 ft. The furnaces are 3 ft. 9 in. by 6 ft. There are 4 boilers with 6 furnaces each.

Ans. 1,188 tons 540 lb.

(467) Describe Howden's closed ash-pit system, and state in what respect it differs from an induced draft system.

(468) Explain (*a*) foaming; (*b*) priming. (*c*) State the dangers attending foaming and priming.

(469) Describe the three methods of firing.

(470) How may a fire be (*a*) cleaned? (*b*) lighted? (*c*) banked?

(471) Show how you would arrange for the systematic firing of two boilers with three furnaces each, performing the operations in the following order: (1) Firing; (2) slicing; (3) leveling; (4) cleaning the grate from below. Make a

rough sketch similar to Fig. 243, Art. **876,** and number the furnaces.

(472) (*a*) What is combustion ? (*b*) What are the elements which usually enter into combustion ?

(473) What is meant by the expressions—element, compound, mixture ? Give examples of each.

(474) (*a*) How many B. T. U. are developed by the perfect combustion of a pound of carbon ? of a pound of hydrogen ? (*b*) Give a rule for computing the heat of combustion of a fuel composed of carbon and hydrogen.

(475) What is the minimum quantity of air, in cubic feet, required for the complete combustion of 17 pounds of a fuel which has the following chemical composition: carbon, 94 parts; hydrogen, .5 part; oxygen and ash, 5.5 parts ?

Ans. 2,467.7 cu. ft.

(476) How many B. T. U. would be given out by the perfect combustion of a pound of the fuel of question 475 ?

Ans. 13,940 B. T. U.

(477) A pound of a certain coal gives out 13,675 B. T. U. How many pounds of water would it evaporate from and at 212° ?

Ans. 14.16 lb., nearly.

(478) From the atomic weights of the elements, find the composition by weight of the substances represented by the following symbols: (*a*) HNO_3; (*b*) N_2O_3; (*c*) H_2SO_4.

$$\text{Ans.}\quad (a)\begin{cases}\text{H, } 1.59\%.\\ \text{N, } 22.22\%.\\ \text{O, } 76.19\%.\end{cases}\quad (b)\begin{cases}\text{N, } 36.84\%.\\ \text{O, } 63.16\%.\end{cases}\quad (c)\begin{cases}\text{H, } 2.44\%.\\ \text{S, } 39.02\%.\\ \text{O, } 58.54\%.\end{cases}$$

(479) (*a*) What are the products of combustion of carbon in air ? (*b*) What is the product if the air supply is insufficient ? (*c*) How many pounds of air are required to burn 8 pounds of carbon to C O gas ?

(480) (*a*) Why should the temperature of the boiler furnace be kept as high as possible? (*b*) What causes the reduction of this temperature? (*c*) What care must be exercised in burning bituminous coals containing hydrocarbons?

(481) Sulphur is a combustible element sometimes found in fuels. When burned with oxygen, the resulting product is sulphurous oxide, $S\,O_2$. Make a scheme for the combustion of a pound of sulphur in air, showing the quantity of air required and the products of combustion. (A similar scheme is shown for carbon and hydrogen in Arts. **835** and **836.**)

(482) (*a*) What effects are sometimes produced when rapid chemical combination takes place? (*b*) Explain the action of the atoms when hydrogen and oxygen unite to form water.

(483) (*a*) What is the cause of incrustation? (*b*) Describe the action of carbonate of lime, sulphate of lime, and chloride of sodium in regard to dissolving in water.

(484) What danger may be caused by incrustation?

(485) (*a*) State for what reasons carbonate of soda is often introduced into marine boilers. (*b*) State whether the formation of sulphate of lime scale can be prevented by the use of the blow-off cock.

(486) (*a*) Why should the surface blow-off cock be frequently used when the water contains carbonate of lime? (*b*) State the use of zinc in marine boilers.

(487) (*a*) Name the different kinds of internal corrosion, and state where they may be looked for. (*b*) State the causes of internal and external corrosion and how they may be prevented.

(488) (*a*) Name the cause of collapse of furnaces. (*b*) Name the cause of fracturing of plates.

(489) What is the object of a hydrostatic test, and how is it applied?

(490) State the precautions you would take when connecting boilers, and give your reasons.

(491) What would you do in case the water got dangerously low?

(492) A water-tube boiler is to evaporate 6,500 lb. of water per hour, burning best anthracite coal. If the height of the smokestack is 64 ft., what should be (*a*) the grate area? (*b*) The heating surface? Assume a ratio of 42 : 1.

Ans. { (*a*) 49.52 sq. ft.
(*b*) 2,079.84 sq. ft.

(493) State the causes of boiler explosions

(494) Describe the construction of a tube stopper.

(495) What is the object of a boiler trial?

(496) Describe the standard method of starting and stopping a boiler trial.

(497) (*a*) Describe the barrel calorimeter. (*b*) What is meant by the quality of steam?

(498) In a test with a barrel calorimeter, the weight of the cold water was 400 lb., and the temperature rose from 44° to 135°. The weight of the steam run in was 35.2 lb. The steam pressure was 90 lb. per sq. in., gauge. Find the quality of the steam. Ans. 95.08%.

(499) The following data were observed during a boiler trial:

Duration of test......................	10 hours.
Average steam pressure..............	64 lb.
Average temp. of feed-water.........	132°.
Coal burned..........................	12,000 lb.
Percentage of ash.....................	4.81%.
Water fed to boiler..................	114,962.7 lb.
Quality of steam....................	98%.

Work up these data and find the evaporation under actual conditions (*a*) per pound of coal; (*b*) per pound of combustible; and also the equivalent evaporation (*c*) per pound of coal; (*d*) per pound of combustible.

Ans. { (*a*) 9.389 lb.
(*b*) 9.863 lb.
(*c*) 10.463 lb.
(*d*) 10.992 lb.

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